Properties of optical instruments

Projection optical systems

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Instruments :
optical systems designed for a specific function

- **Projection systems: real image**
  (object real or at infinity)

Examples: videoprojector, camera

- **Ocular systems: image at infinity**
  (object real or at infinity)

Examples: eyepiece, microscope, astronomical telescope with eyepiece

Characteristics of instruments

	Object at finite distance	Object at infinity
Size of the image	Magnification gy	Focal length
Aperture	Numerical aperture (object or image space) Pupil diameter	F-number
Resolution	Diffraction,Aberrations Pixel of detector	similar
Field of view	Bright field, total field, vignetting (associated with detector)	Similar
Depth of field	Depth of field (associated with pixel size)	Hyperfocal distance (associated with pixel size)

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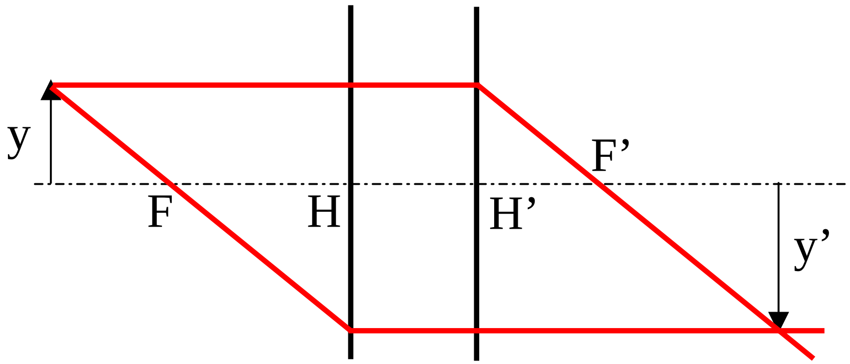
1-Size of Image

Example Webcam : Object and Image at finite distances

Object with size y , Image with size y'

→ Magnification

$$g_y = \frac{y'}{y}$$



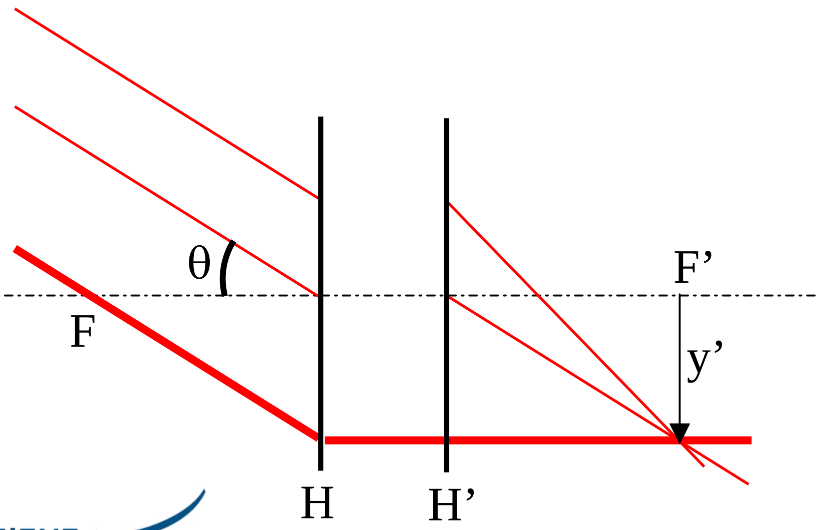
1-Size of Image

Zoom : Object at infinity, Image at finite distance

Object with angular size θ, Image with size y'

➔ The « magnification » is the focal length of the zoom lens

$$f' = \frac{y'}{\theta}$$



Size of image for a combined system

Example of a zoom made of two lenses

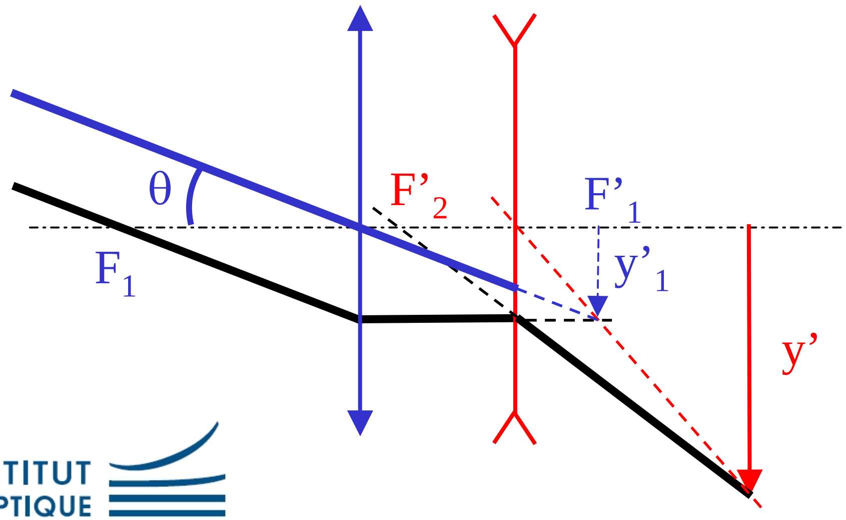
1st lens — 2nd lens

Object at infinity → intermediate image → final image

size θ — size y'_1 — size y'

magnif $f'_1 = y'_1/\theta$ — magnif $g_{y2} = y'/y'_1$

Overall magnification: $f' = y'/\theta = y'_1/\theta \,.\, y'/y'_1 = f'_1 \,.\, g_{y2}$



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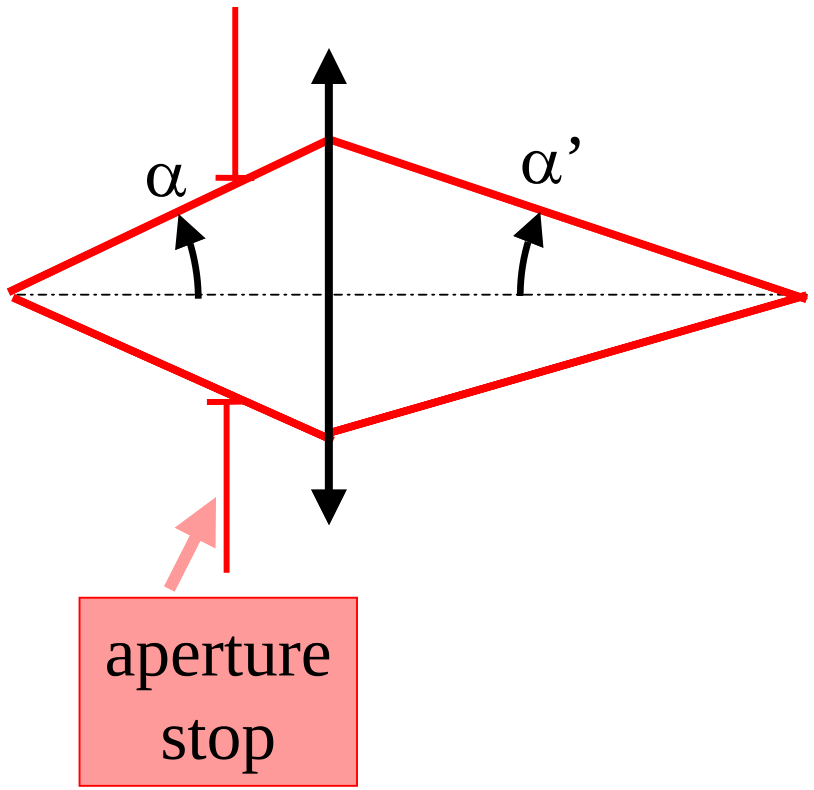
2. Stops and apertures

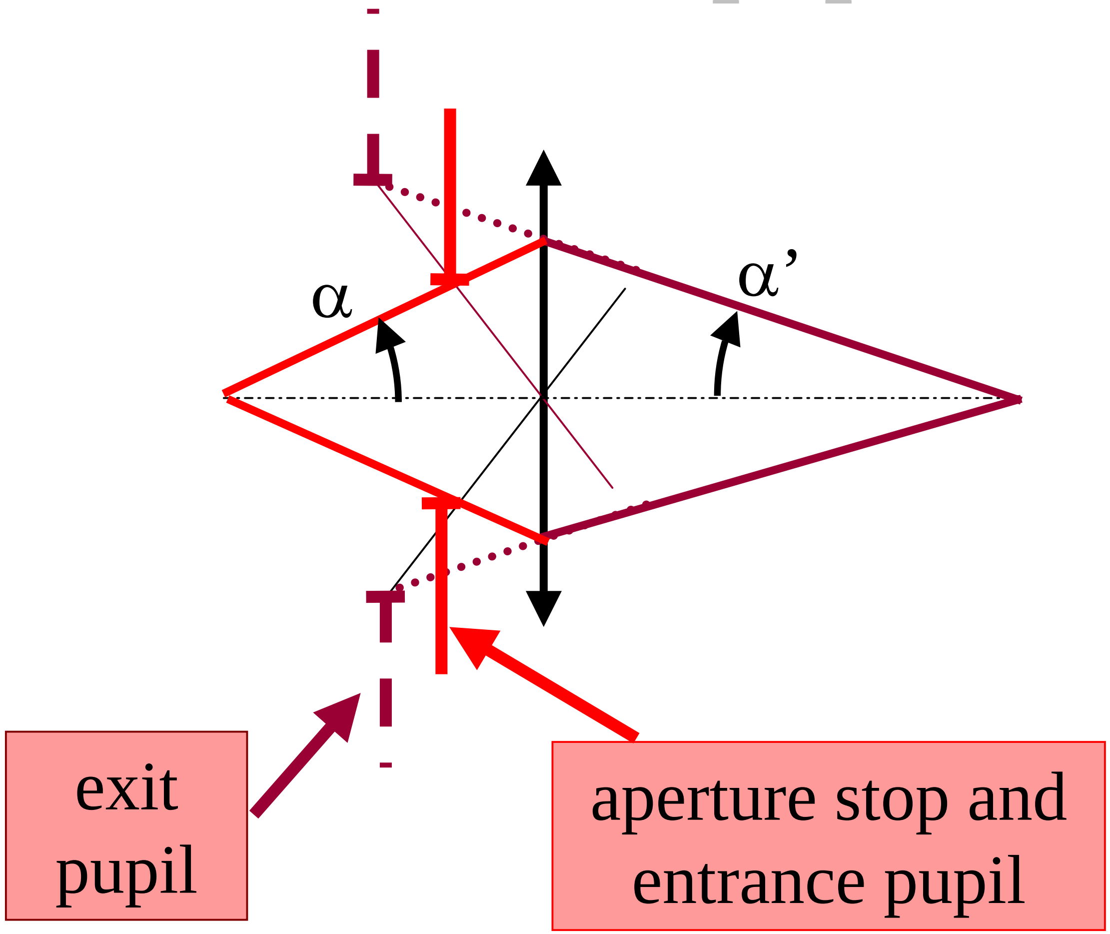


➔ Numerical aperture

$$NA_{obj} = n \sin\alpha$$
$$NA_{img} = n' \sin\alpha'$$

connected to the amount of light entering the system, to the resolution and depth of field

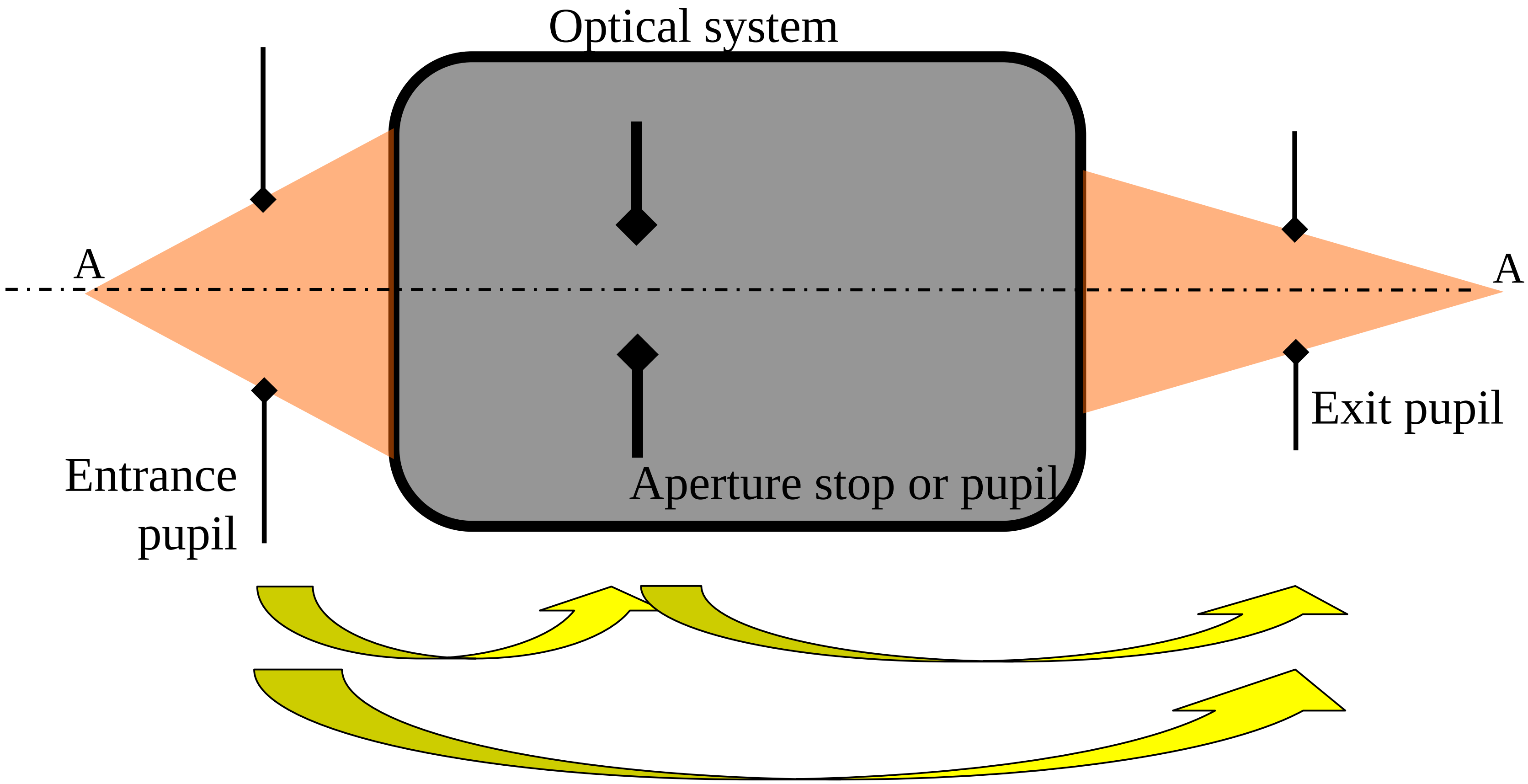
Entrance and exit pupils: an example



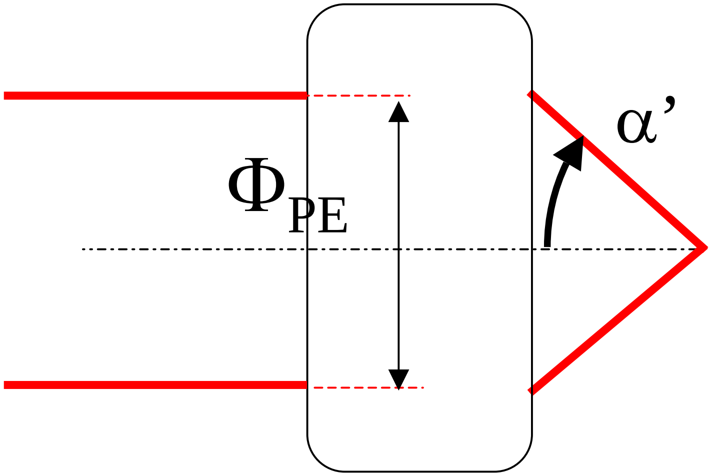
Conjugates of the aperture stop:

- entrance pupil in object space
- exit pupil in the image space

Entrance and exit pupil in general



Aperture for an object at infinity



➔ Numerical aperture

$$NA_{image} = \sin\alpha'$$

➔ f-Number

$$N = \frac{f'}{\Phi_{PE}}$$

Φ_{PE} diameter of the entrance pupil

The objective is said to be «open at f/N » or «open at N»

Relationship between NA and f-number?

Abbe sine condition for an object at infinity:

$$-h\theta = y'\sin\alpha'$$

$$h = \frac{\Phi_{PE}}{2}$$

$$y' = f'\theta$$

Numerical aperture in image space

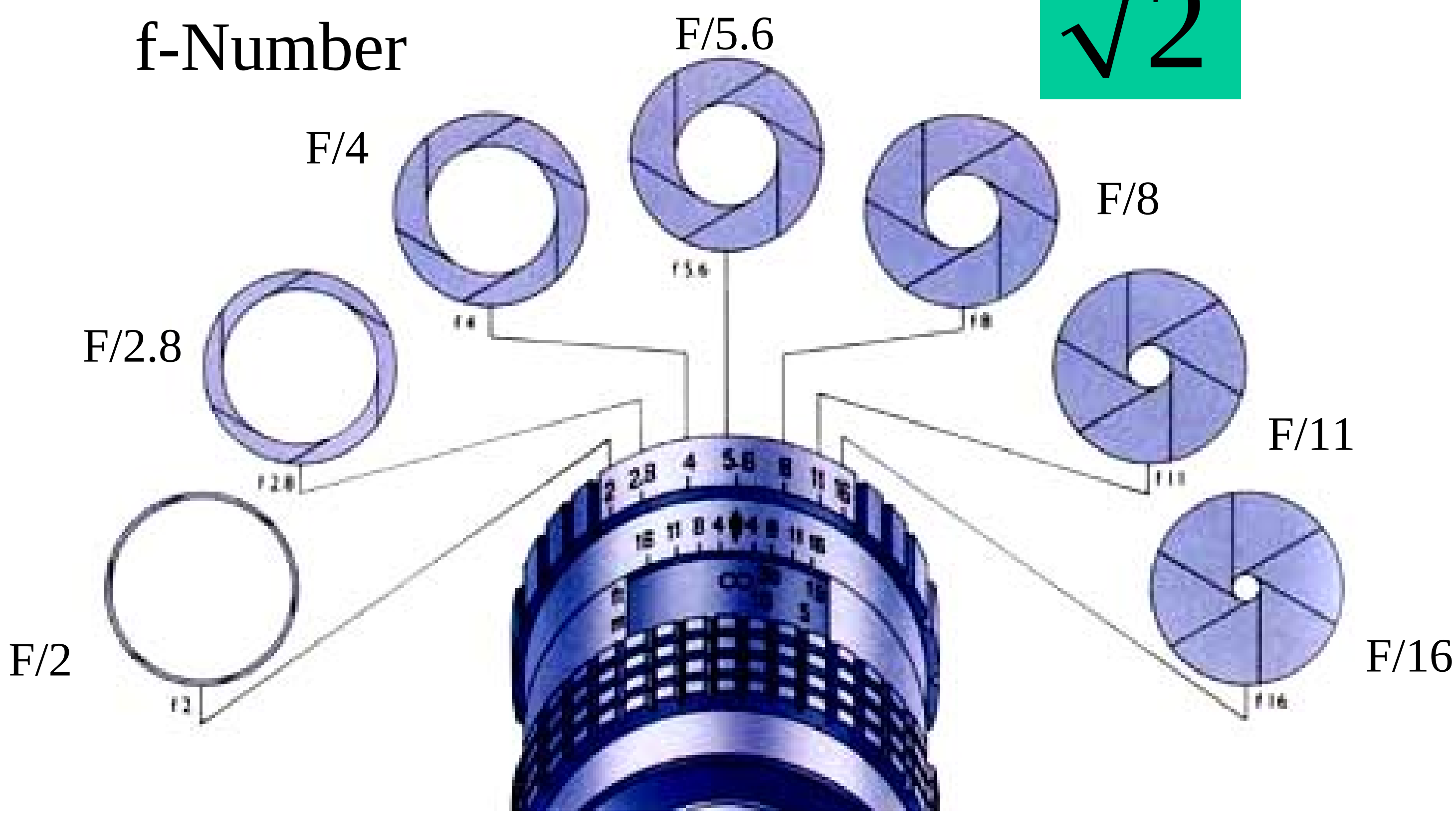
$$\left|\sin(\alpha')\right| = \frac{\Phi_{PE}}{2f'} = \frac{1}{2N}$$

N **: f-number**

F-number for a camera lens

Ratio between values of N:

$\sqrt{2}$



$$E = \tau\pi L_{objet} / 4 N^2$$

Illumination varies by factor of 2 between positions

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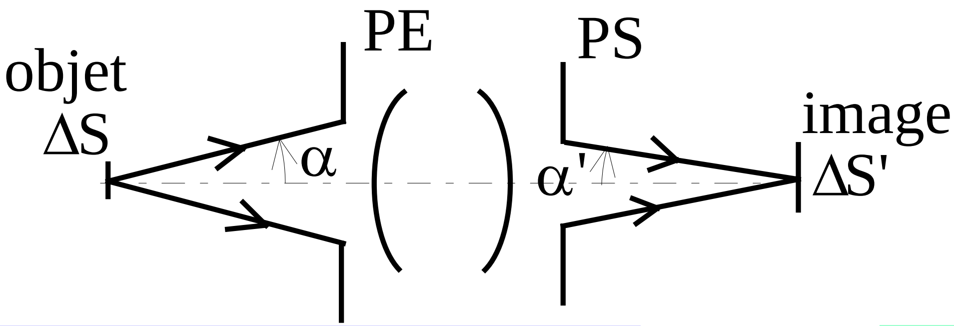
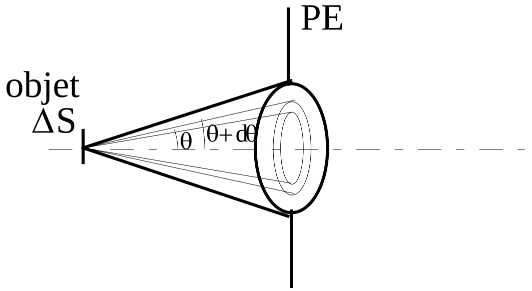
Reminder about radiometry

Flux entering an instrument:





$$G=\int_0^{\alpha}\Delta S\cos\theta . 2\pi\sin\theta d\theta=\pi\Delta S\sin^2\alpha$$

$$\Phi_{\text{entering}}=LG=\pi L\Delta S\sin^2\alpha$$

$$\Phi_{\text{exiting}}=L'G'=\pi L'\Delta S'\sin^2\alpha'$$

$$\Phi_{\text{exiting}}=\tau\Phi_{\text{entering}}$$

Abbe sine condition $\Rightarrow$ $G=G'$

$\Rightarrow$ Luminance is conserved: $L'=\tau L$

For a small surface *dS'* of the image:

$$d\Phi_{\text{exiting}}=\pi L' dS'\sin^2\alpha'=\pi\tau L dS'\sin^2\alpha'$$

Illumination on the image:

$$E=\frac{d\Phi_{\text{exiting}}}{dS'}=\tau\pi L\sin^2\alpha'$$

$$E=\frac{\pi\,\tau\,L}{4N^2}$$

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3. Resolution

→ What is the smallest distance between two object points that we can separate ?

The resolution of a projection system can be limited by :

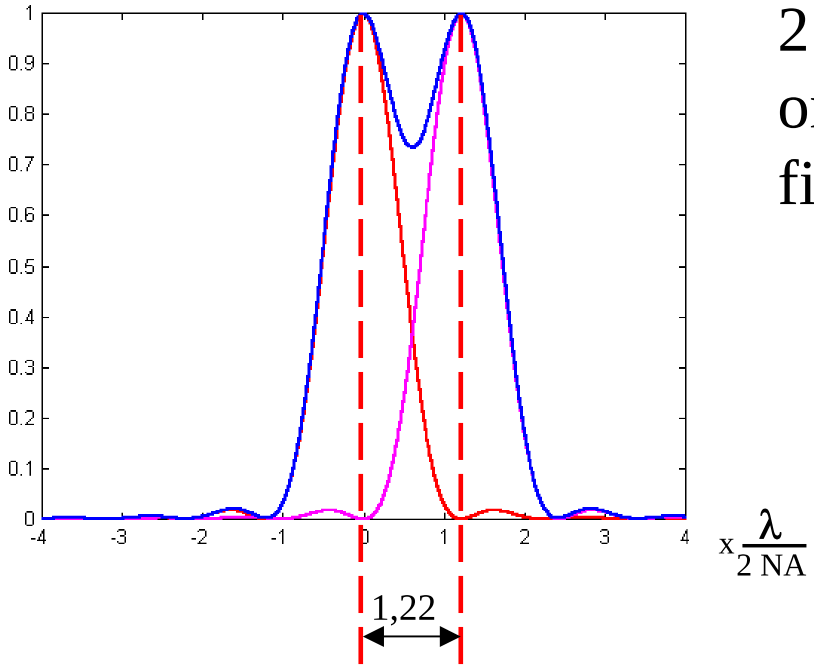
- diffraction
- detector (emulsion grain, pixel,…)
- geometrical aberrations

a. Diffraction limit: depends on aperture

→ When can we distinguish two images when the object is lit with incoherent light ?

Rayleigh's criterion :

2 Airy spots can be separated if one maximum coincides with the first minimum of the other one



Airy function :

Fourier transform of a disc

$$I \propto \left(\frac{2J_1(Z)}{Z} \right)^2 \qquad Z = 2\pi \frac{x}{\lambda} NA$$

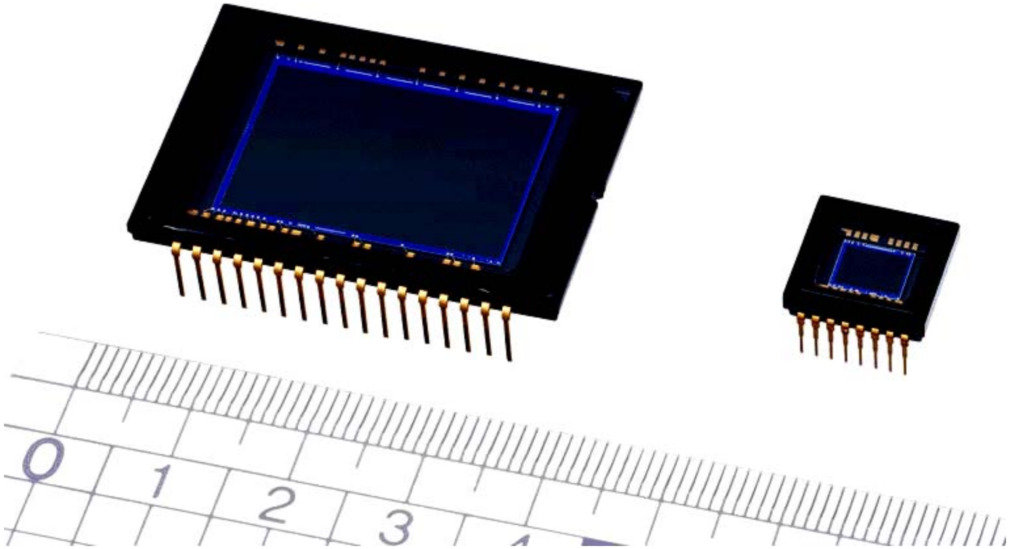
Resolution = radius of the first dark ring = 1,22 λ / 2 NA

b. Detector's resolution

For a projection system (real image)

➔ finite size of the detector's pixels

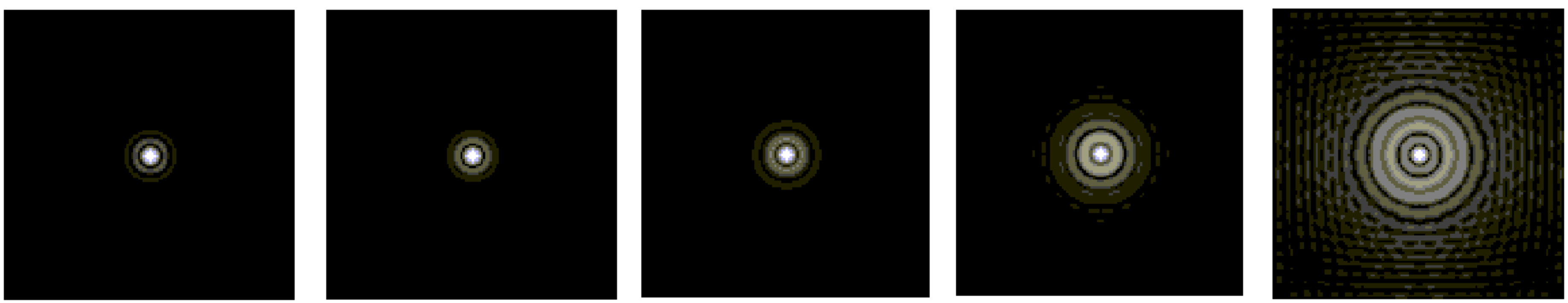
Ex : CCD pixel (~10µm), emulsion grain (5-30µm)



c. Effect of geometrical aberrations

Aberration will always increase the size of the image compared to the diffraction limit

➔ example : spherical aberration



Diffraction limit | SphAb 0,5λ | SpAb 1λ | SpAb 2λ | SpAb 5λ

Increasing spherical aberration

See optical design course…

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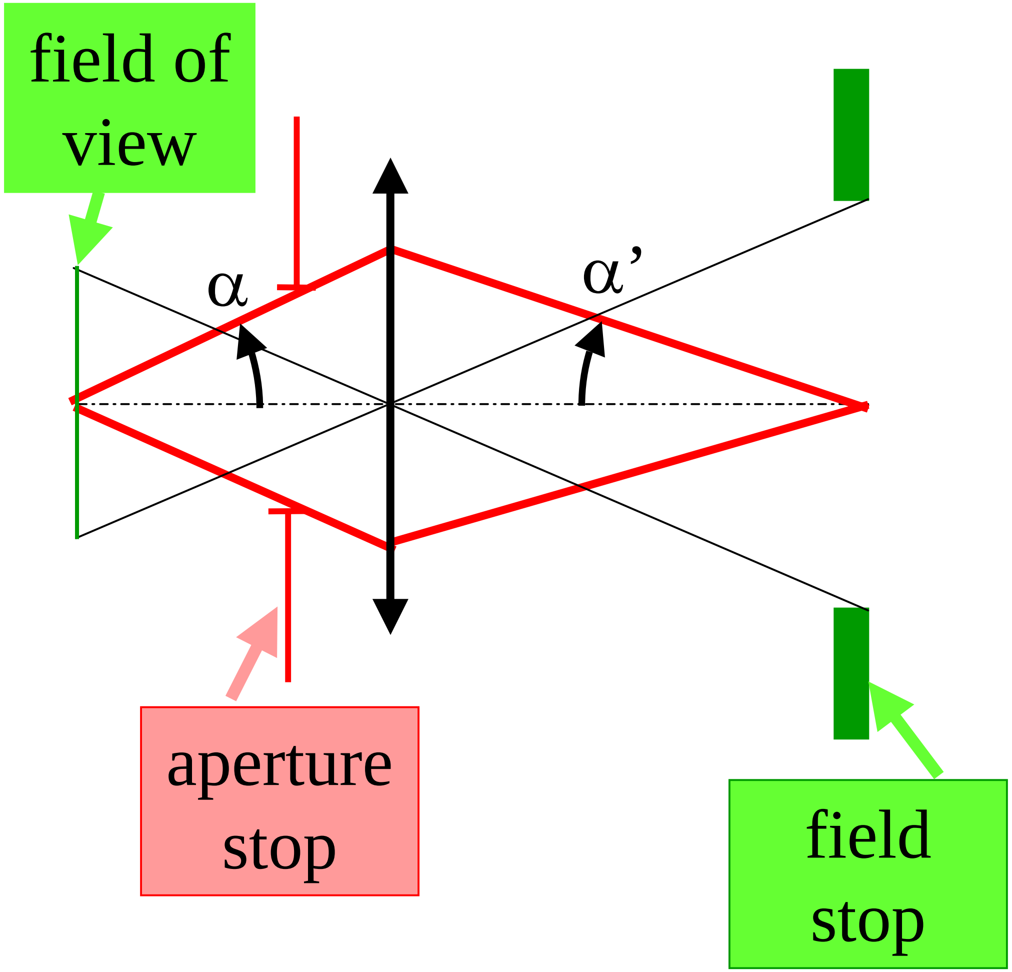
4. Field of view



➔ Field of view

= Portion of the object seen through the system

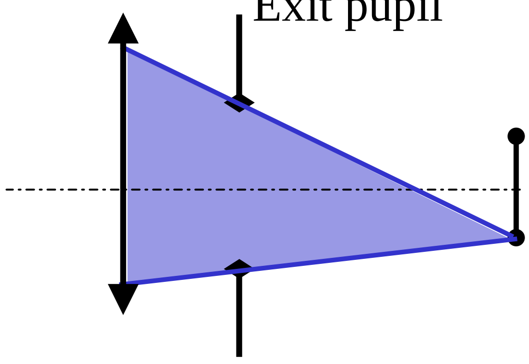
For a projection system

The field of view is usually limited by the detector's size

Bright field over the whole detector

➔ *No other vignetting*

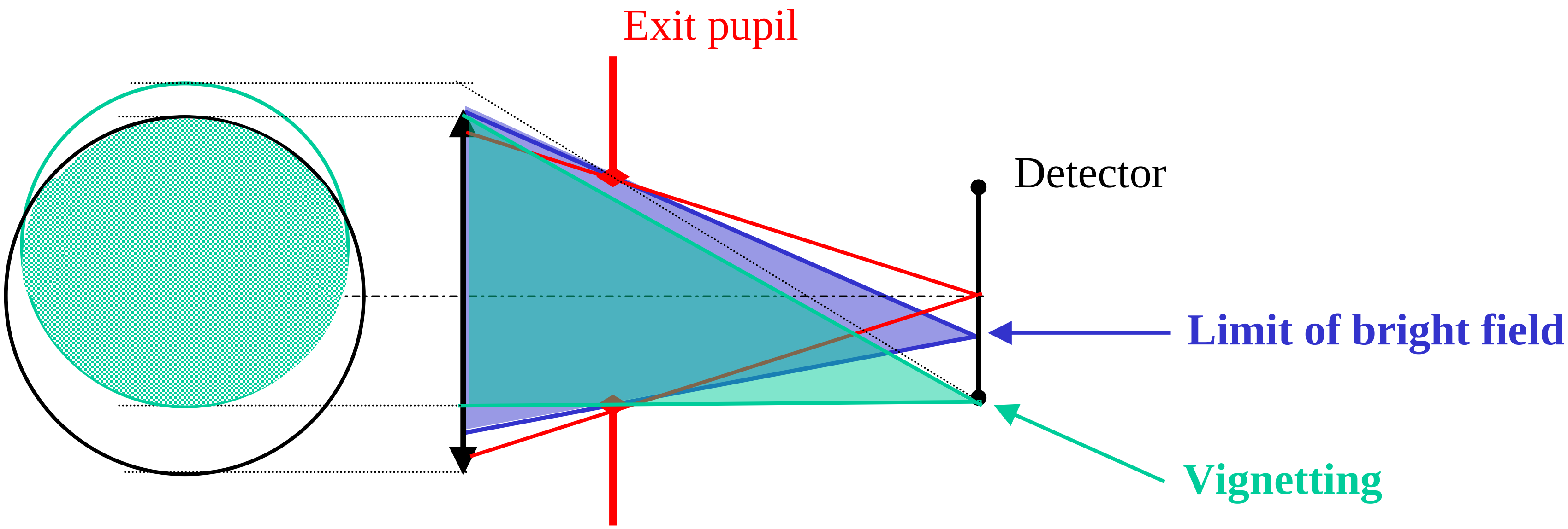
➔ *Requires a large enough diameter for the optics*



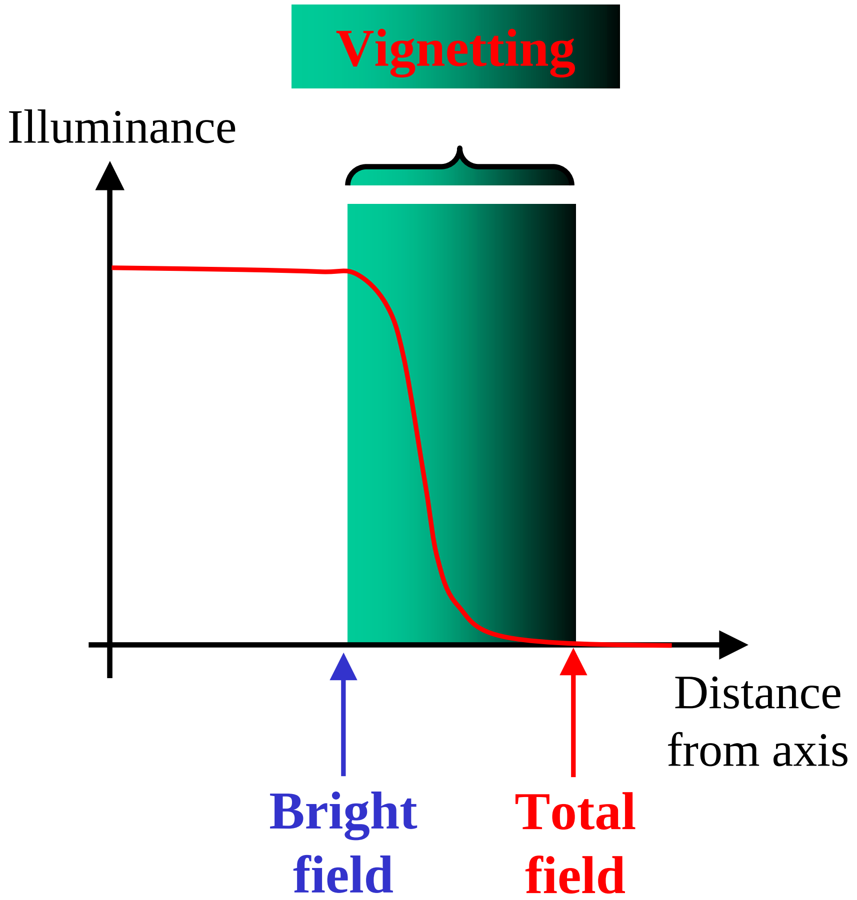
See examples in problems

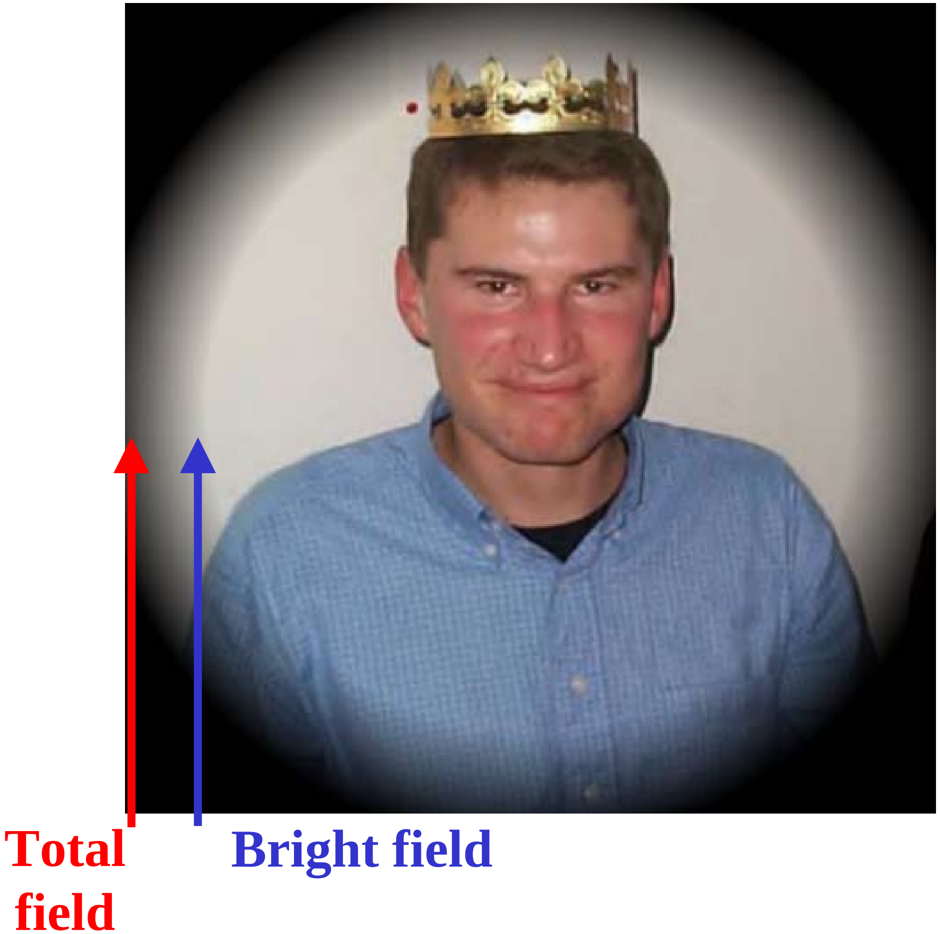
Vignetting

If the size of the lens (or another stop) limits the field of view

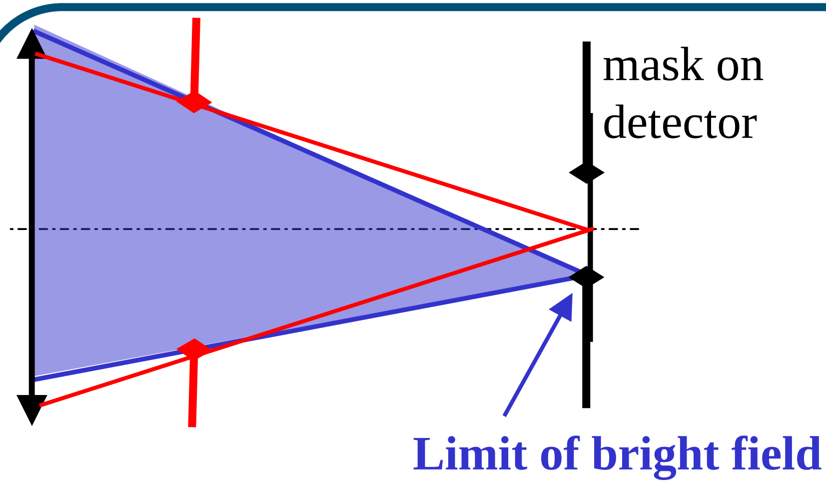


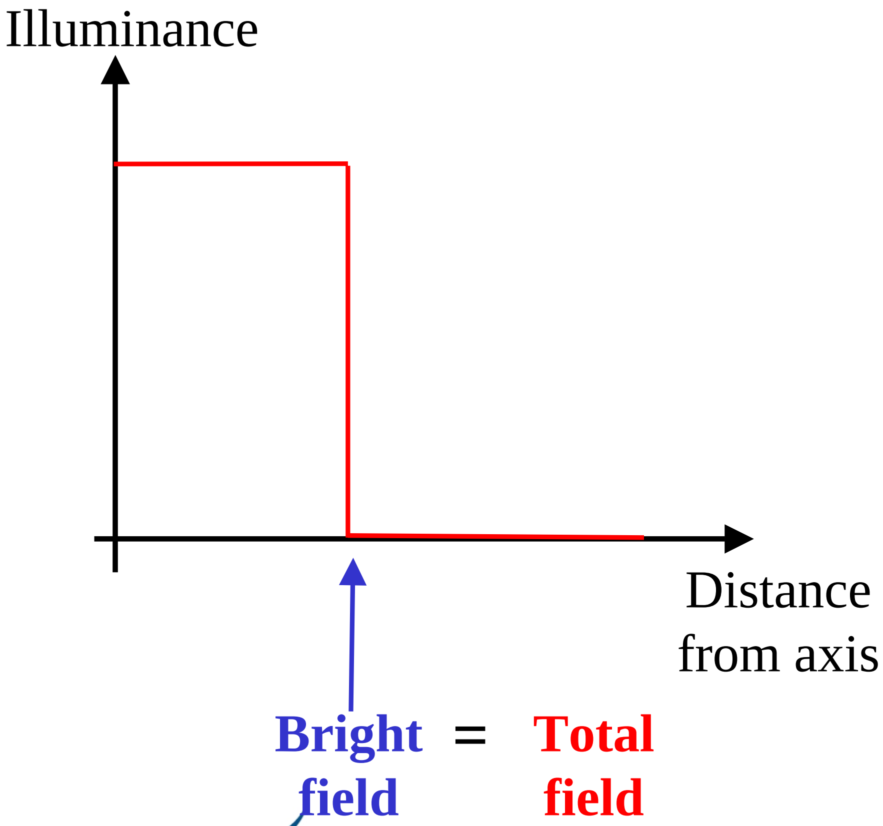
Vignetting: between bright field and total field of view





Without vignetting





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5. Depth of field

Distance along the optical axis where we get a « clear » image

Larger aperture

F-number: N = 2.8

Smaller aperture

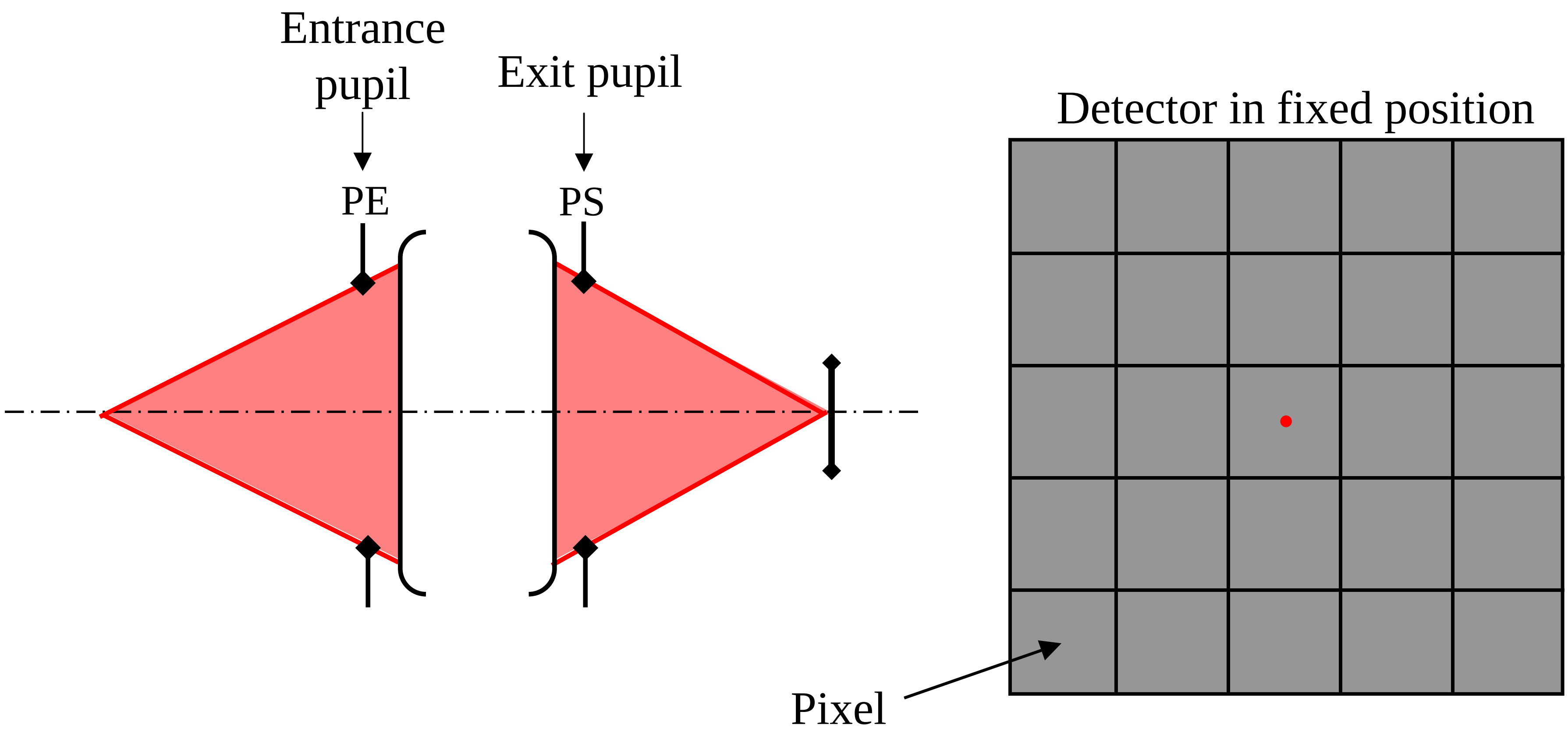
F-number: N = 8

How does the depth of field vary?

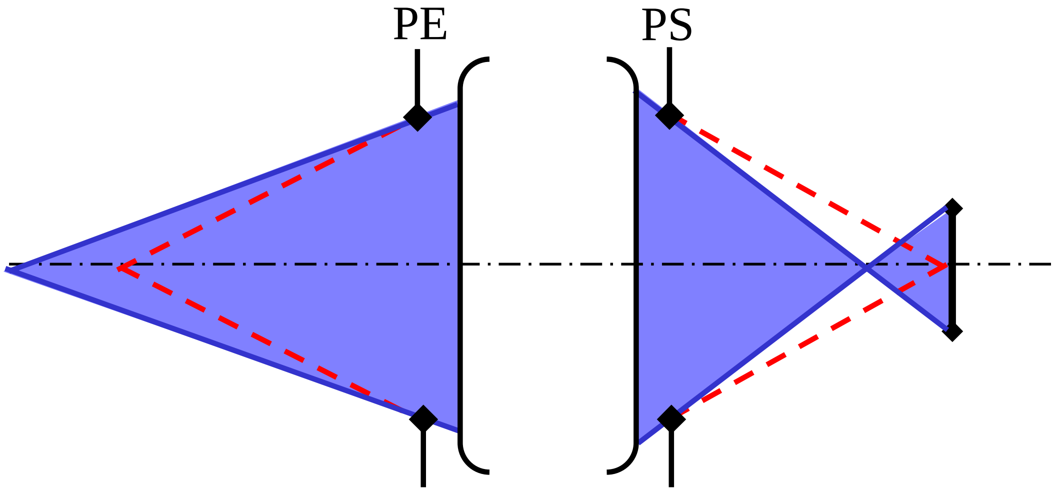
When taking a picture, the depth of field :

- Increases when you reduce the aperture (increase f-number)
- Increases with distance to object
- Depends on the detector’s resolution (or whatever limits the resolution: diffraction, aberrations)

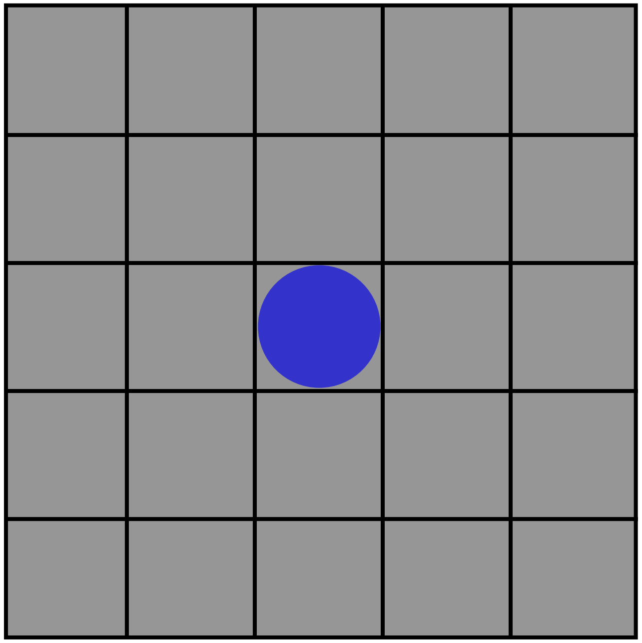
Determination of depth of field



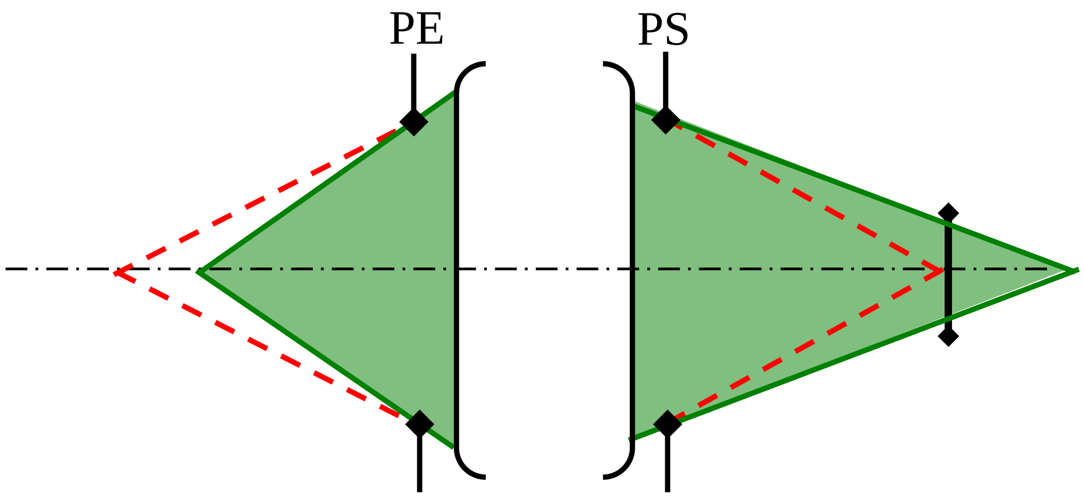
The detector cannot distinguish a point image from an image with the size of one pixel



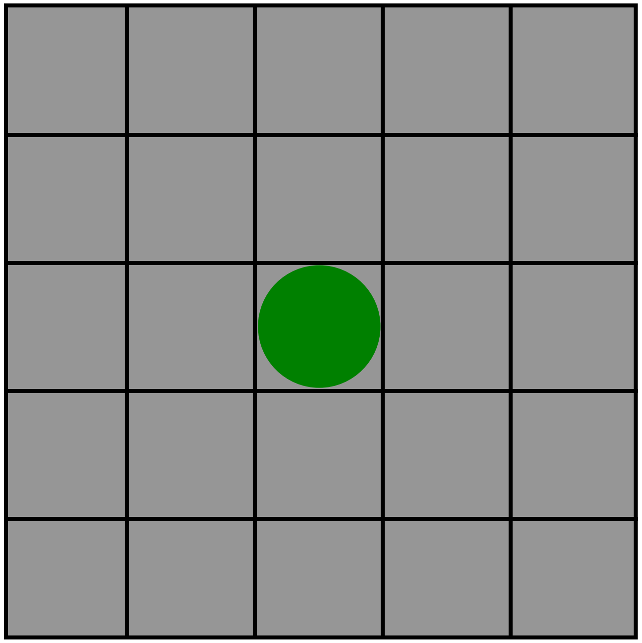
Detector in fixed position



The detector cannot distinguish a point image from an image with the size of one pixel

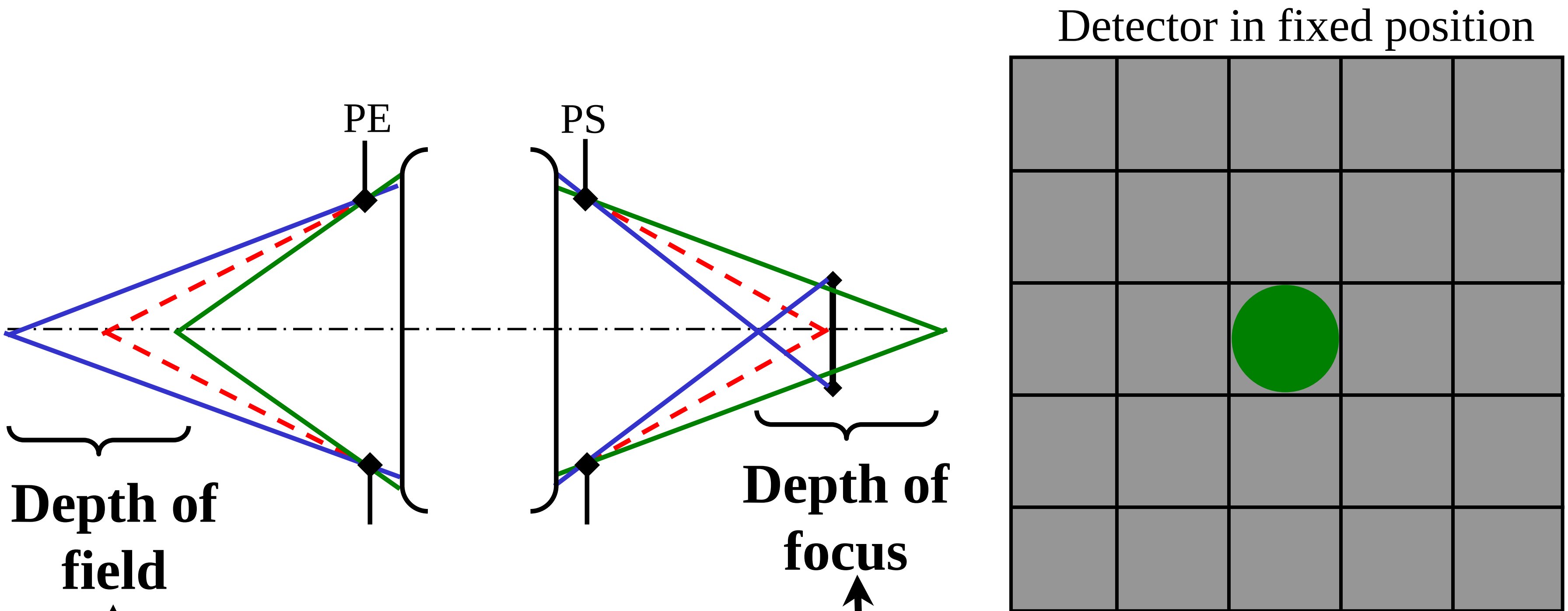


Detector in fixed position



The detector cannot distinguish a point image from an image with the size of one pixel

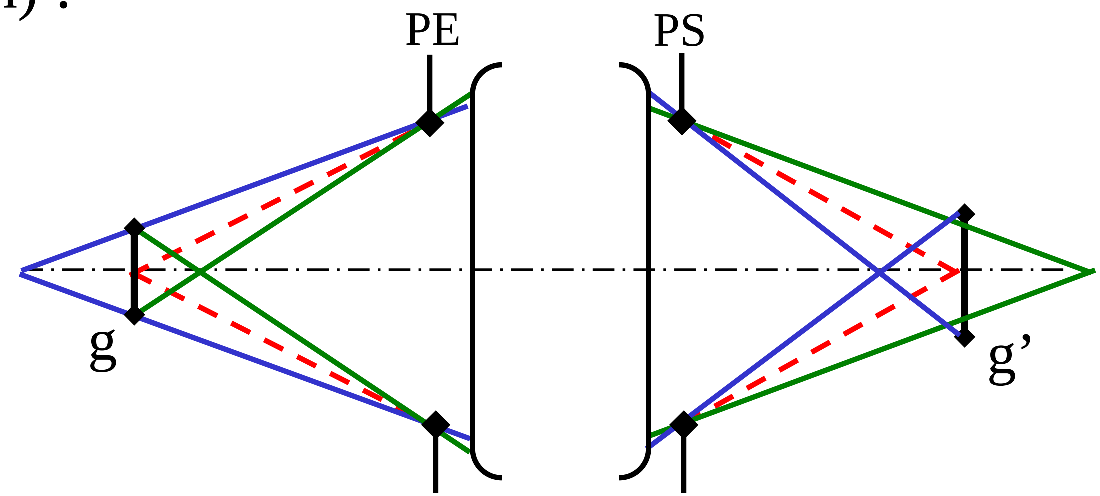
Relationship between depth of field and depth of focus



Edges of this depth of focus defined for spot size = pixel size

Edges of depth of field
in object space = conjugates to edges of the field in image space

→ If g’ is the resolution in the image space (usually pixel size, but the calculation remains valid for diffraction limit or any other limitation) :



Corresponding resolution in object space : $g = g' / g_y$

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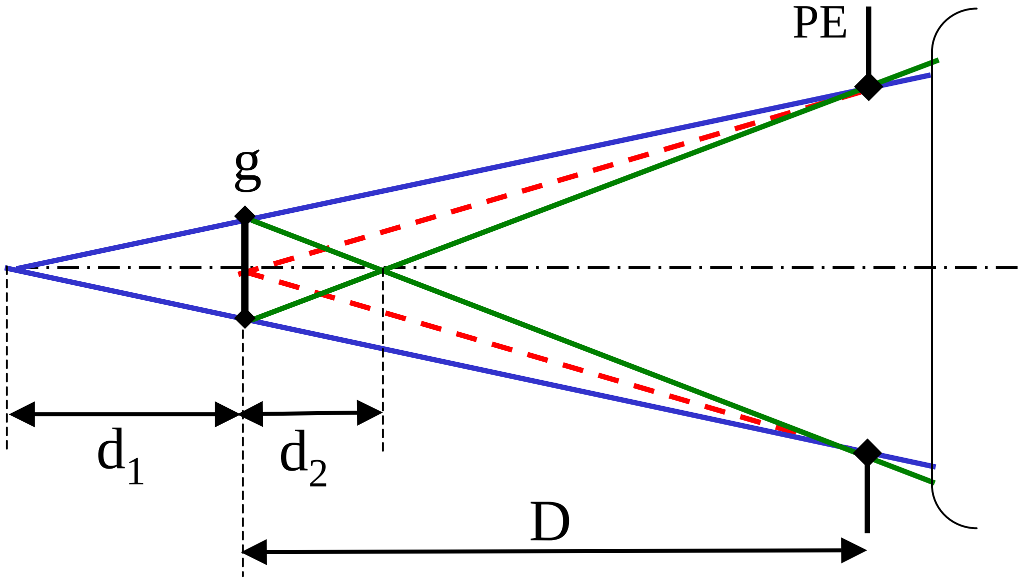
→ Detail of what happens in the object space :



$g = g'/g_y$

$= $ resolution in obj. space

Φ = dia entrance pupil

$$d_1 = \frac{g\,D}{\Phi - g}$$

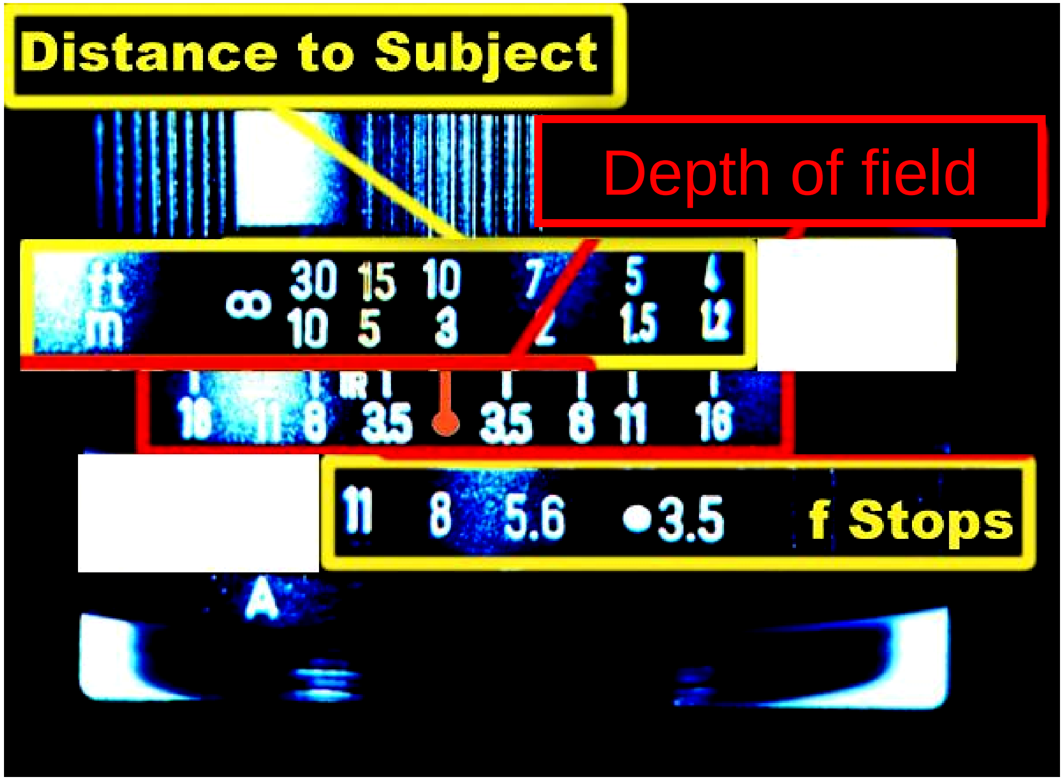
$$d_2 = \frac{g\,D}{\Phi + g}$$

Depth of field

$$d = d_1 + d_2 = \frac{2\,\Phi\,g\,D}{\Phi^2 - g^2}$$

Note that the image does not suddenly become blurry on the edge of the depth of field (→ rough estimate…)

Example for a photographic objective



Object at 3 m, Aperture N=8, depth of field from 1.7 to 10 m

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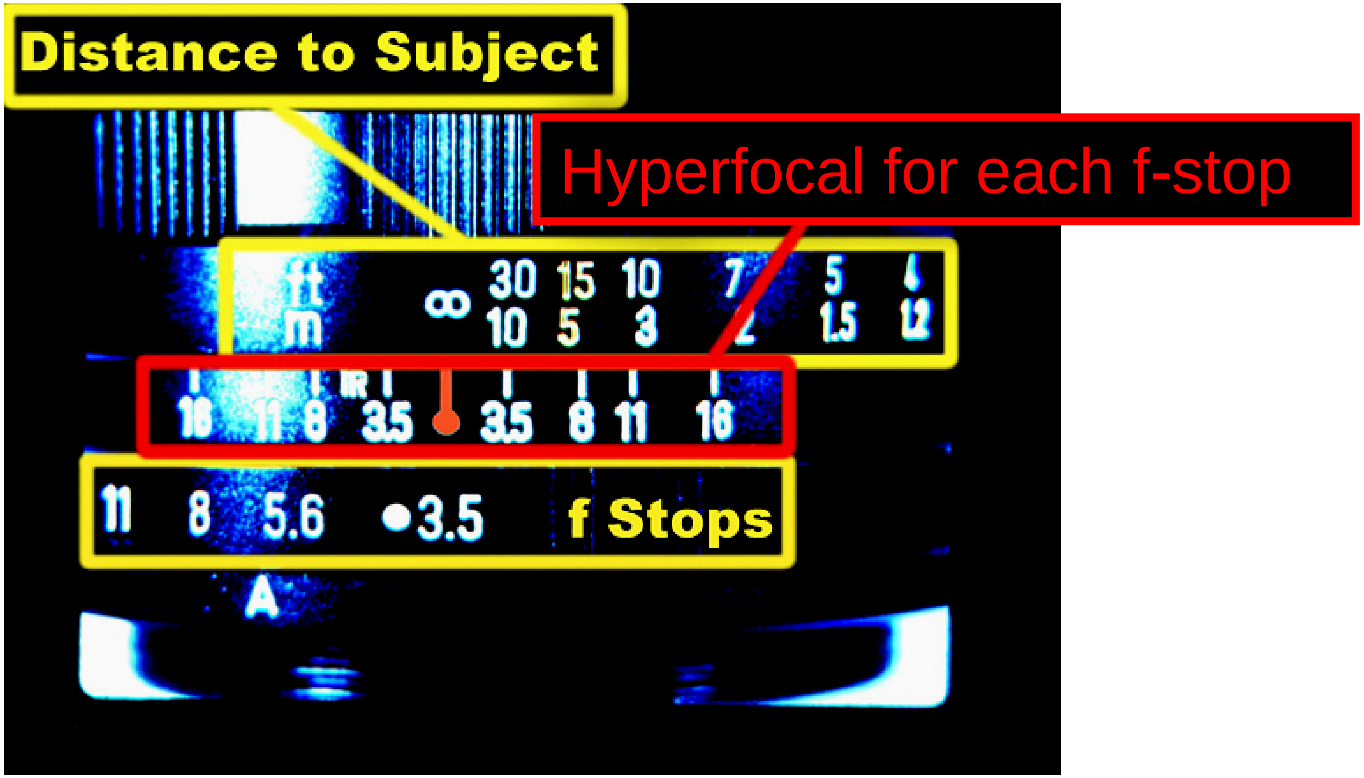
Hyperfocal distance

= depth of field for an object at infinity

Photographic objective



Object at infinity, Aperture N=3.5, hyperfocal distance 10 m