Image formation in the paraxial approximation – part 2

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & e/n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -P_1 & 1 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

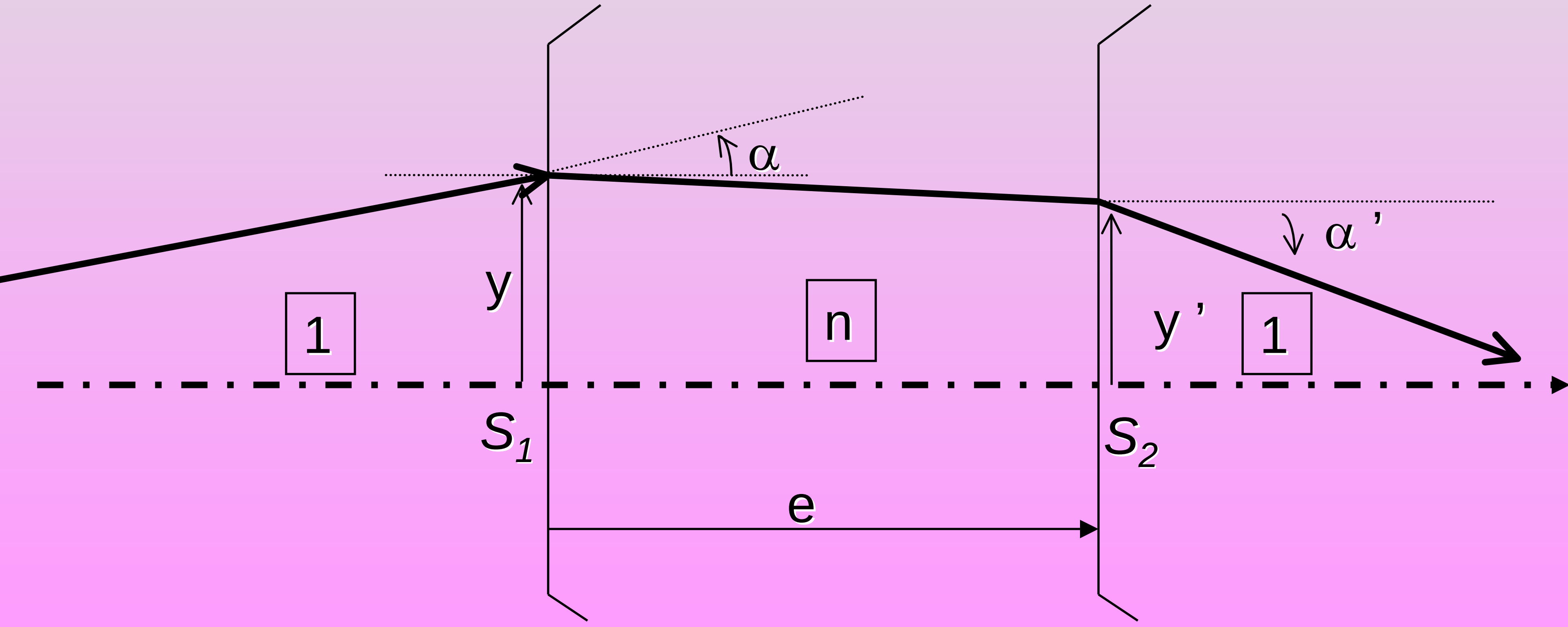


Image formation in the paraxial approximation

- 7- Thick lens

a : Matrix for a thick lens

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 - \frac{e}{n}P_1 & \frac{e}{n} \\ -P & 1 - \frac{e}{n}P_2 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

$$\boxed{\frac{1}{f'} = P = P_1 + P_2 - \frac{e}{n}P_1 P_2}$$

«Gullstrand's formula » !

$$\boxed{\frac{1}{f'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + e\frac{(n-1)^2}{nR_1R_2}}$$

Image formation in the paraxial approximation

- *b- Case of a thin lens :*

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix}_{E \to S} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

where

$$\boxed{P = \frac{1}{f'}}$$

$$\boxed{\frac{1}{f'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Image formation in the paraxial approximation

- Thin lens :

Gaussian formula : $\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f'}$

Focal points and constructions

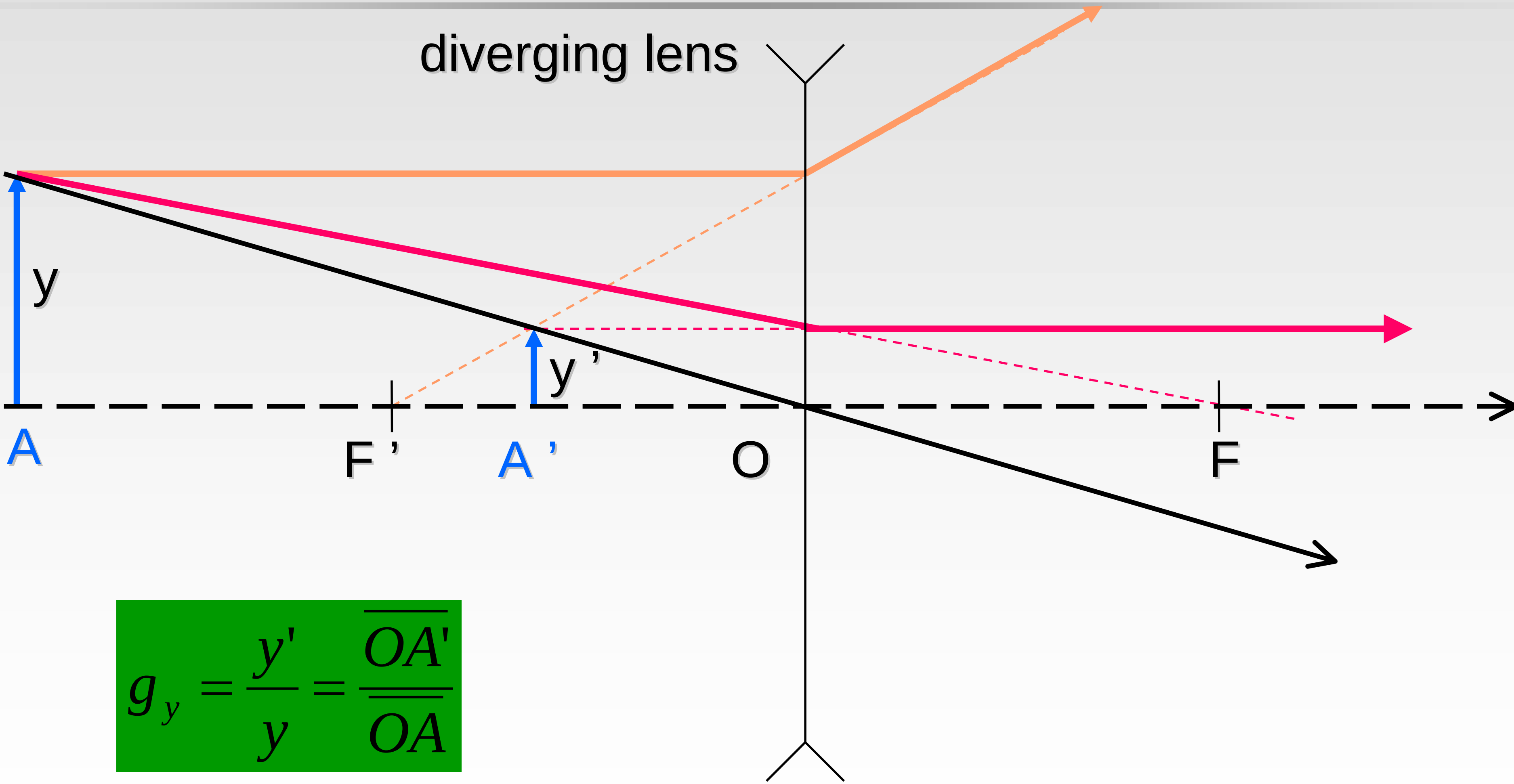
Magnifications

Newtonian formula

Lagrange invariant

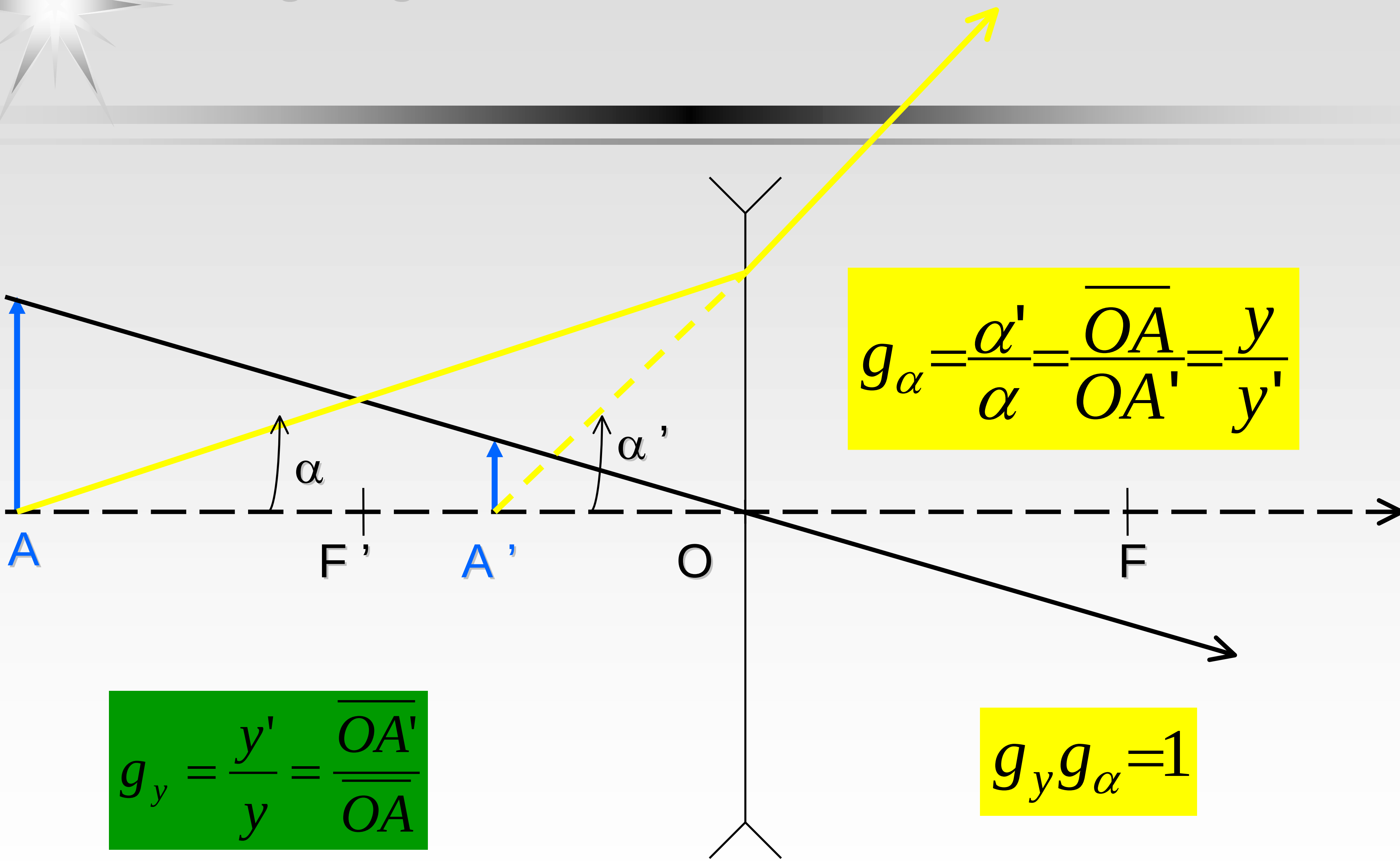
$$g_y = \frac{y'}{y} = \frac{\overline{F'A'}}{\overline{F'O}} = -\frac{\overline{F'A'}}{f'}$$

$$g_y = \frac{y'}{y} = \frac{\overline{FO}}{\overline{FA}} = -\frac{f}{\overline{FA}}$$



$$g_y = \frac{y'}{y} = \frac{\overline{OA'}}{\overline{OA}}$$

Lagrange’s formula



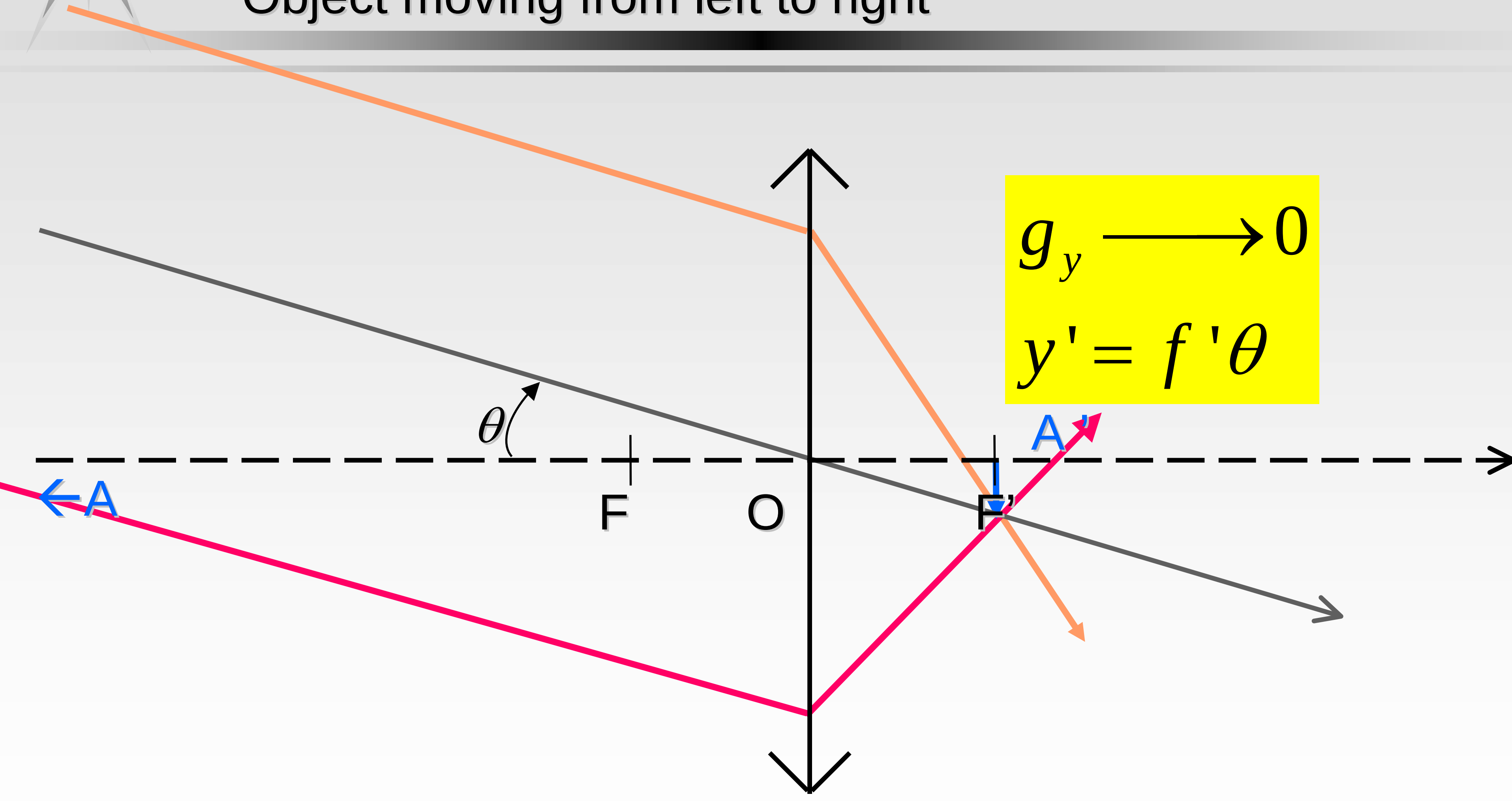
$$g_\alpha = \frac{\alpha'}{\alpha} = \frac{\overline{OA}}{OA'} = \frac{y}{y'}$$

$$g_y = \frac{y'}{y} = \frac{\overline{OA'}}{\overline{OA}}$$

$$g_y g_\alpha = 1$$

Constructions for converging lens

Object moving from left to right

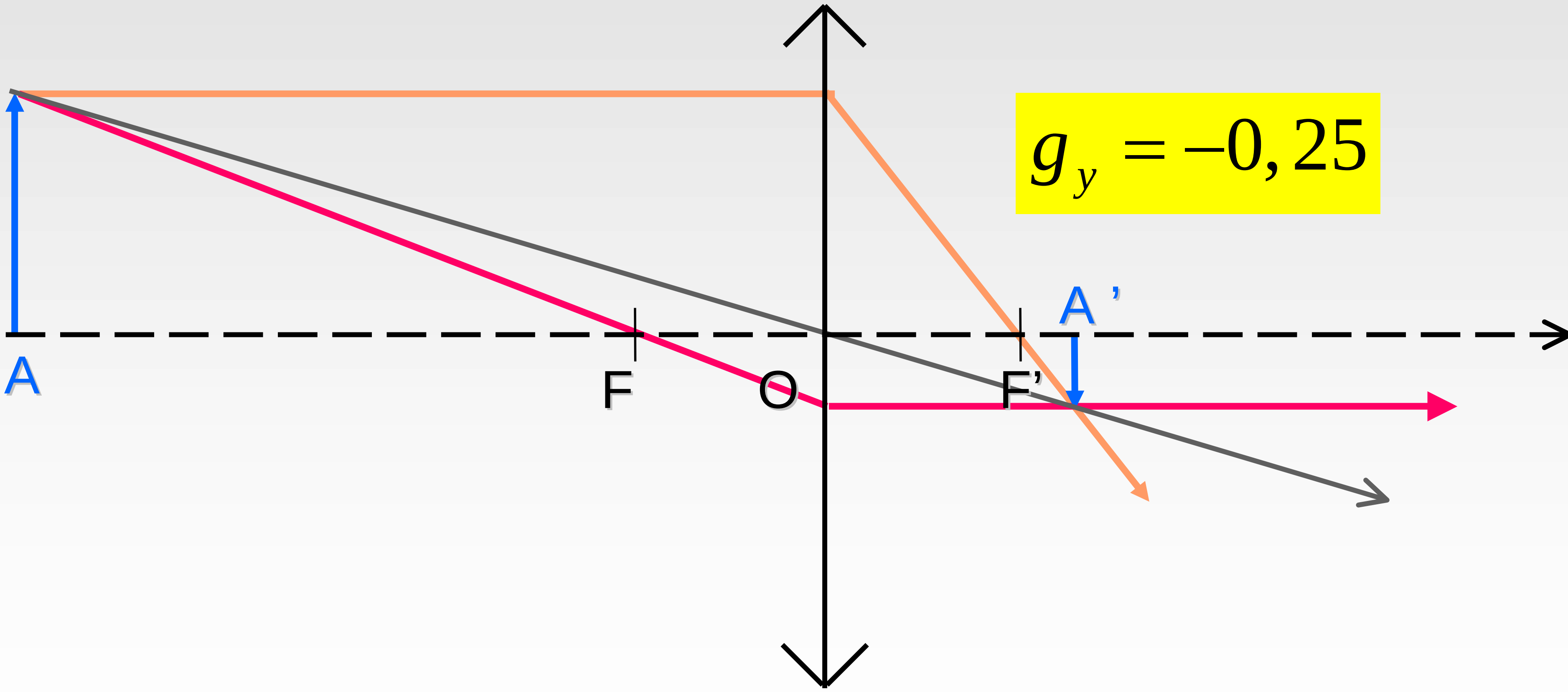


$$g_y \longrightarrow 0$$

$$y' = f'\theta$$

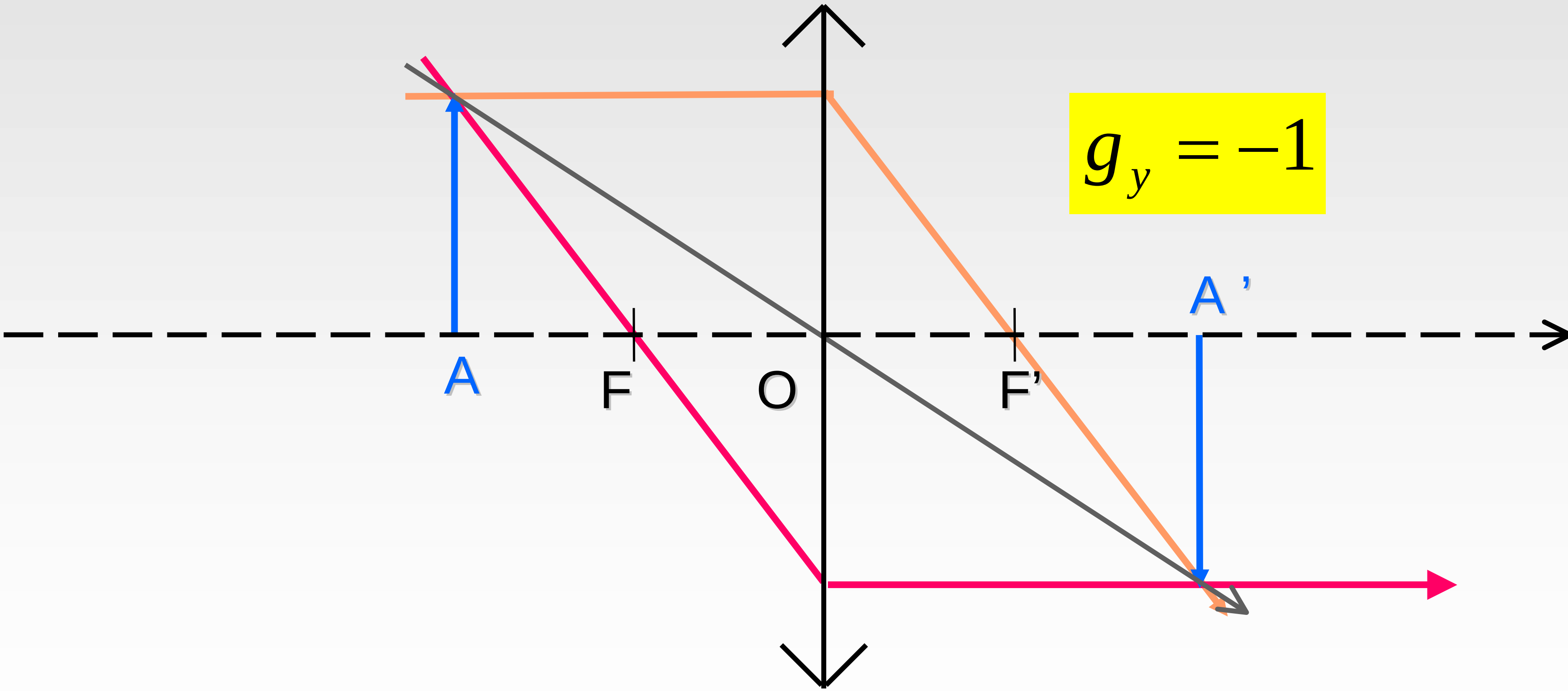
Object at infinity – Image at 2nd focal point

Converging lens



$$g_y = -0,25$$

Converging lens

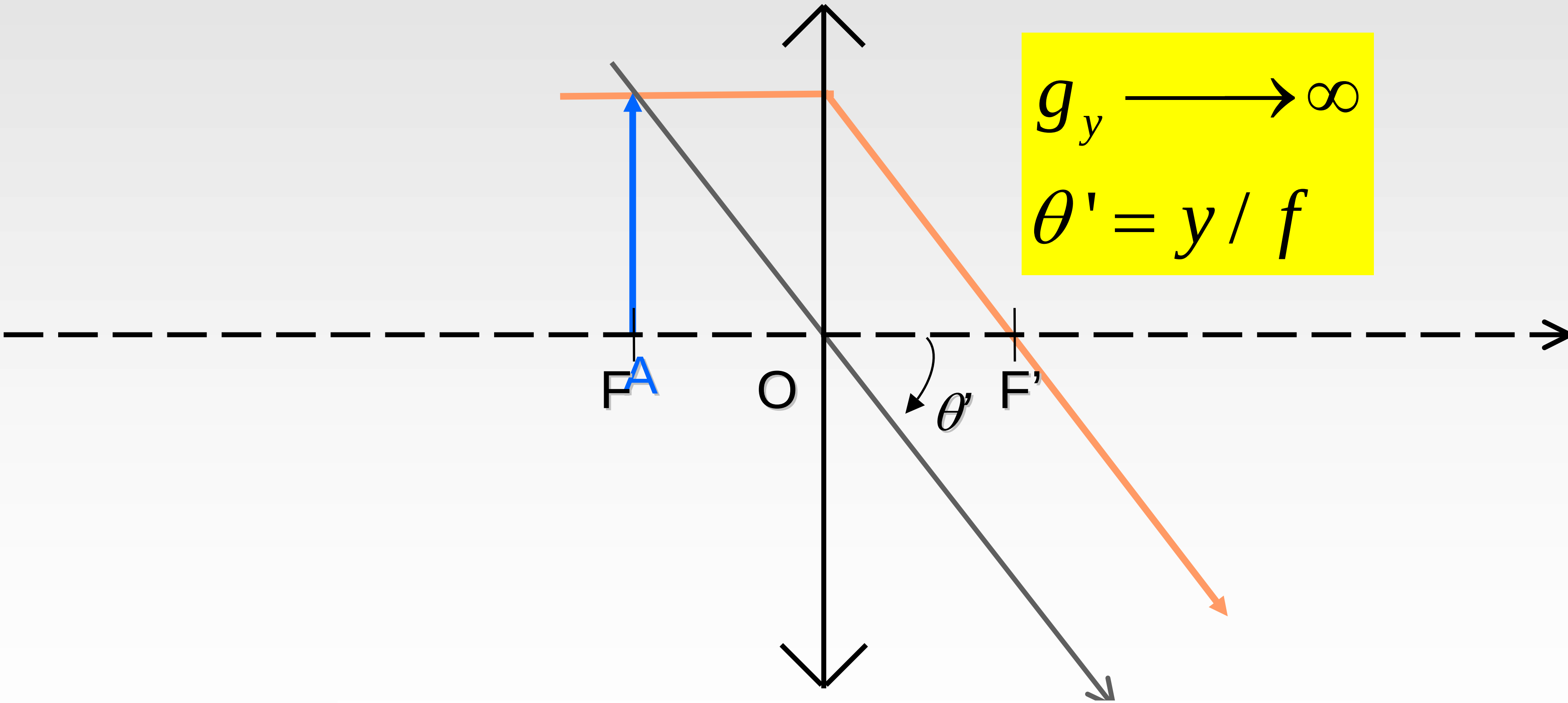


$$g_y = -1$$

Conjugate points: 2f-2f

4f : minimum distance between REAL conjugate points

Converging lens

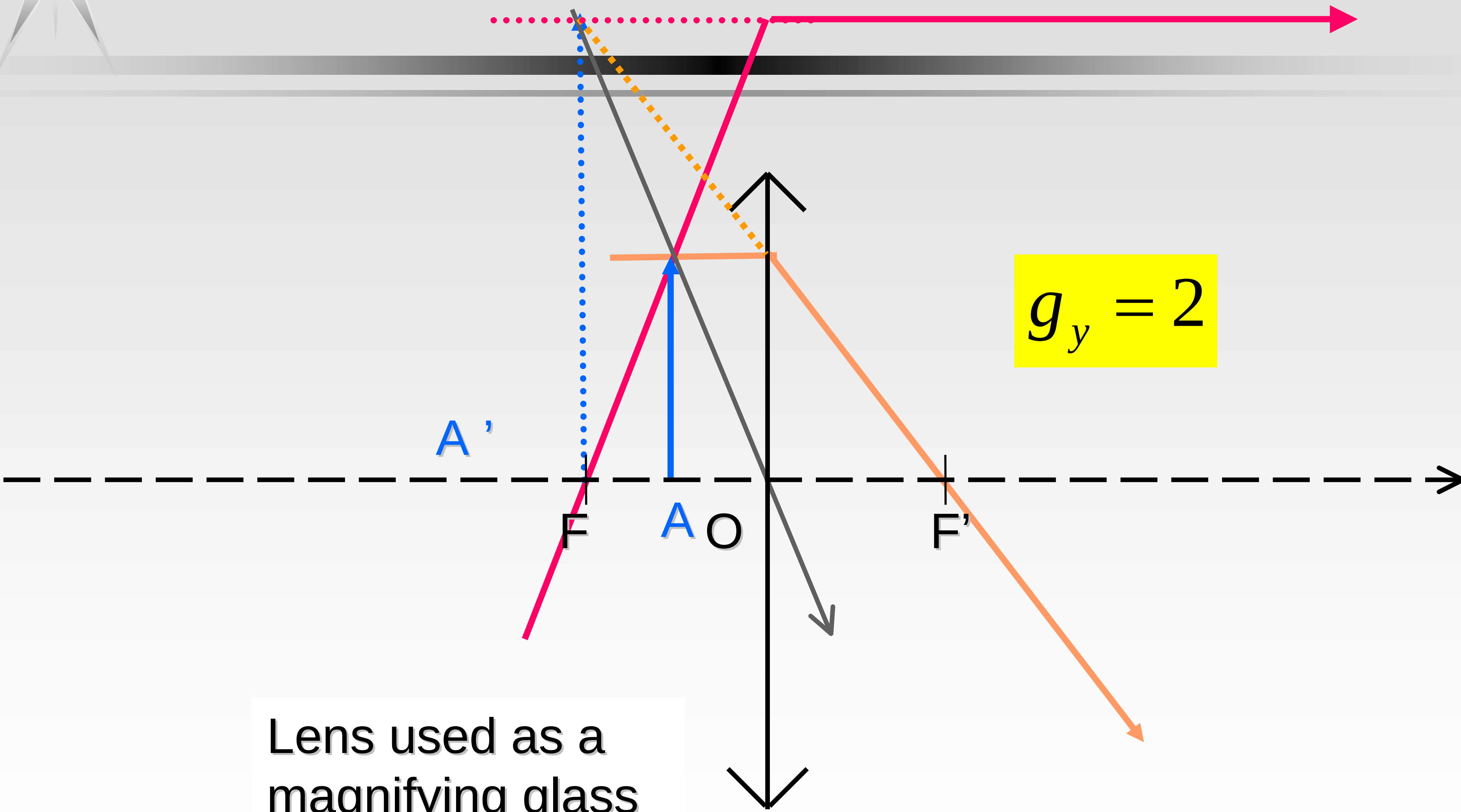


$$g_y \longrightarrow \infty$$

$$\theta' = y / f$$

Conjugate points: 1st focal point- infinity

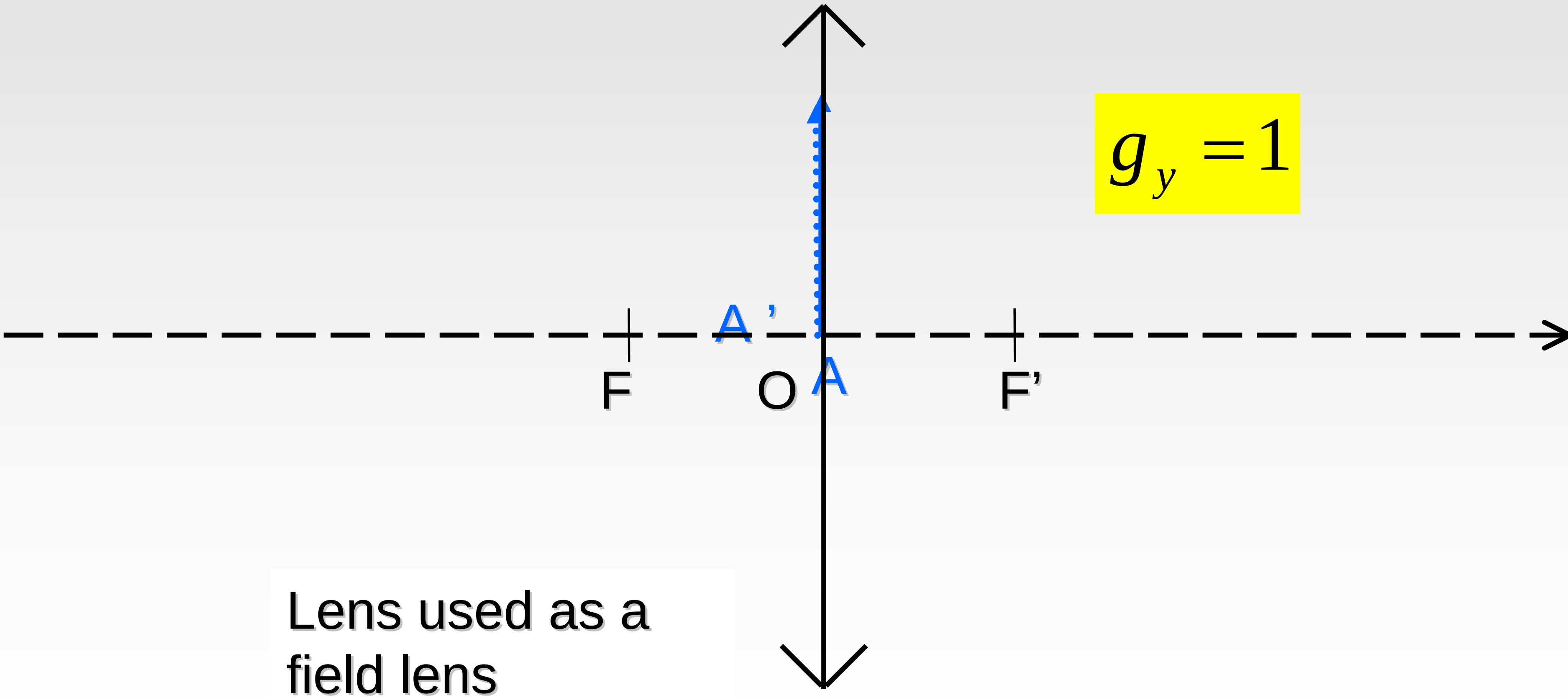
Converging lens



$$g_y = 2$$

Lens used as a magnifying glass

Converging lens



$$g_y = 1$$

Lens used as a field lens

Converging lens

$$g_y = 0{,}5$$

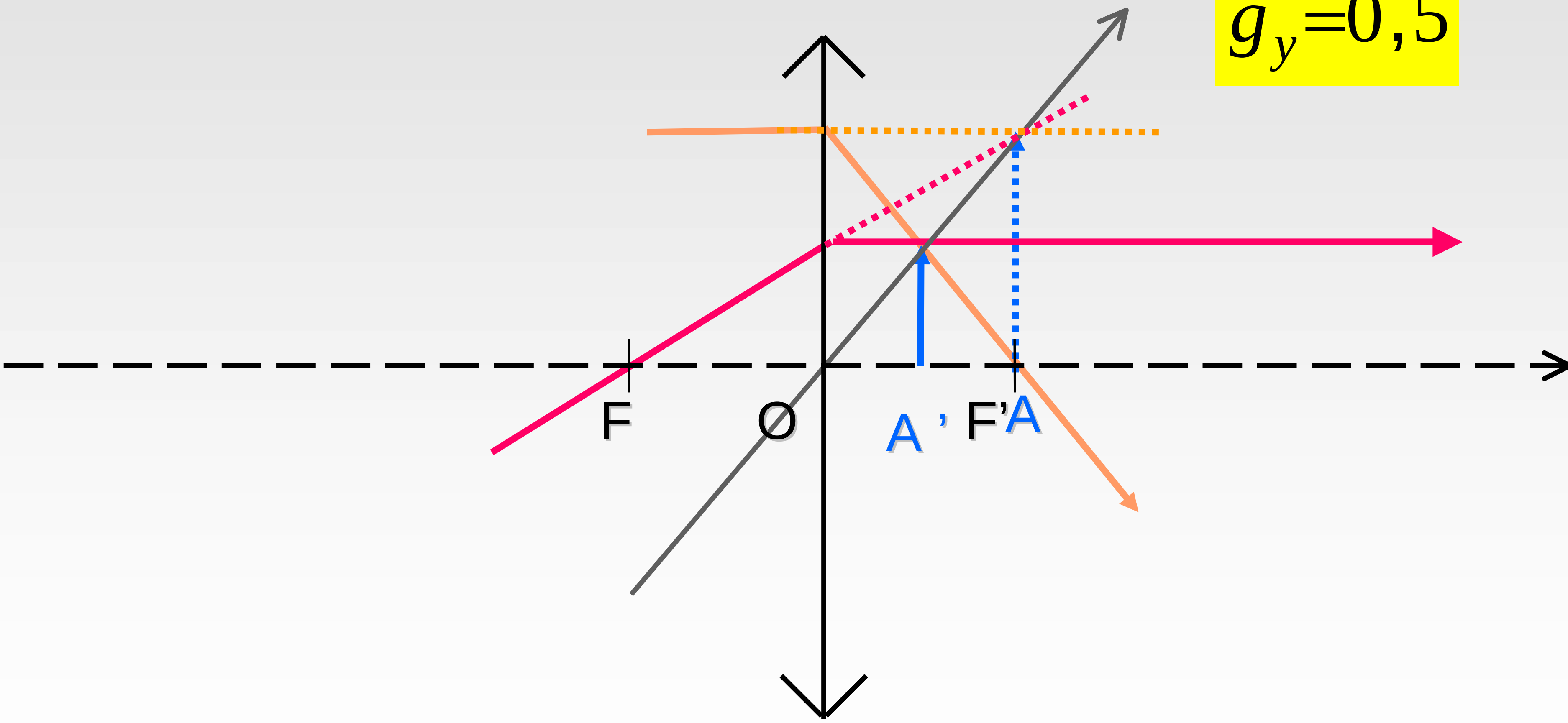
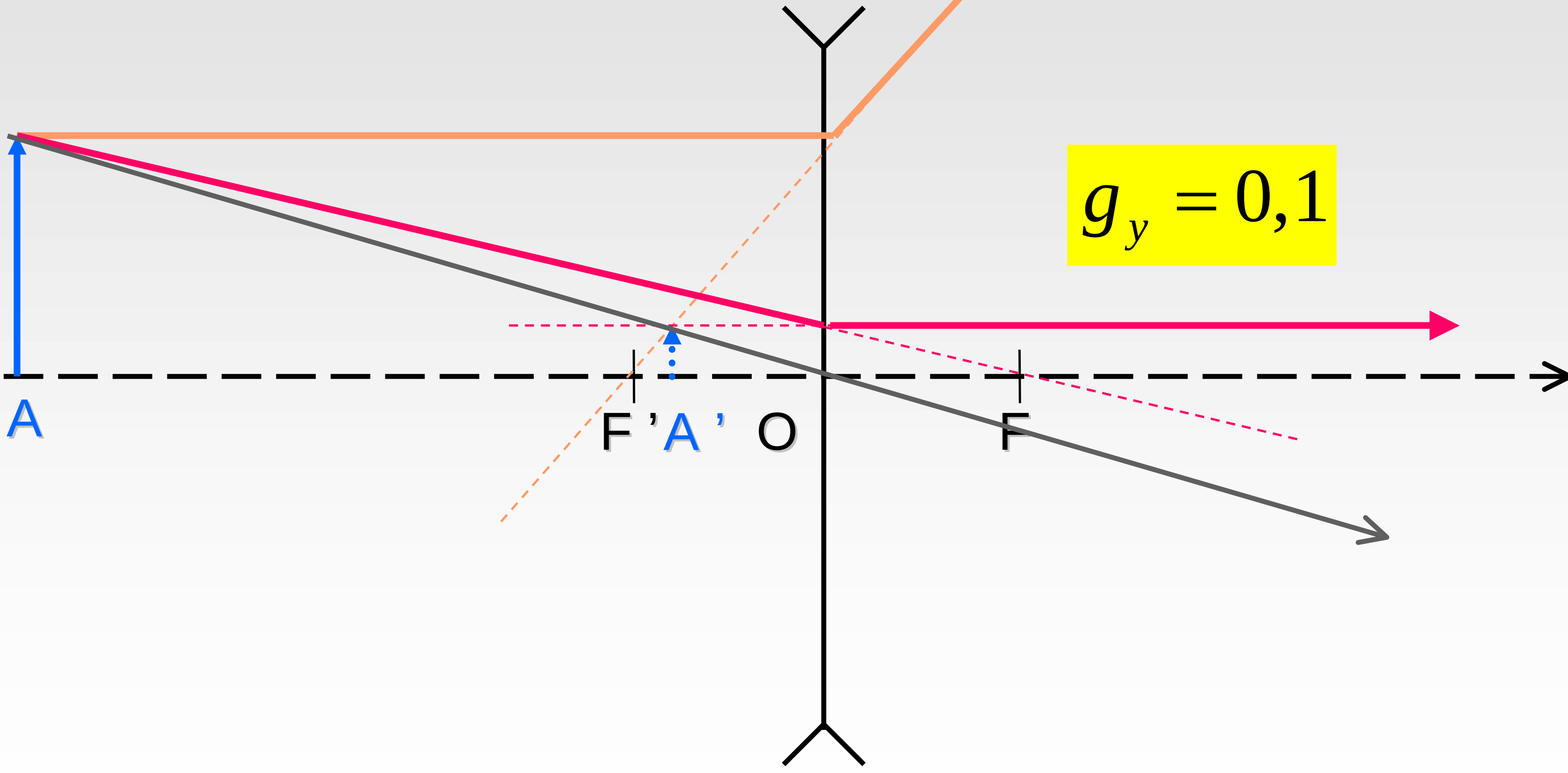


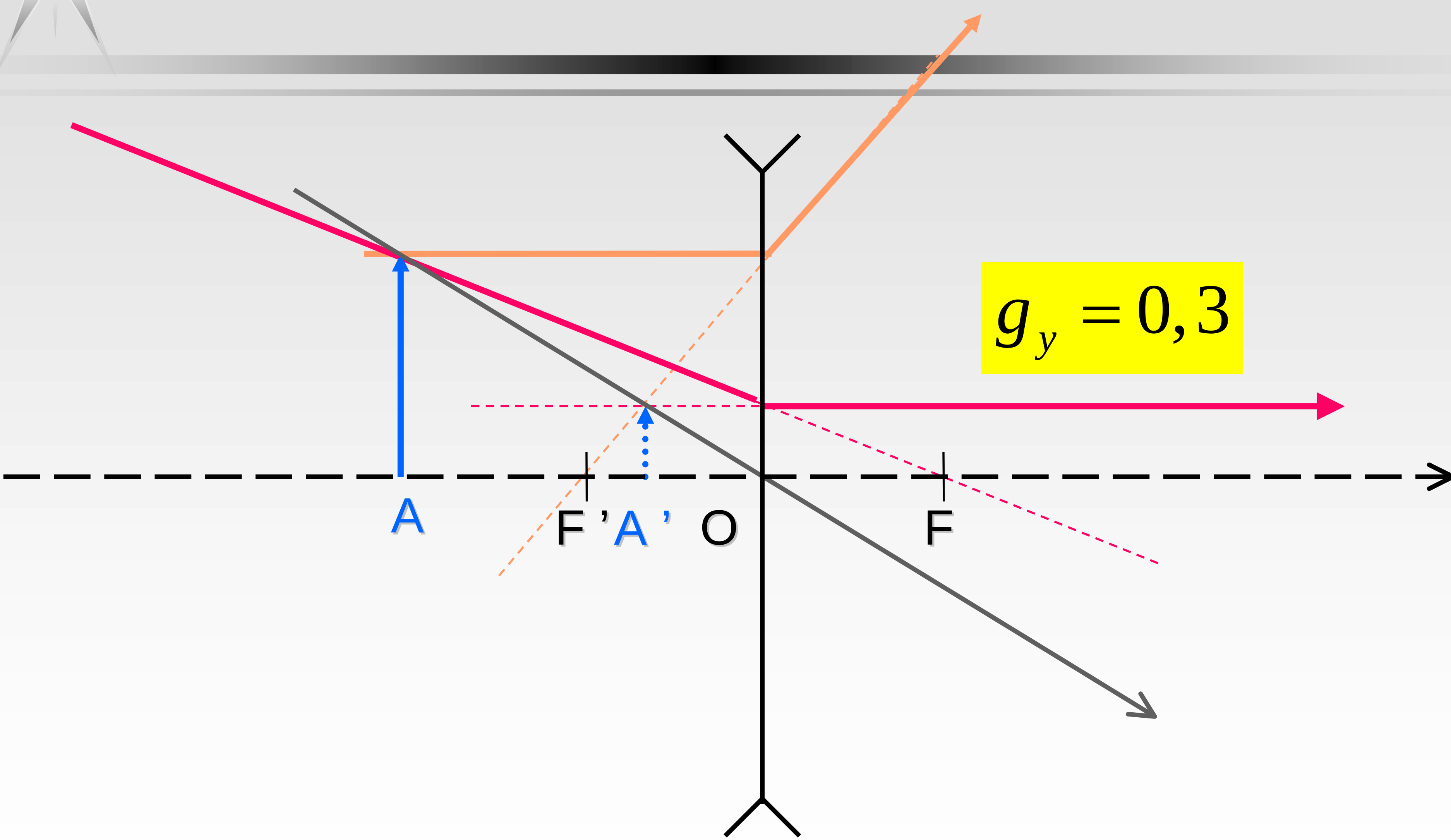
Image always moved in the same direction as object (g_x>0)

Constructions for diverging lens

Object moving from left to right

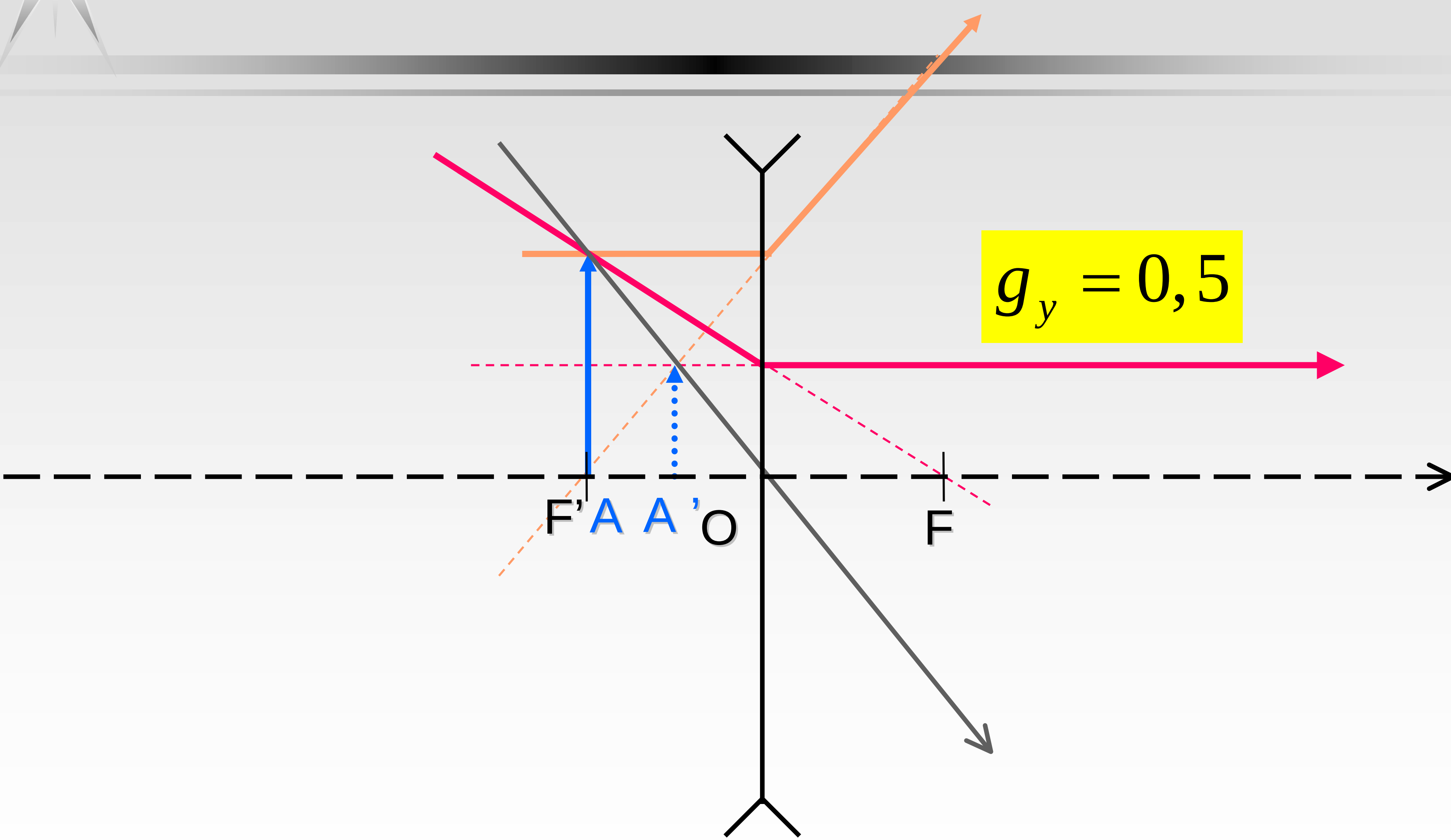


Diverging lens



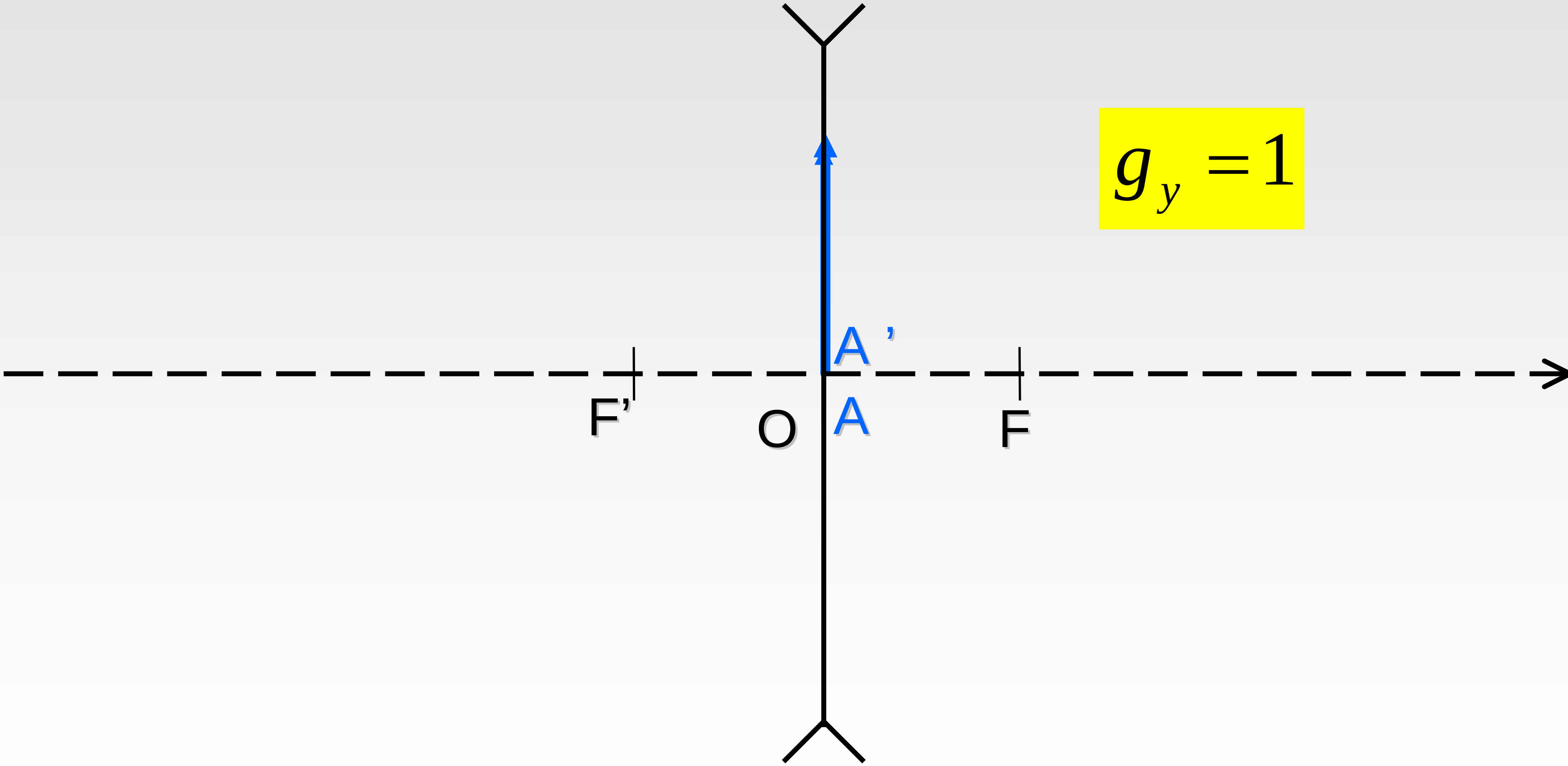
$$g_y = 0,3$$

Diverging lens



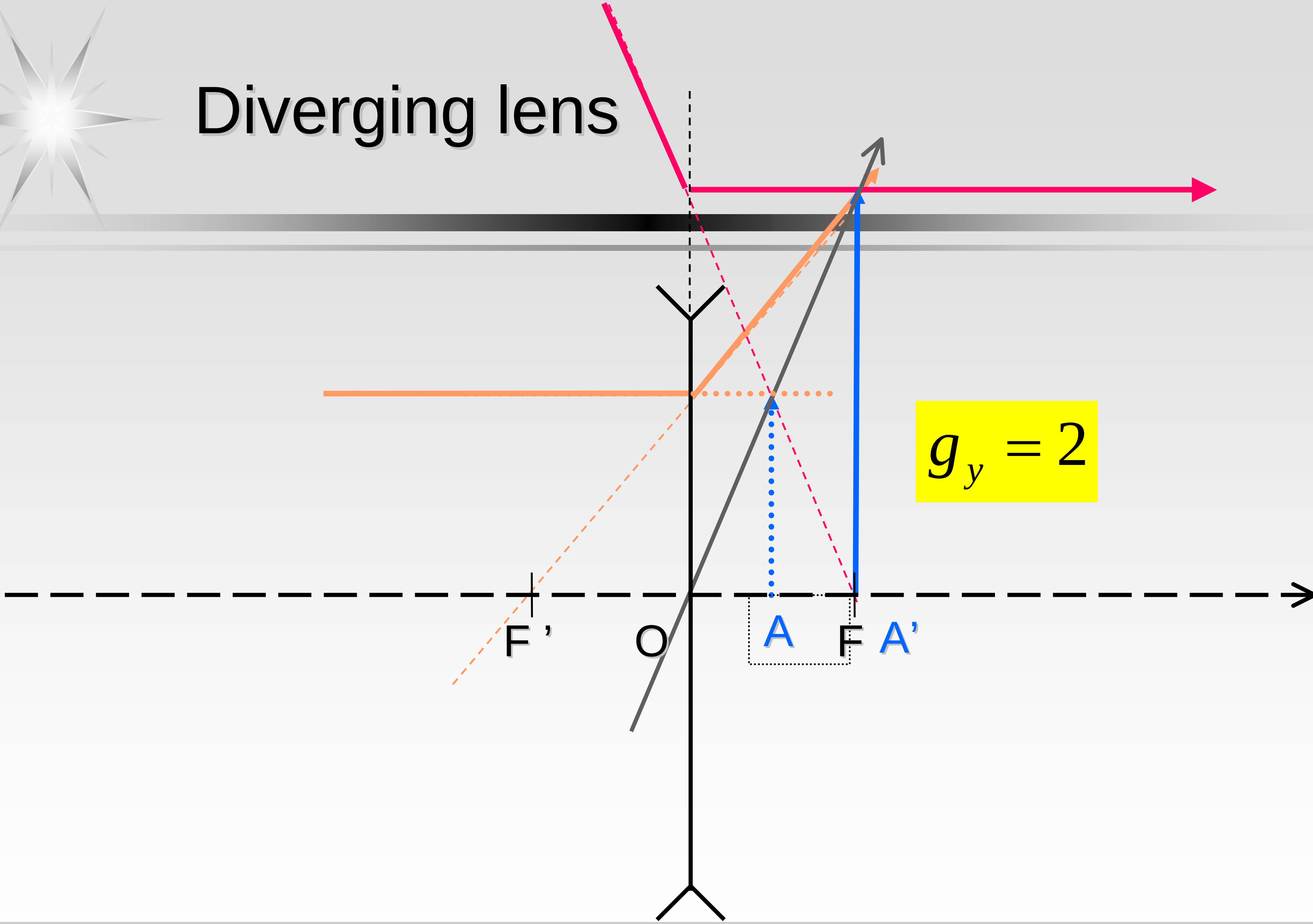
$$g_y = 0,5$$

Diverging lens



$$g_y = 1$$

Diverging lens



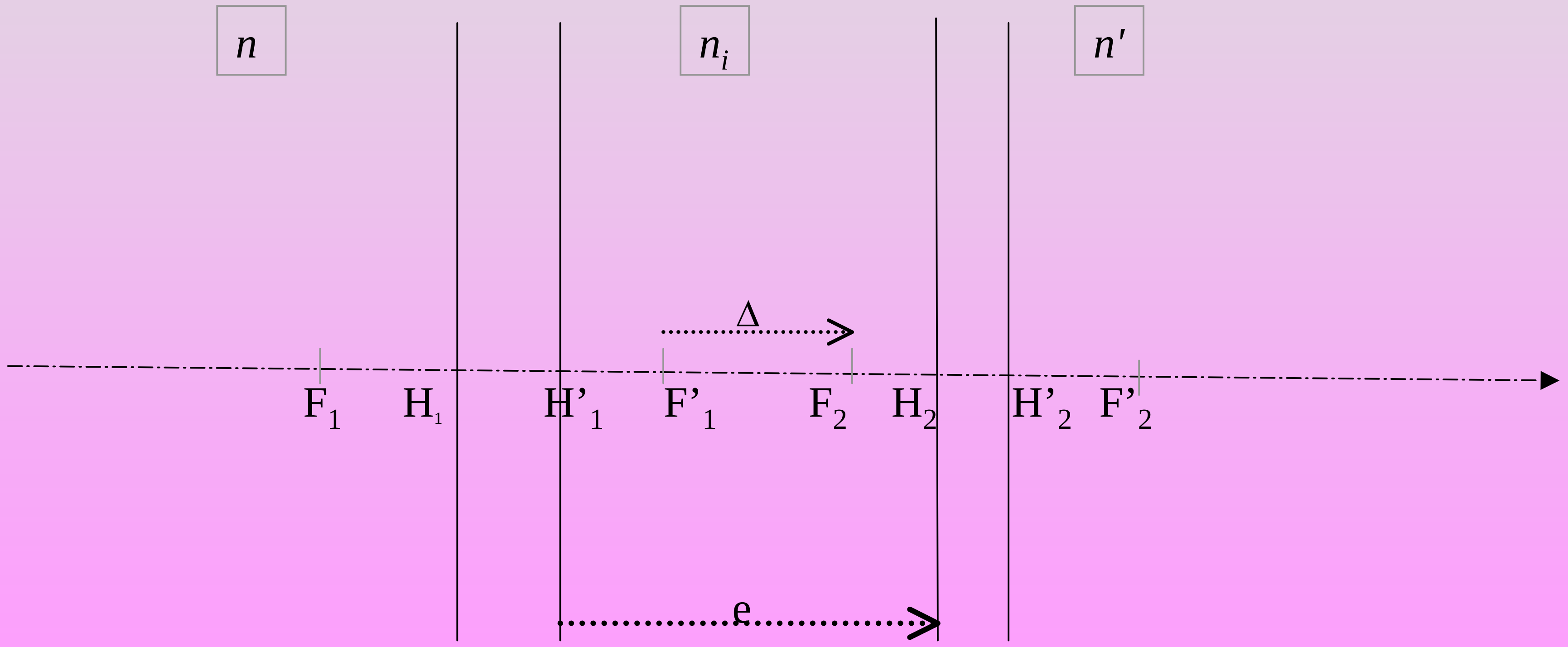
$$g_y = 2$$

Here also: Image moved in the same direction as object (g_x>0)

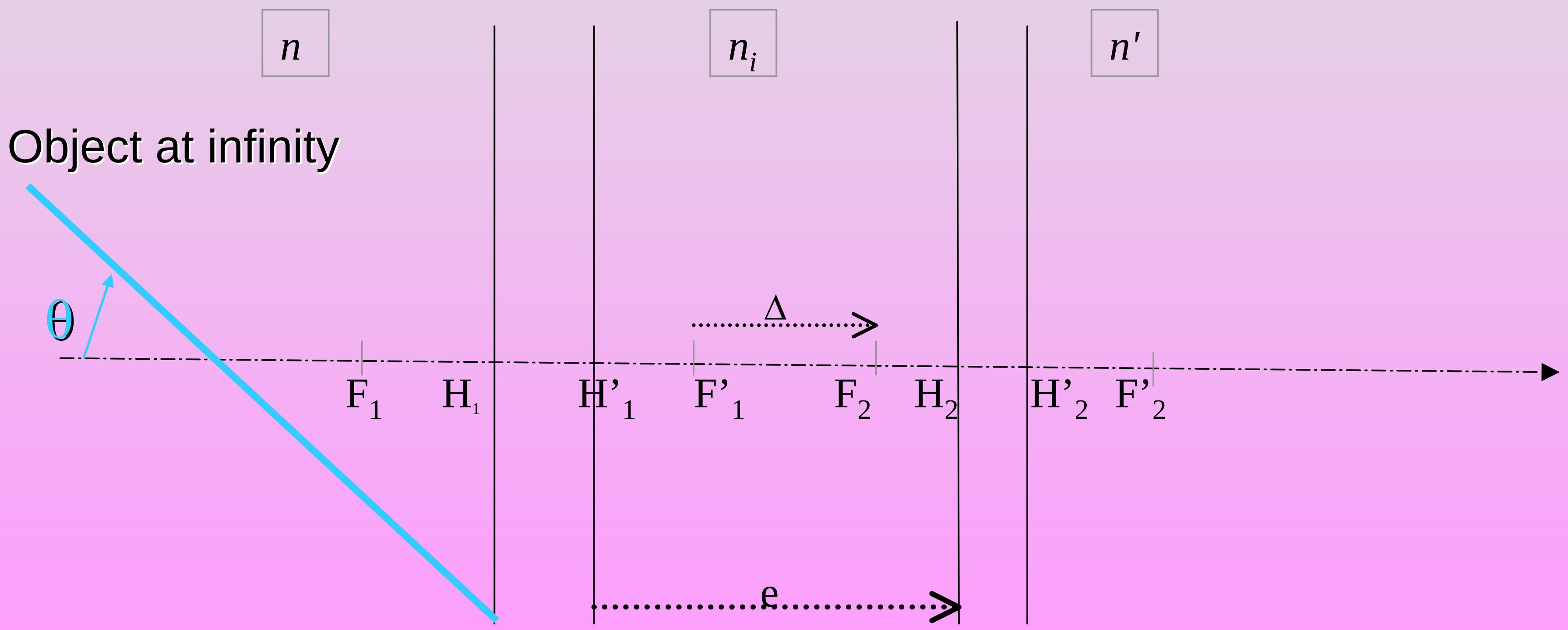
A few problems on combining 2 focal systems

1a - Calculate the matrix of the whole system and deduce the power of the whole system as a function of the powers of the 2 subsystems.

1b- Construct the positions of the two foci of the whole system F and F'



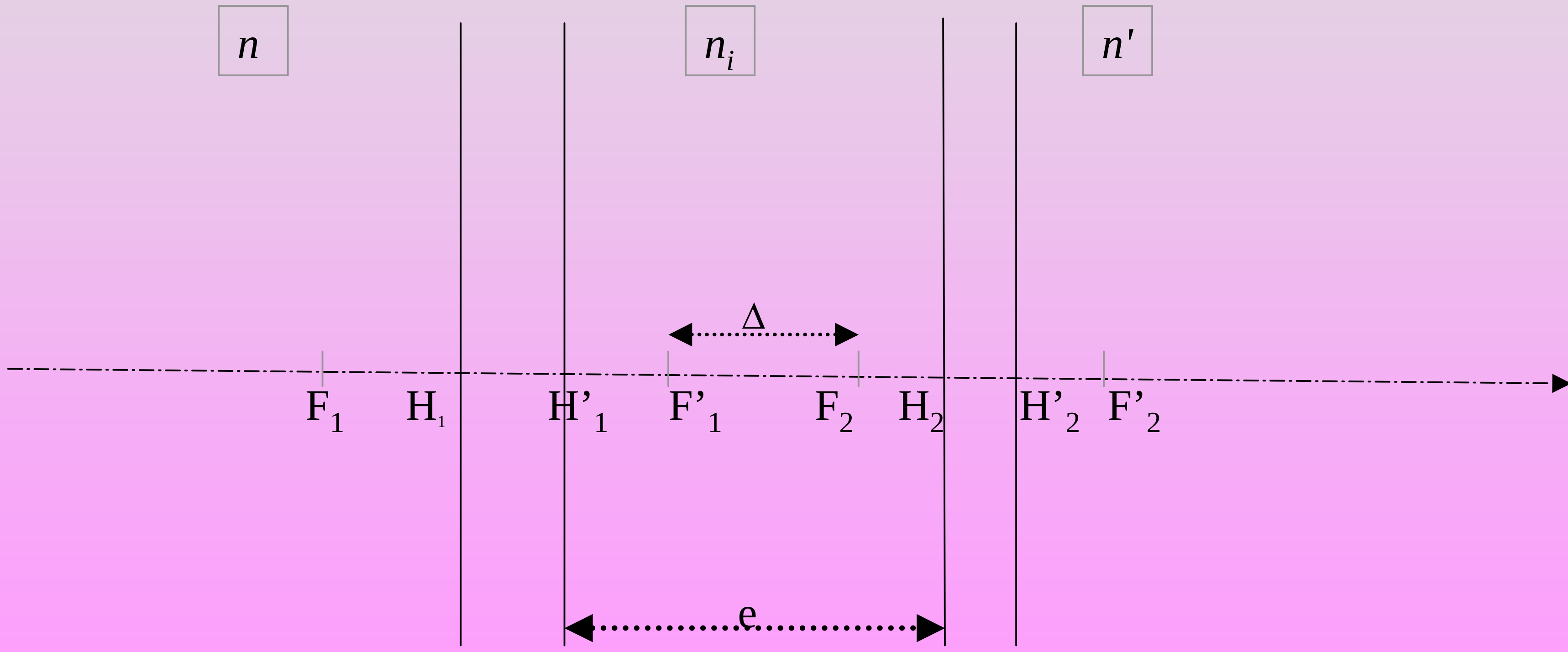
2 - Construct the image of an object of angular size θ located at infinity. Calculate the size of the intermediate image, and the magnification through the 2nd system. Deduce an expression of the focal length of the whole system.



ANSWERS

1a-

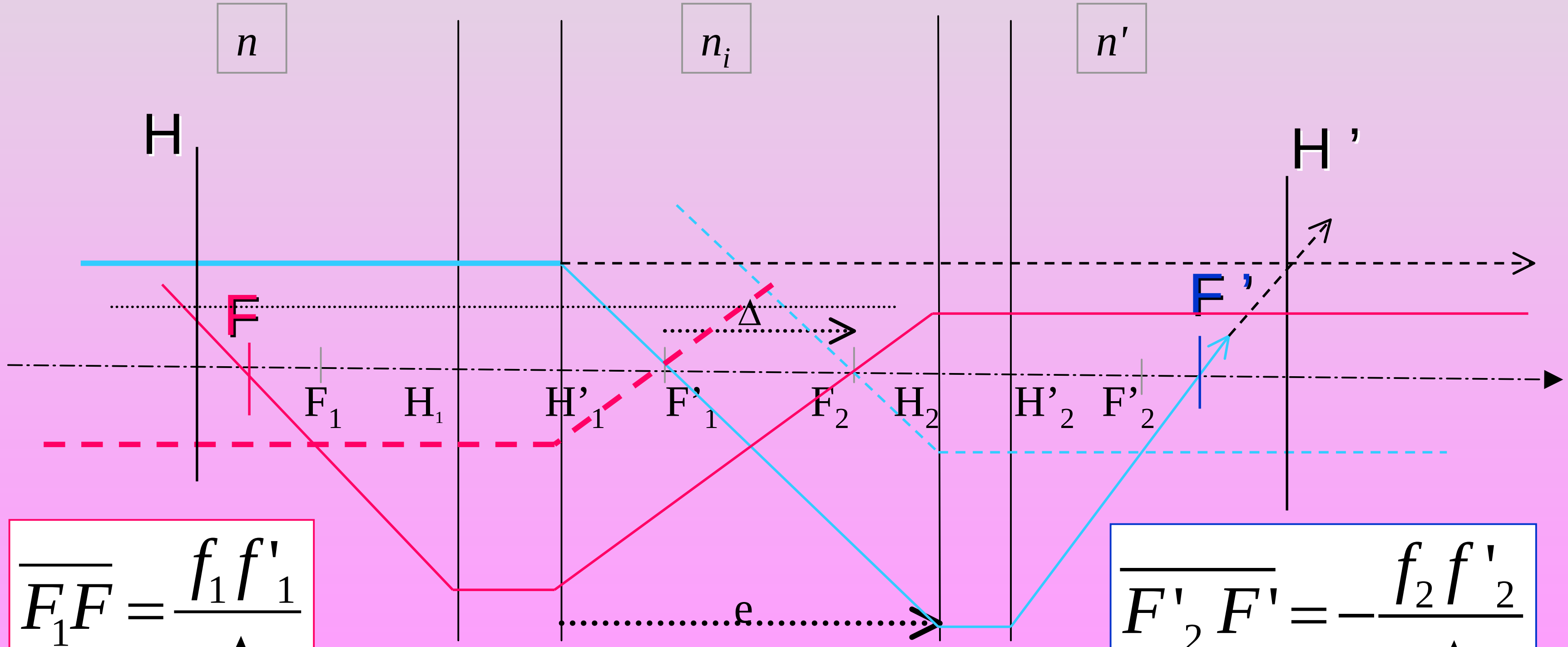
$$\begin{bmatrix} y' \\ n'\alpha' \end{bmatrix} = \underset{H_2 \to H'_2}{\begin{bmatrix} 1 & 0 \\ -P_2 & 1 \end{bmatrix}} \underset{H'_1 \to H_2}{\begin{bmatrix} 1 & e/n_i \\ 0 & 1 \end{bmatrix}} \underset{H_1 \to H'_1}{\begin{bmatrix} 1 & 0 \\ -P_1 & 1 \end{bmatrix}} \begin{bmatrix} y \\ n\alpha \end{bmatrix}$$



$$P = P_1 + P_2 - \frac{e}{n_i} P_1 P_2$$

1b-

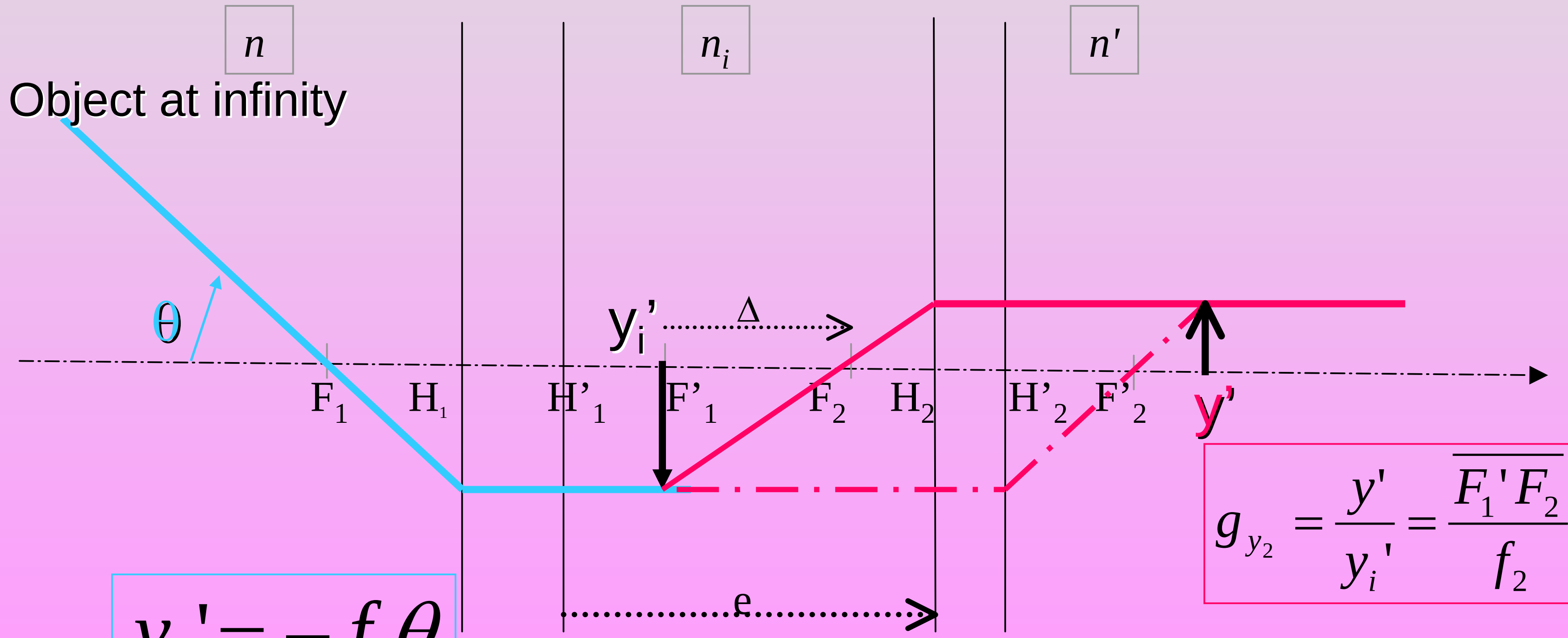
$$P = P_1 + P_2 - \frac{e}{n_i} P_1 P_2$$



$$\overline{F_1F} = \frac{f_1 f'_1}{\Delta}$$

$$\overline{F'_2F'} = -\frac{f_2 f'_2}{\Delta}$$

2 - Combination of 2 optical systems : « Magnification method »



$$g_{y_2} = \frac{y'}{y_i'} = \frac{\overline{F_1'F_2}}{f_2}$$

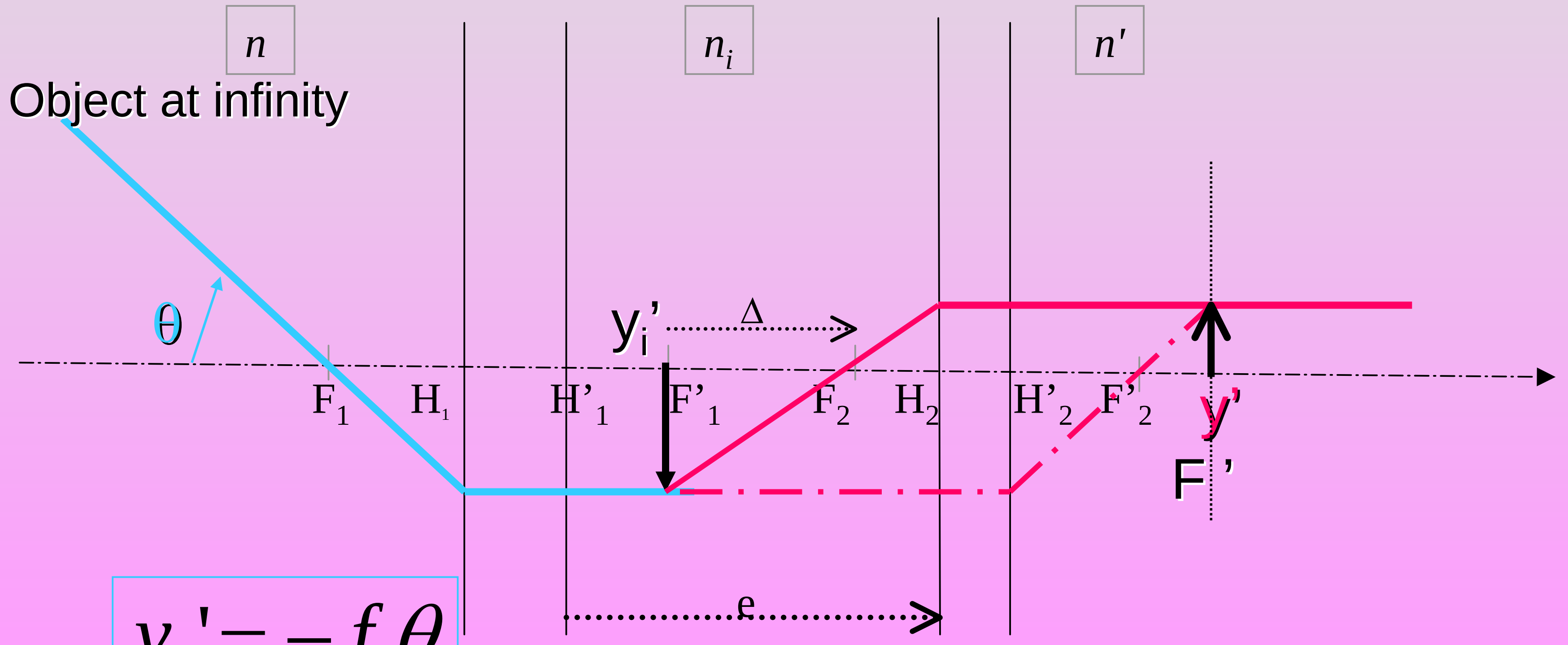
$$y_i' = -f_1\theta$$

$$y' = -f\theta \quad \text{and} \quad y' = g_{y_2} y_i' = -g_{y_2} f_1 \theta$$

$$f = g_{y_2} f_1$$

$$g_{y_2} = \frac{y'}{y_i'} = \frac{f_2}{\Delta}$$

$$g_{y_2} = -\frac{\overline{F'_2 F'}}{f'_2}$$



$$y_i' = -f_1\theta$$

$y' = -f\theta$ and $y' = g_{y_2} y_i' = -g_{y_2} f_1 \theta$