Ray Optics

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- 30 teaching hours (every wednesday 9-12am)

including lectures, problems in class and regular assignments, as many labs as possible, tutoring

(see NW's homepage on www.atomoptic.fr)

- Reference books (available at the Institut d'Optique library):

« Optics » by E. Hecht (chap5 Geometrical optics-paraxial theory)

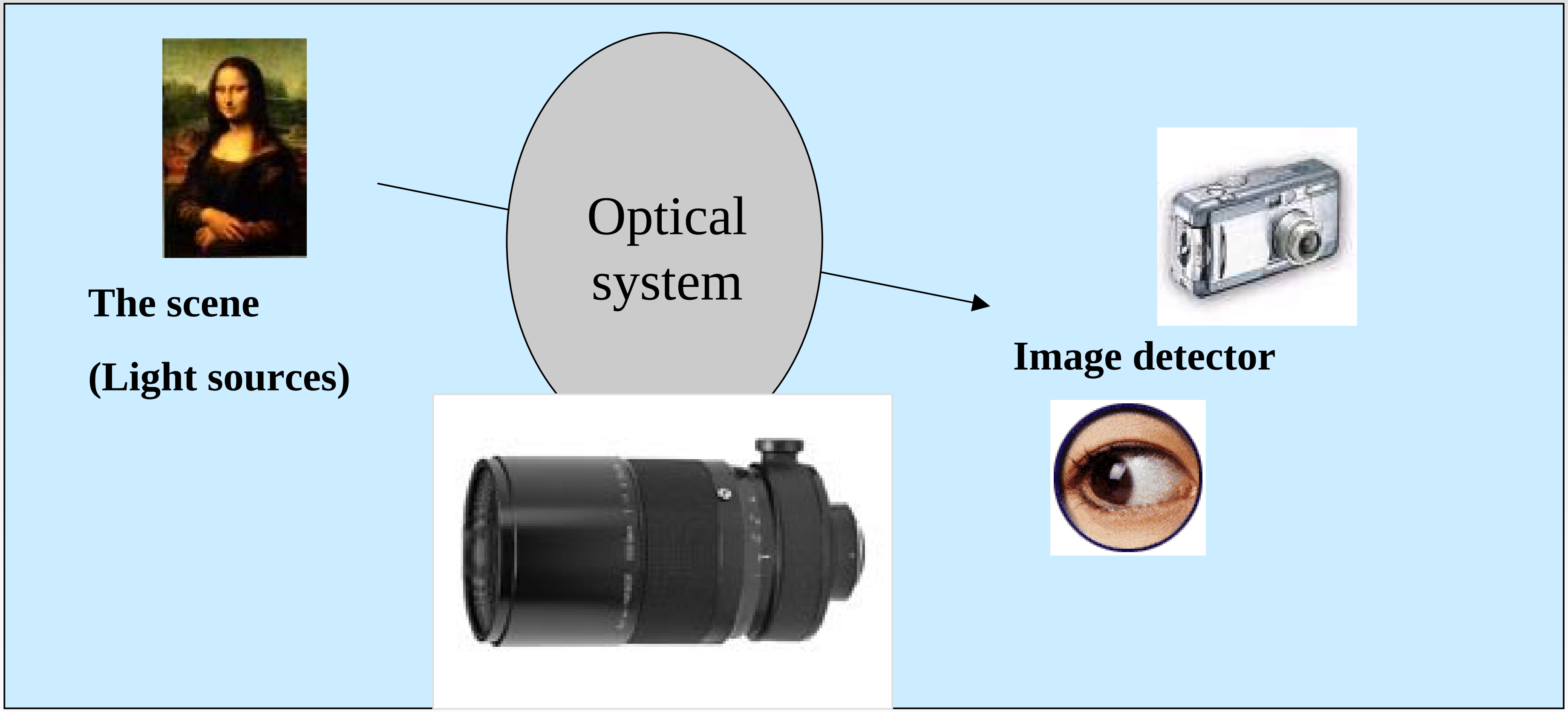
« Modern Optical Engineering » by W. J. Smith (chap 2-4-5-6-9)

My lecture notes « Ray optics » translated in english, in print, also available on the webpage

Subject covered in this course:

Image formation and optical instruments

in the paraxial approximation

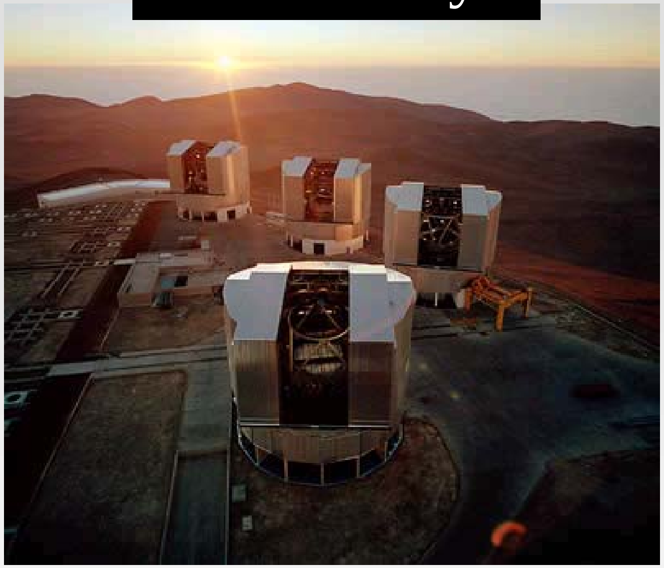


complementary to « sources and detectors » in 1st semester,

and followed by « optical design » in 2nd semester

Applications of complex optical systems

Astronomy

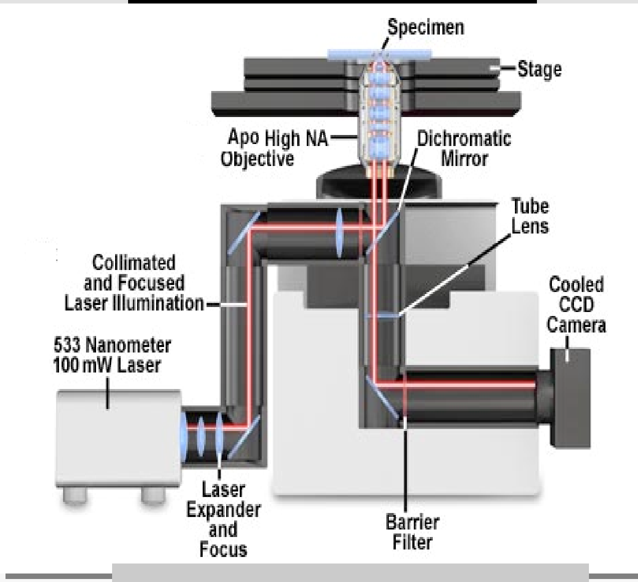


Very Large Telescope with adaptive optics, Chile

Photography

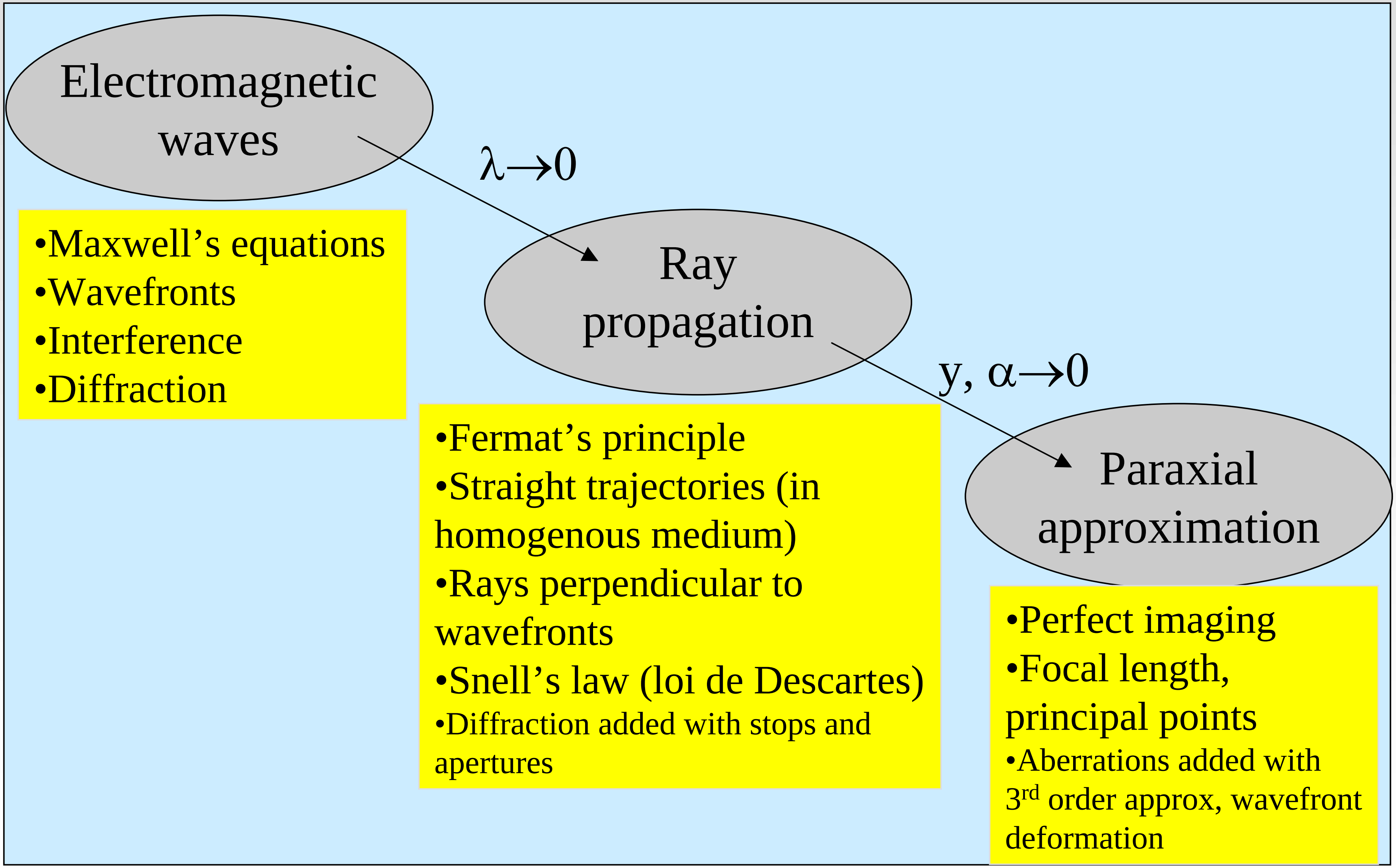
SPOT satellite, earth observation

Microscopy



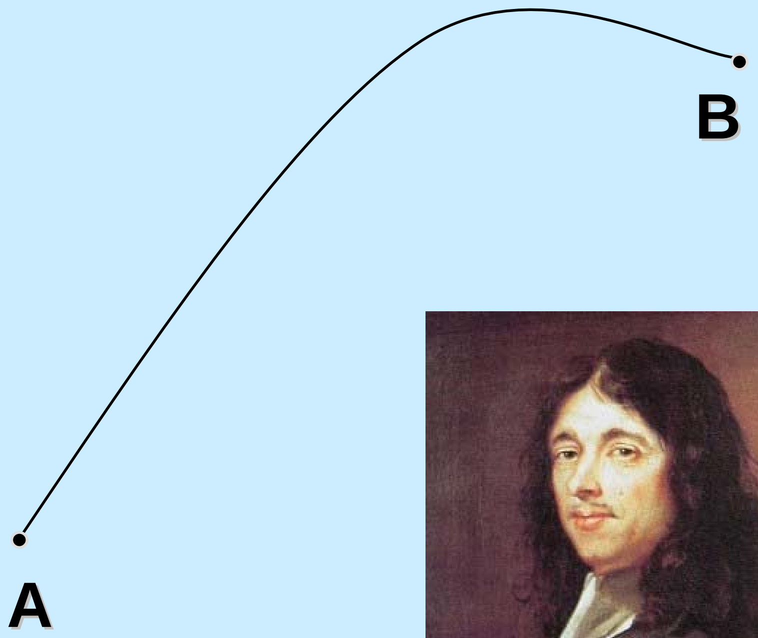
Single molecule fluorescence microscopy

Simplifications of light propagation



Ray propagation: Fermat's principle

The optical path length (taking into account the index of refraction along the path) is extremum.



$$L = \int_A^B n\,ds$$

$$\delta L = 0$$

Pierre de Fermat (1601-1665)

Snell's law (or 'loi de Descartes')

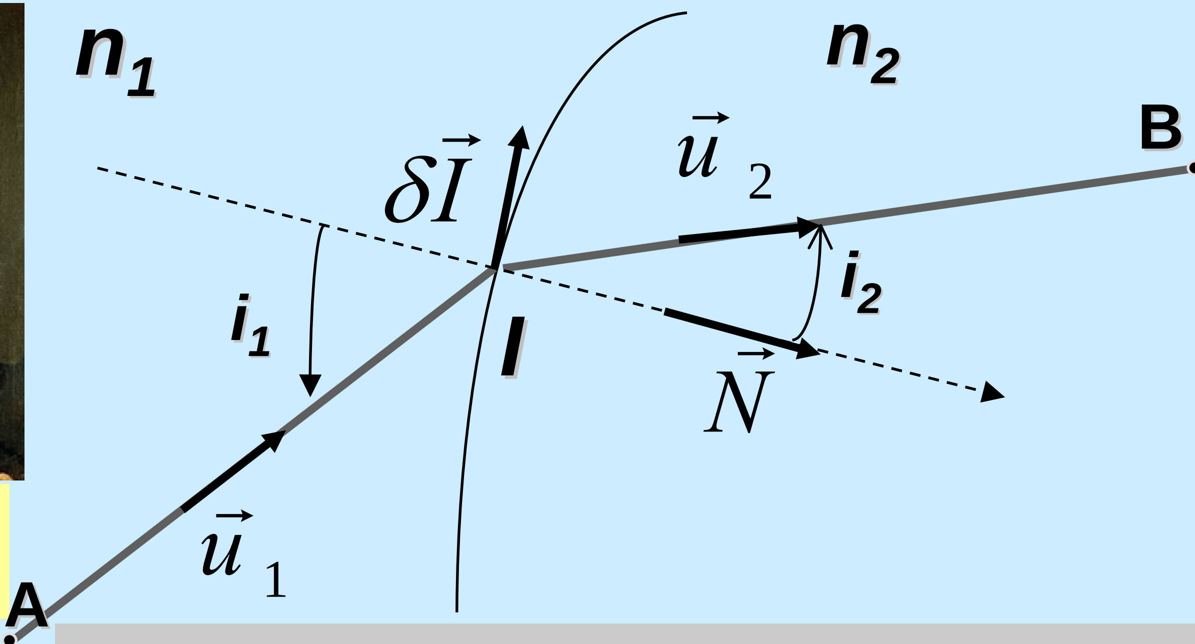
Fermat :

$$L(AB) = n_1 AI + n_2 IB$$

$$\delta L(AB) = n_1 \overrightarrow{u_1}\overrightarrow{\delta I} - n_2 \overrightarrow{u_2}\overrightarrow{\delta I} = \left(n_1 \overrightarrow{u_1} - n_2 \overrightarrow{u_2}\right)\overrightarrow{\delta I} = 0$$

$$n_1 \overrightarrow{u_1} - n_2 \overrightarrow{u_2} = a\overrightarrow{N}$$

René Descartes (1596-1650)



Willebrord Snell (1580-1626)

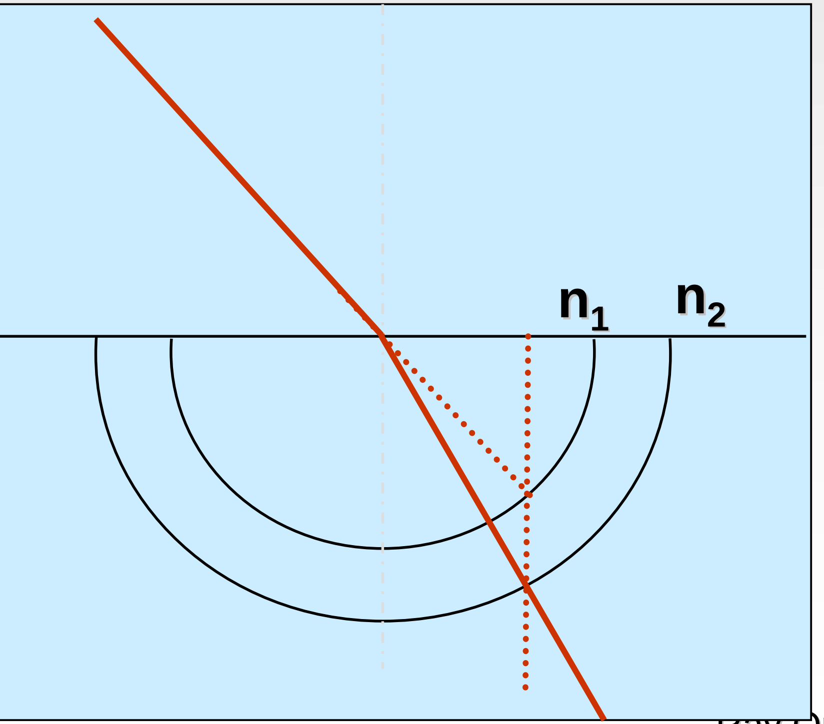
$$n_1\vec{u}_1 - n_2\vec{u}_2 = \left(n_1\cos(i_1) - n_2\cos(i_2)\right)\vec{N}$$

in the incidence plane: $n_1\sin(i_1) = n_2\sin(i_2)$

Construction of refracted rays

Based on Snell's law

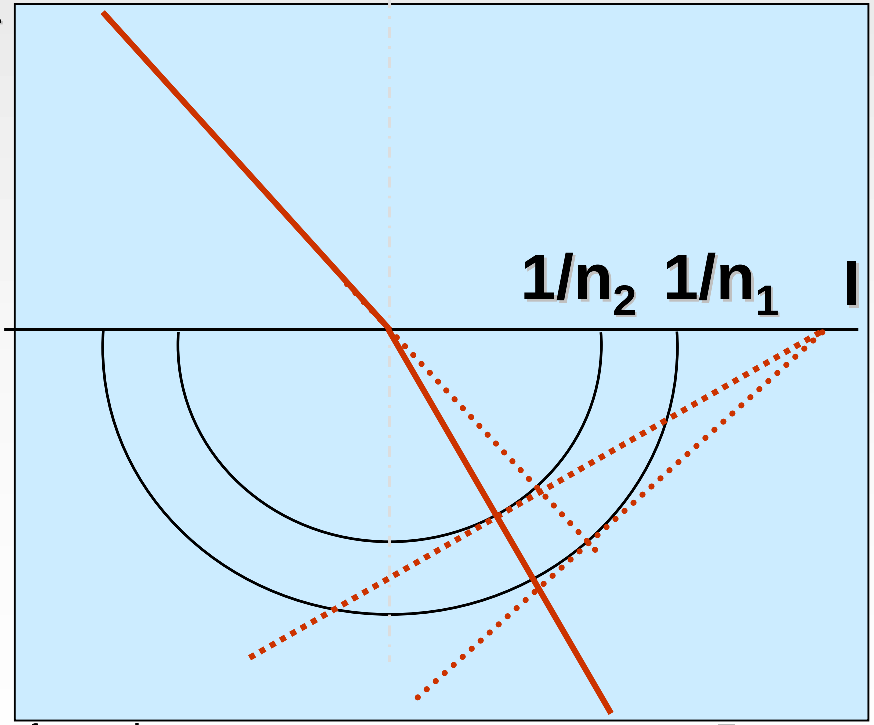
Index surfaces



$n_2 > n_1$

Based on Huyghens's theory

Velocity surfaces



Total internal reflection

$n_1 > n_2$

$n_1 \sin i_{critical} = n_2$

Glass-air interface :

$i_{critical} = 42°$

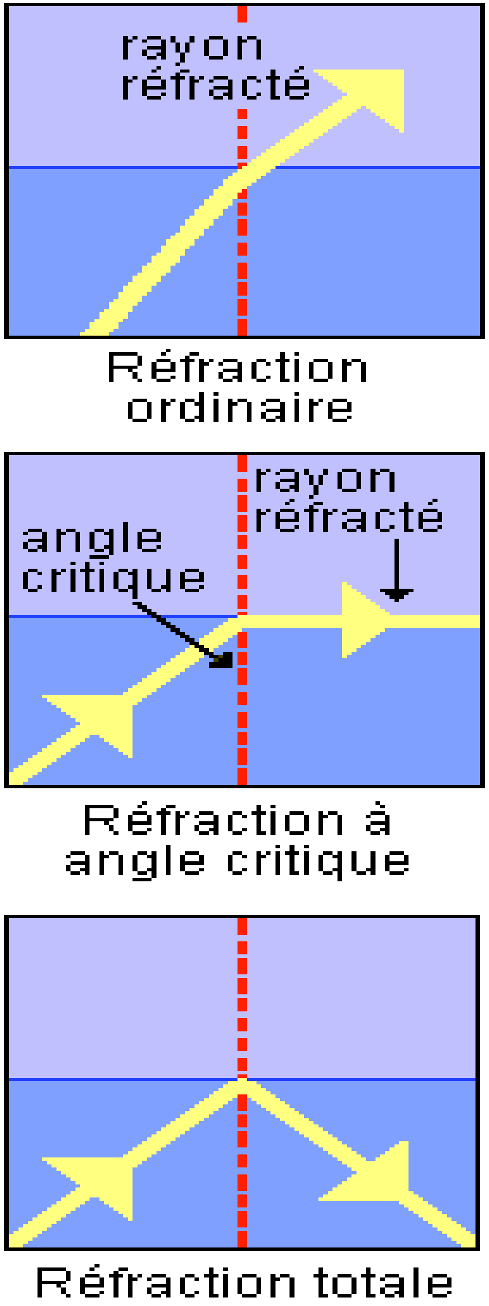
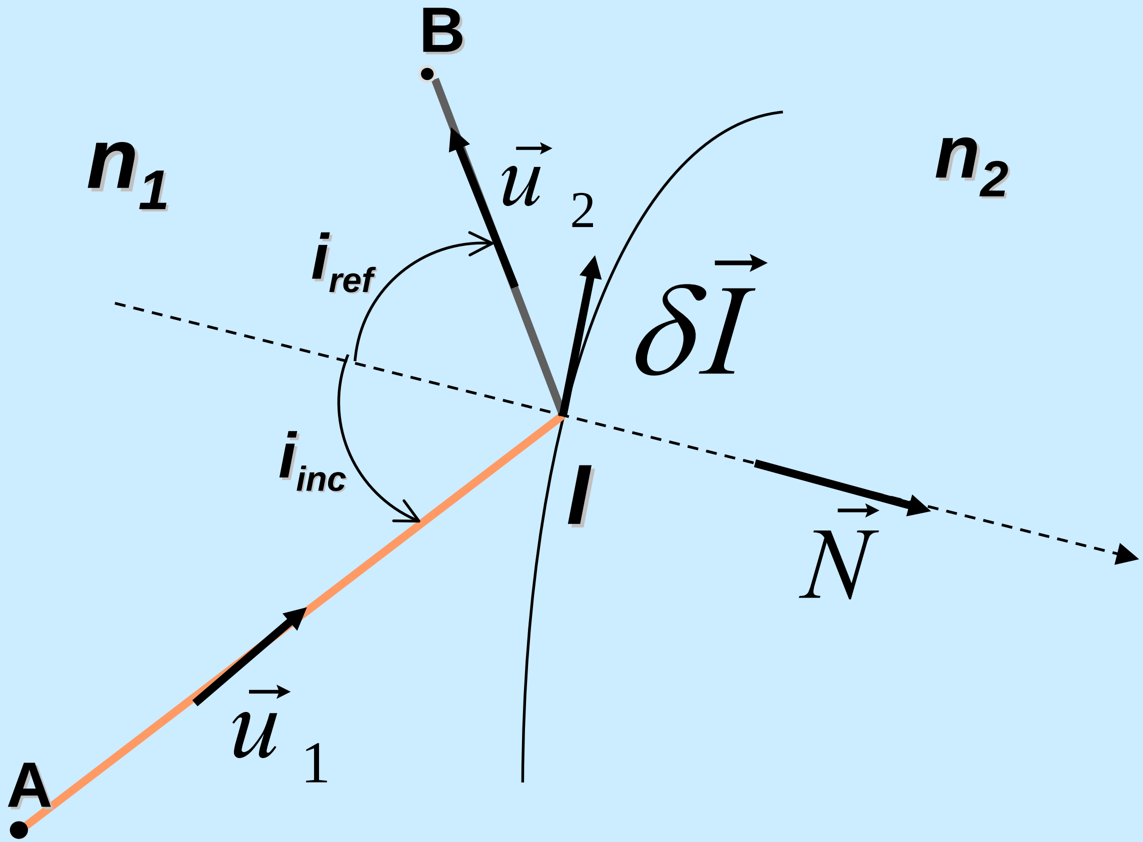


Figure 6
Angle critique de réfraction

Snell's law for reflection



$$\vec{u}_1 - \vec{u}_2 = 2\cos(i)\vec{N}$$

in the incidence plane: $i_{ref} = -i_{inc}$

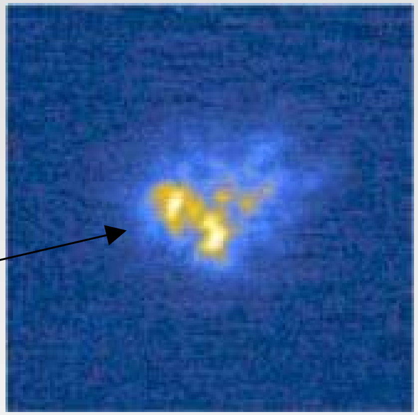
Image formation

Quality of an optical system

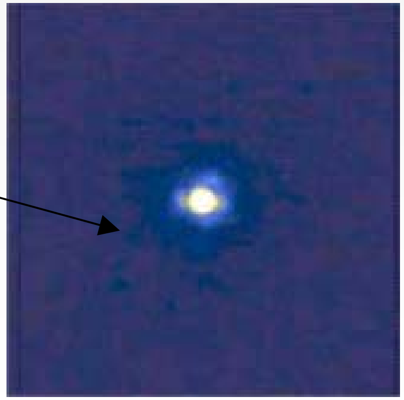
Stigmatism (perfect imaging) :

The image of a point source is a point.

EXAMPLE: image of a star



Without adaptive optics



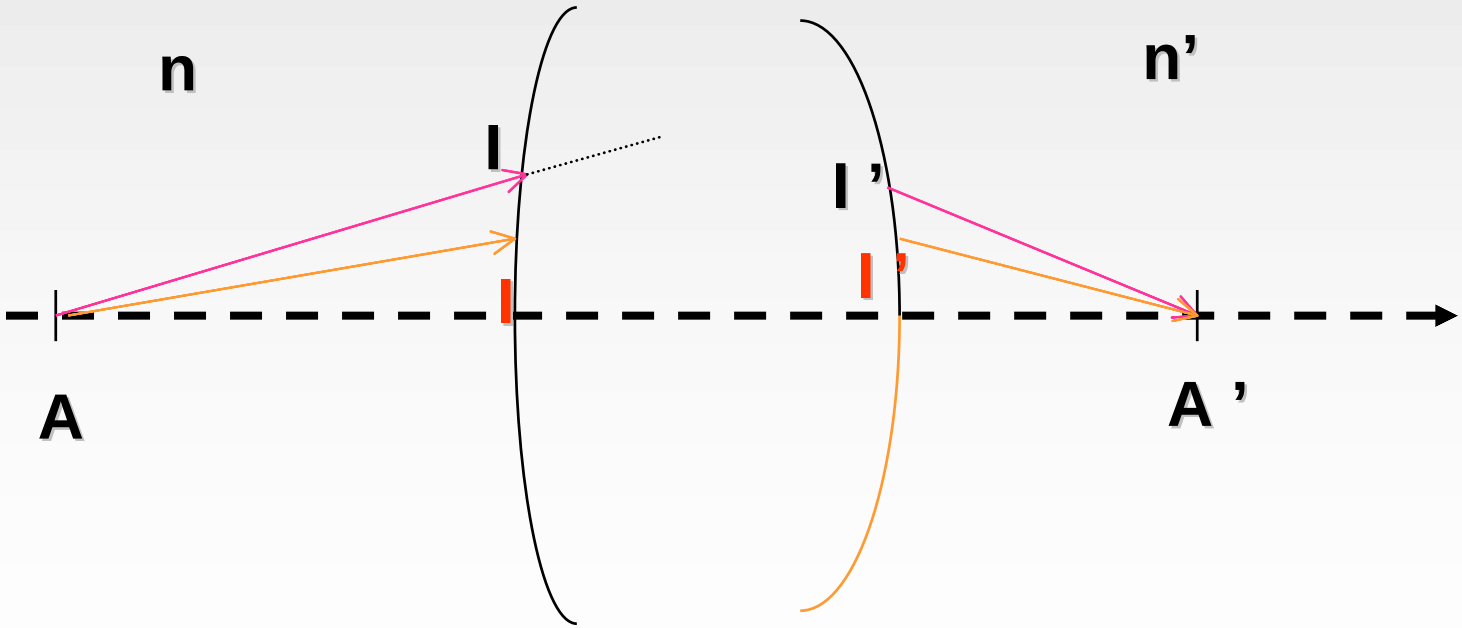
With adaptive optics

Stigmatic condition in terms of optical path :

If a system is perfectly stigmatic for A (object) and A'(image of A), then :

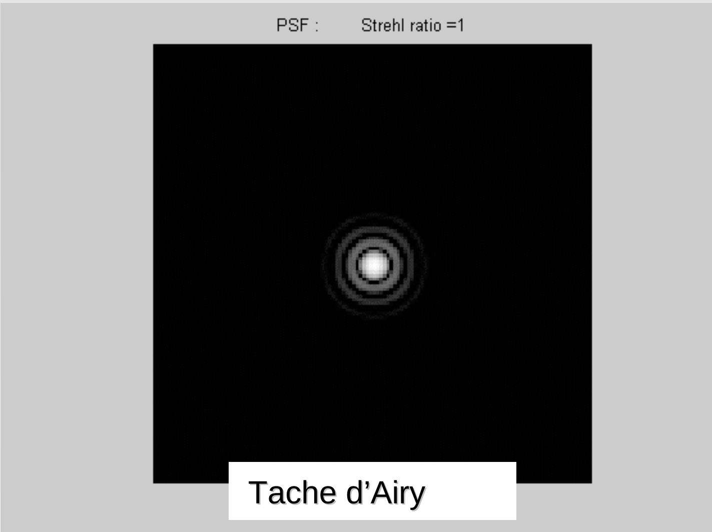
L (A A ') = Constant

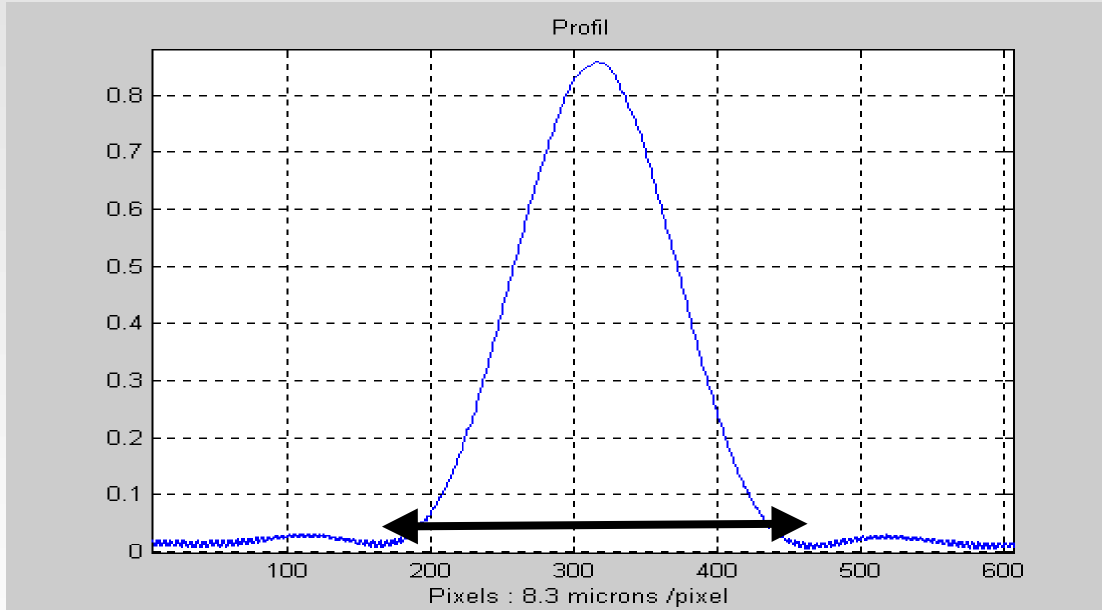
for any ray coming from A passing through the optical system (Fermat's principle).



Is perfect stigmatism really necessary?

NO, because even an ideal optical system is limited by diffraction





For a point source at infinity :

$$\Phi_{Airy} = 2.44\lambda N = 2.44\lambda \frac{f'}{D_{\text{entrance pupil}}}$$

Image of a point source : Airy function

Why perfect stigmatism is not necessary

An optical system is always limited by diffraction

+ there is the limitation due to the image detector :

Grain size (or pixel size) of the detector

•Some optical systems do not require perfect imaging !!

Lighting systems (search lights, condensers, road signs,..)

Other qualities of an optical system

- flat image (no field curvature)
- Constant magnification (no distortion)
- Achromatism
- Sufficient luminous flux
- Uniform illumination

Do simple systems make perfect images?

No, unfortunately !!!

Even a plane refractive surface or a ***spherical mirror***

Same for simple lenses: ***planconvex lens****(there is a better orientation),* ***biconvex lens***

Plane refractive surface

Spherical mirror

Object at infinity on axis

center of
curvature

Object

Are there simple optical systems that are perfectly stigmatic ?

Yes!

but only for a ***specific pair*** *of conjugate points*

Stigmatic points for mirrors

- ☞ Only the plane mirror is always stigmatic, other mirrors are only stigmatic for specific points

  Spherical (center), parabolic (object at infinity), elliptical and hyperbolic mirrors (foci of the conical forms)

- ↗ *Application to telescopes*

Stigmatic points for a refractive surface

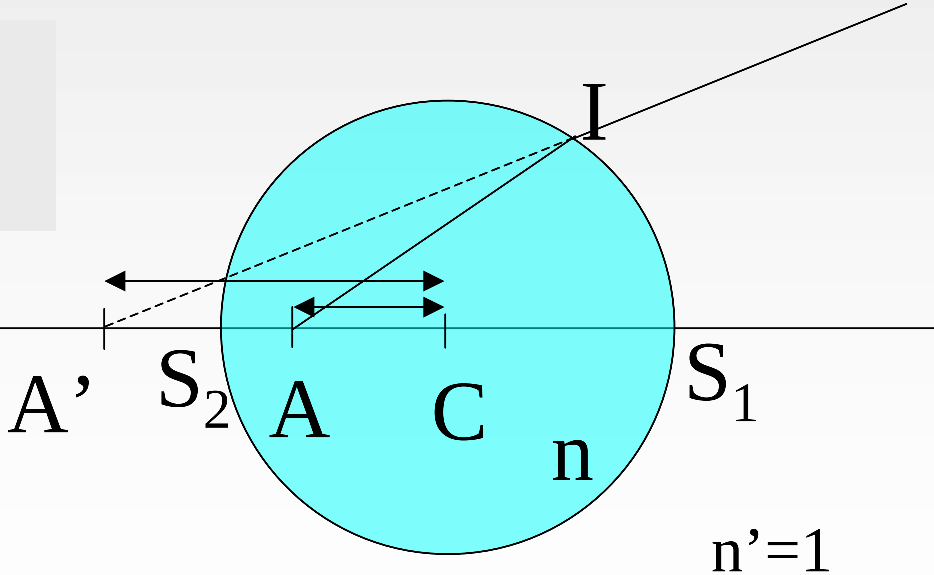
Perfect stigmatism for a refractive surface: n**AI** + n'**IA'**=K **(cst)**

K≠0: Descartes Ovoïds

K=0: IA/IA'=cst $\Rightarrow$ spherical surface

A and A' : one real and one virtual

one inside, one outside the sphere



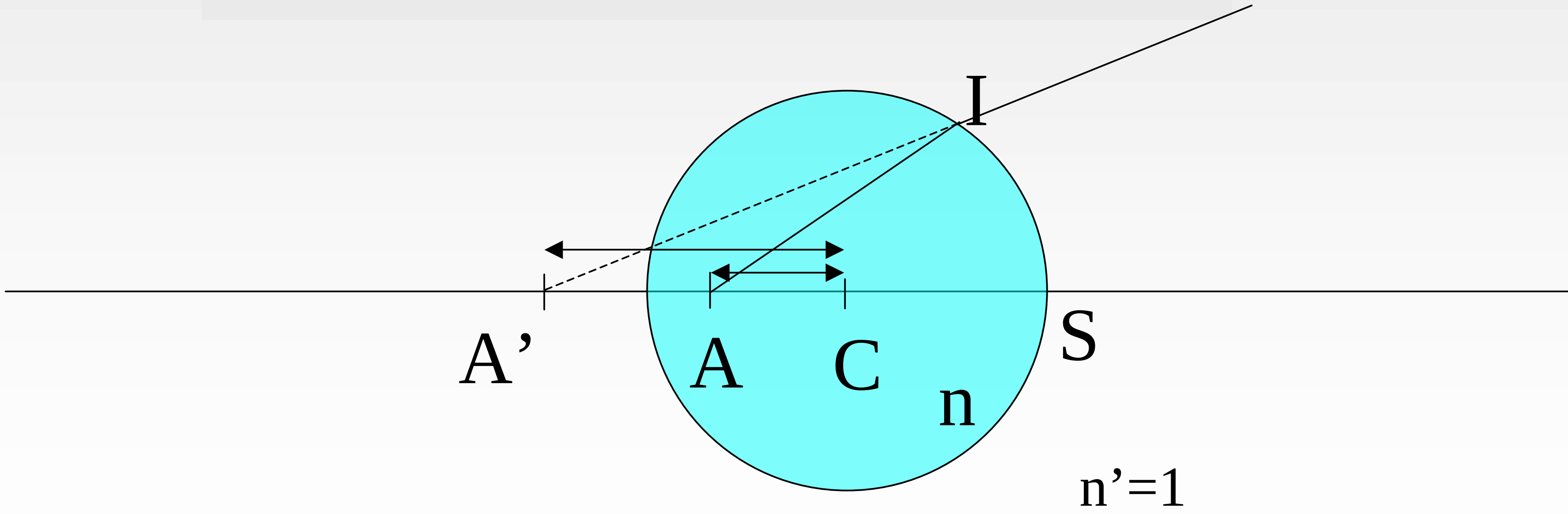
$S_1A/S_1A' = S_2A/A'S_2 = n'/n$

Stigmatic points for a spherical refractive surface

Weierstrass or aplanetic points:

R= **SC**, **CA =R.**n'/n,

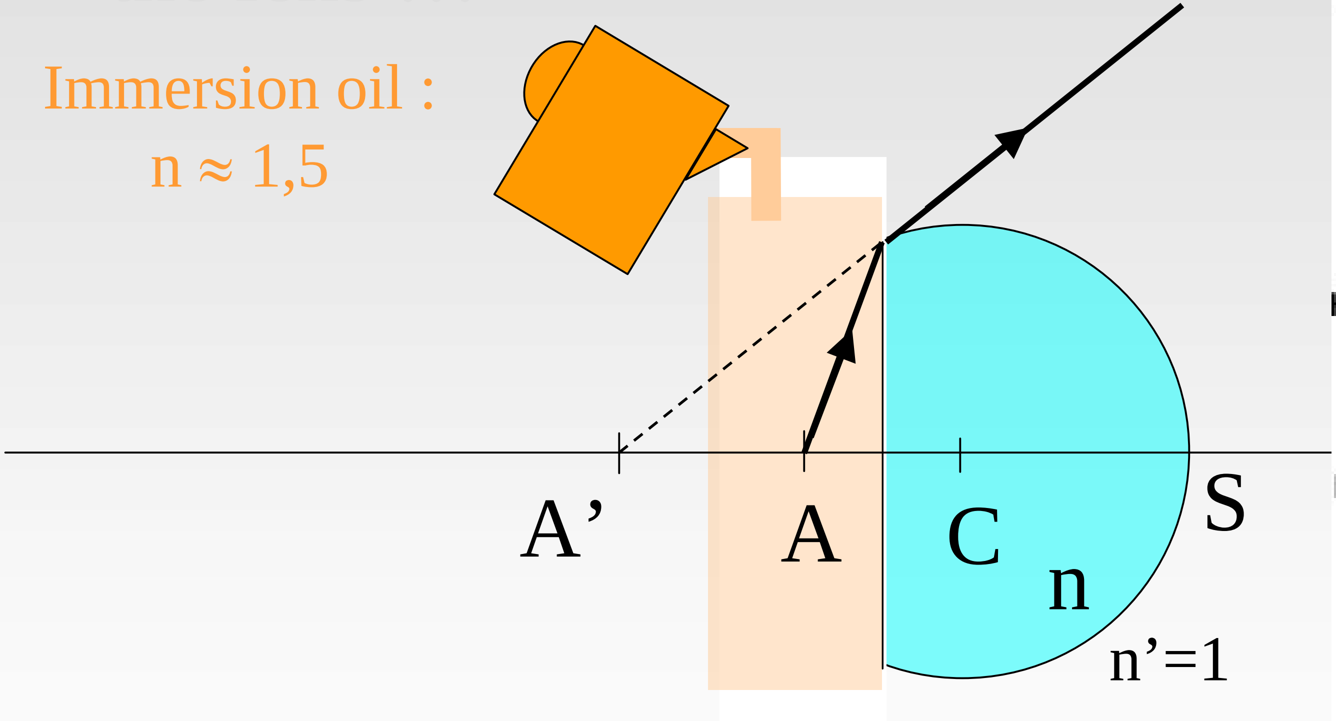
CA'=R.n/n'

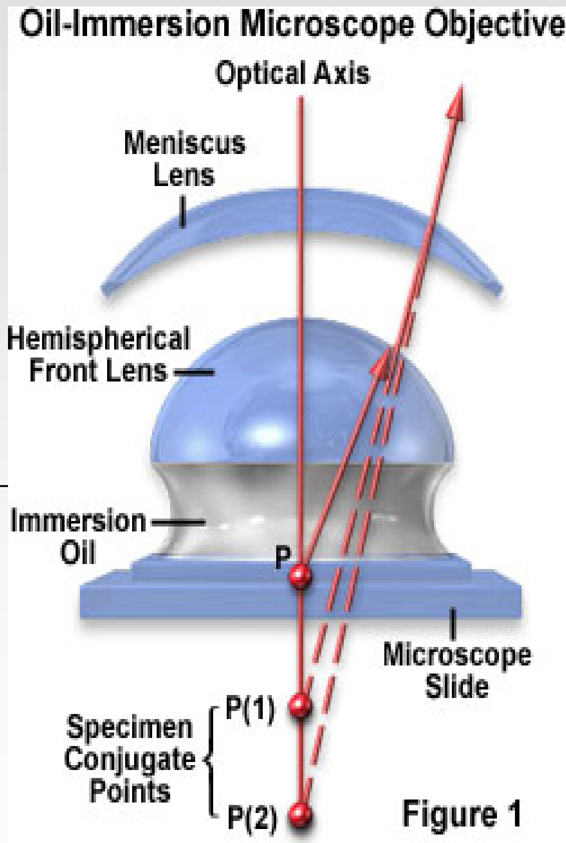


Application to microscope objectives

Problem : place the object at point A… **inside** the lens !!!

Immersion oil : $n \approx 1{,}5$





Large aperture angle in the object plane, reduced after the lens

Other stigmatic lenses

Aspherical surfaces or aspherical lenses

Are there perfect optical systems for several pair of conjugate points?

No , unfortunately !!!

BUT

We can maintain ***approximate stigmatism*** *:*

- either in a plane orthogonal to the axis (aplanetism)

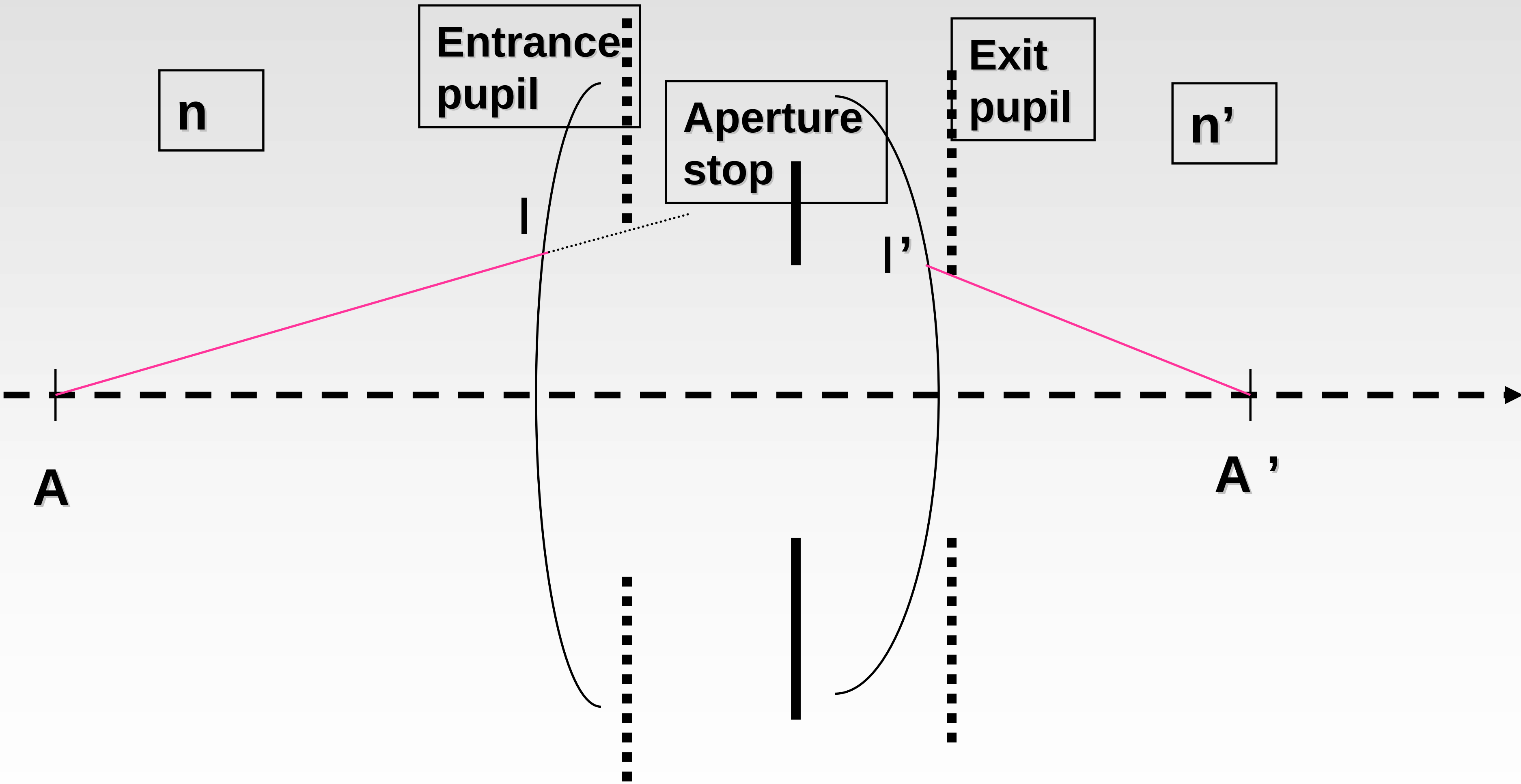
- or along the axis (Herschel Condition)

Aplanetic single surface must be spherical

Approximate stigmatism in a plane: aplanetism
Abbe sine condition

Hypothesis : centered optical system perfectly stigmatic for A and A '

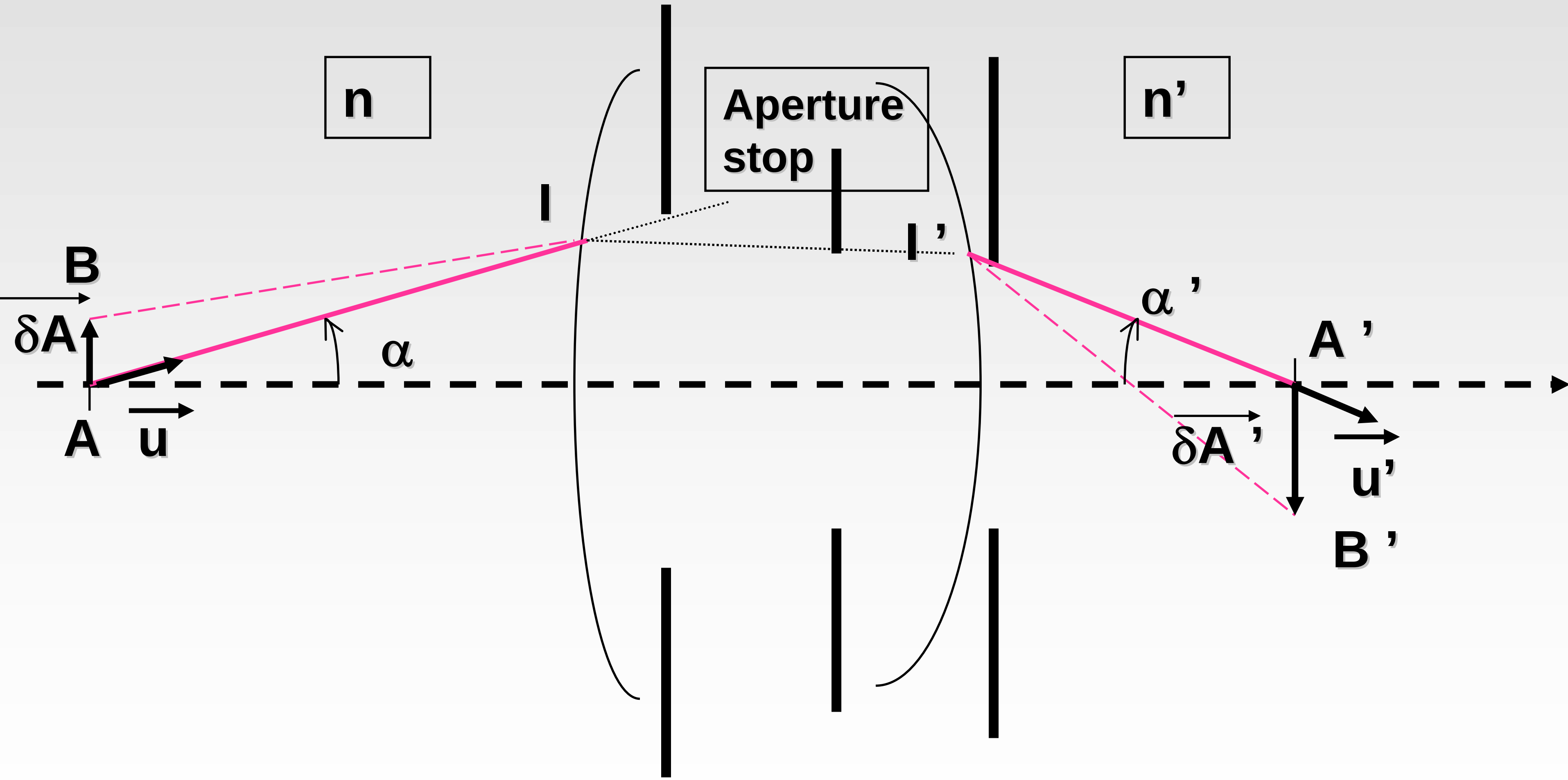
Fermat : $L(AA') = cst \quad \forall \, I$



The system is perfectly stigmatic for B and B' if :

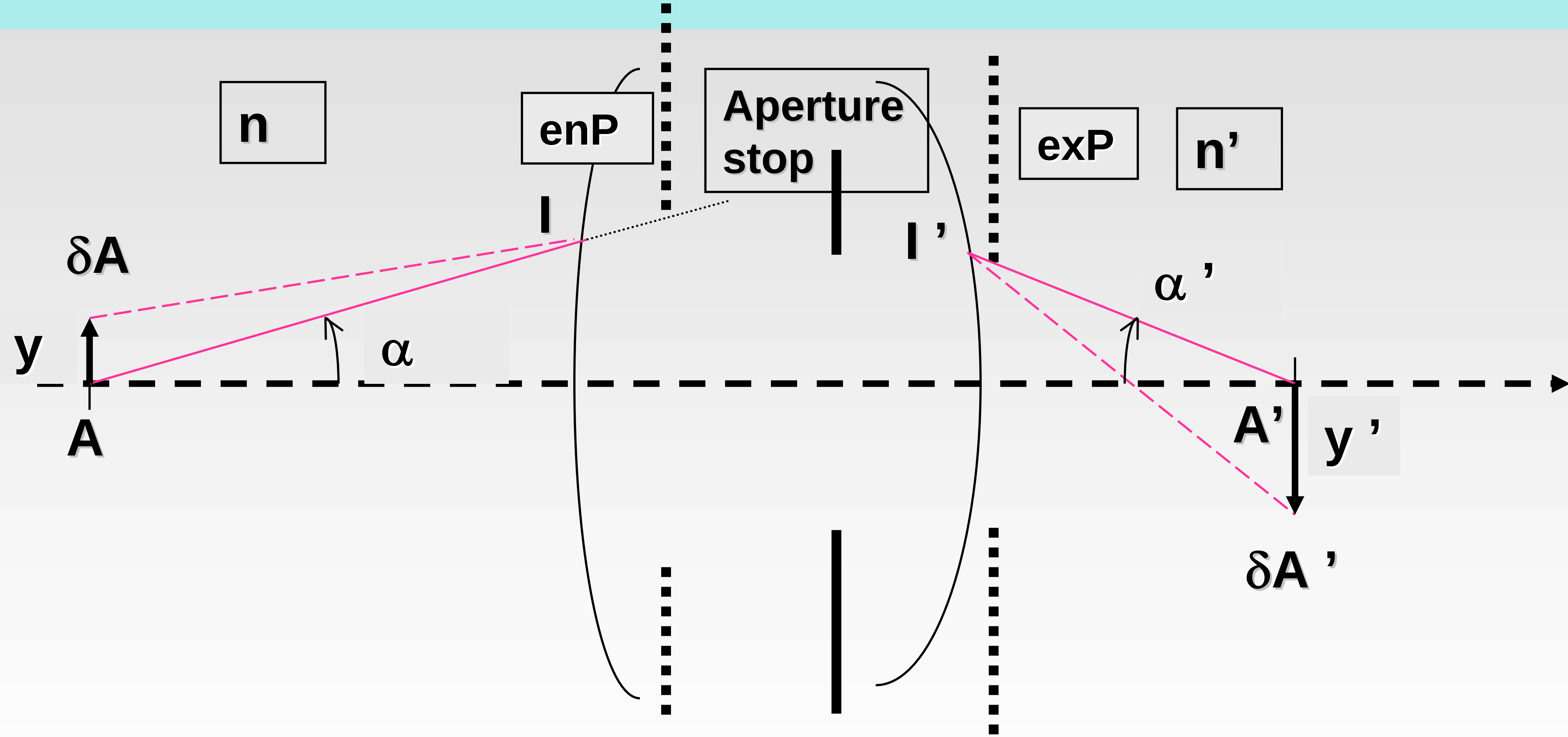
$$L(BB') = cst \qquad \forall\ \mathrm{I}$$

Thus : $\Delta L = L(BB') - L(AA') = cst$



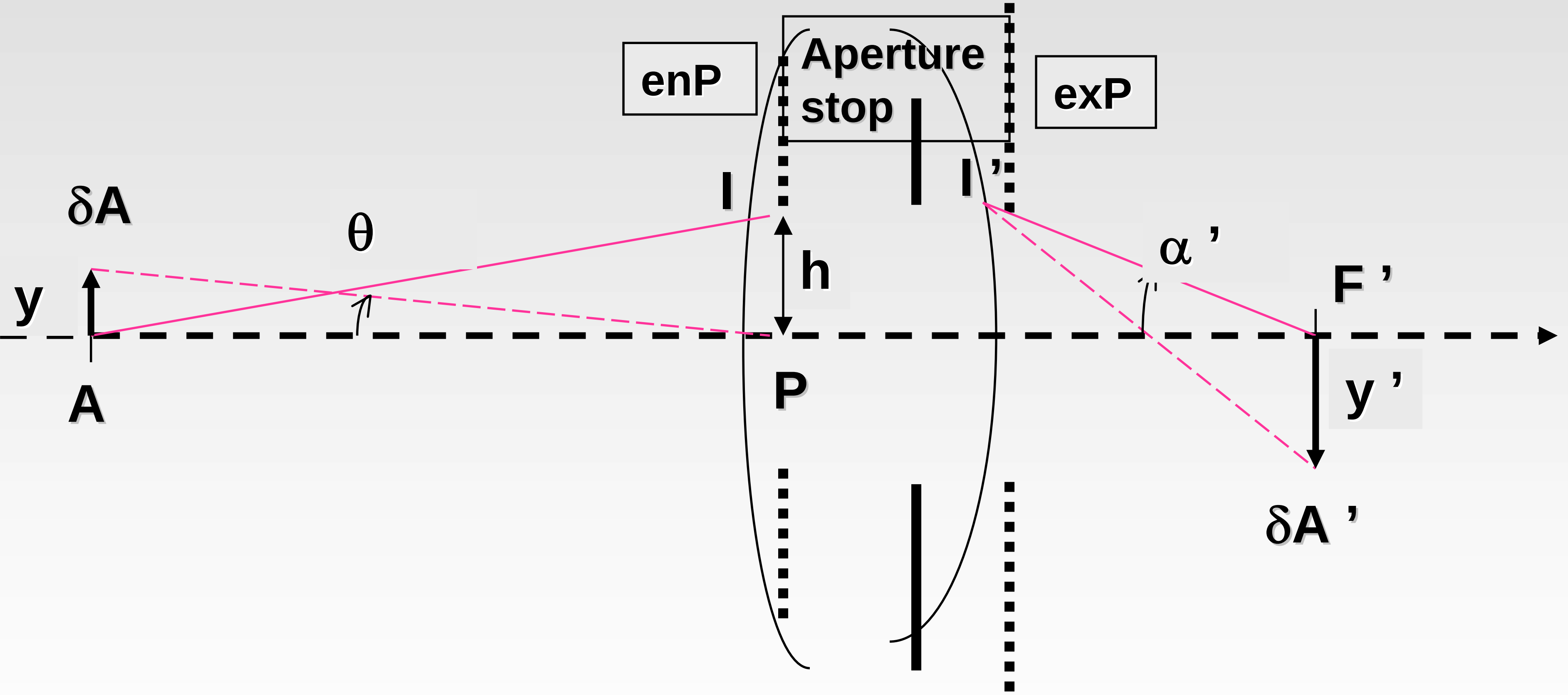
$$\Delta L = -n\,\overrightarrow{\delta A}\cdot\vec{u} + n'\,\overrightarrow{\delta A'}\cdot\vec{u}' = cst$$

Abbe sine condition : a **fundamental theorem** for imaging optical systems



$$n y \sin\alpha = n' y' \sin\alpha'$$

Abbe condition for an object at infinity



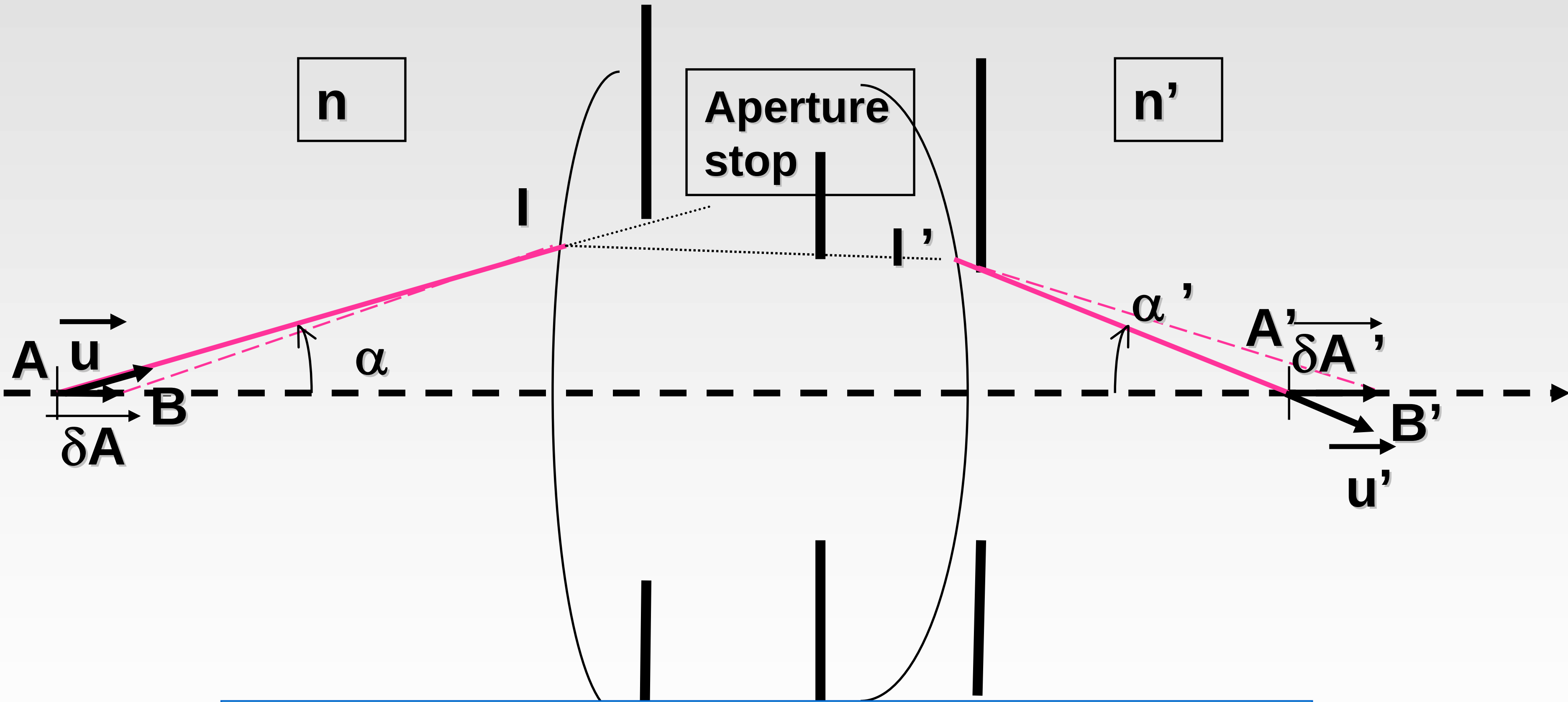
$$-nh\theta = n'y'\sin\alpha'$$

Approximate stigmatism **along the axis**: Herschel’s condition

Now B and B' are slightly displaced along the optical axis :

$$L(BB')=cst \qquad \forall \; I$$

Thus : $\Delta L = L(BB') - L(AA') = cst$



$$\Delta L = -n\overrightarrow{\delta A}\cdot\vec{u} + n'\overrightarrow{\delta A'}\cdot\vec{u}' = cst$$

$$-n\delta x\cos\alpha + n'\delta x'\cos\alpha' = -n\delta x + n'\delta x' \quad (\alpha=\alpha'=0)$$

Herschell condition

Condition for almost perfect imaging along the optical axis:

$$n\,\delta x \sin^2(\alpha/2) = n'\,\delta x' \sin^2(\alpha'/2)$$

$$Herschel \Rightarrow \left|\sin(\alpha/2)/\sin(\alpha'/2)\right| = cst$$

$$Abbe \Rightarrow \sin\alpha'/\sin\alpha = cst$$

$$Herschel + Abbe \Rightarrow \left|\cos(\alpha/2)/\cos(\alpha'/2)\right| = cst = 1\ (\alpha = \alpha' = 0)$$

$$\Rightarrow \alpha = \pm\alpha'$$

Abbe and Herschel conditions cannot be both satisfied in general

Paraxial approximation

- Small object AND Small aperture :
  - ↓ (y,δx)
  - ↓ α

Linearized form of Abbe and Herschel conditions:

$$n y \alpha = n' y' \alpha'$$

Lagrange invariant

$$n \delta x \, \alpha^2 = n' \delta x' \alpha'^2$$

Satisfied for all conjugate points!