Optical instruments with mirrors

part 1 : single plane and spherical mirrors

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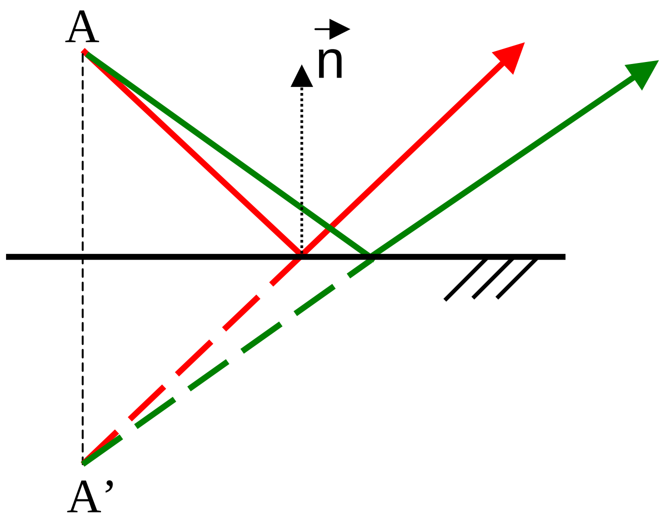
Plane mirrors

Basic properties



Reflected ray :

• in the plane of incidence

• Symmetric to the incident ray with respect to the surface of the mirror

Image A’ of A :

• perfectly stigmatic

• A’ = symmetric to A with respect to the plane of the mirror (real <> virtual)

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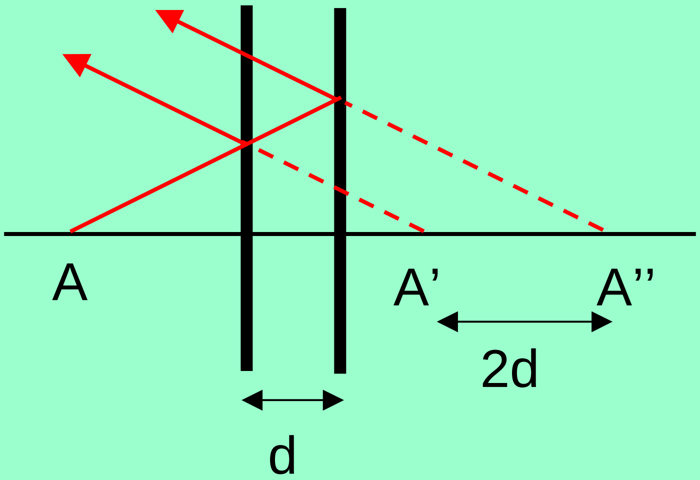
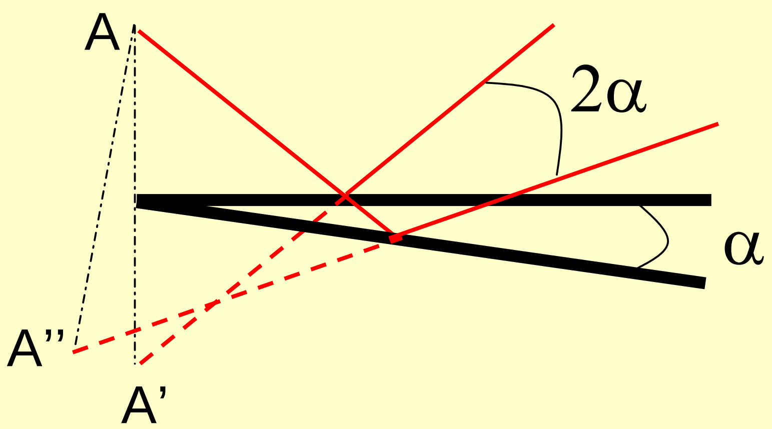
Displacement of a plane mirror

Translation d of the mirror



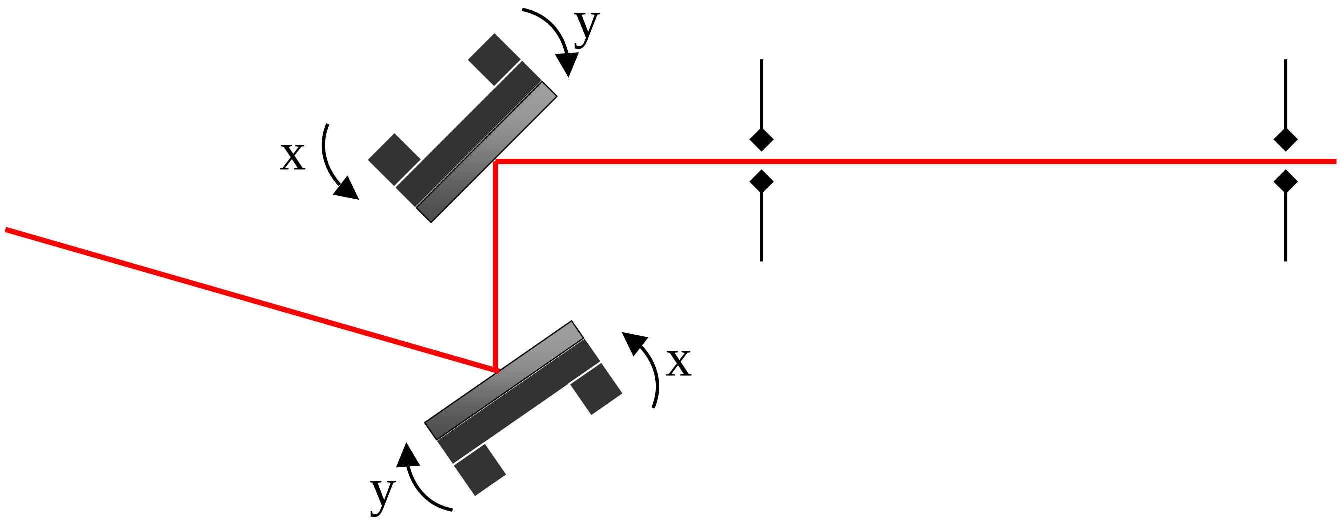
Rotation by an angle α of the mirror



Systems with plane mirrors

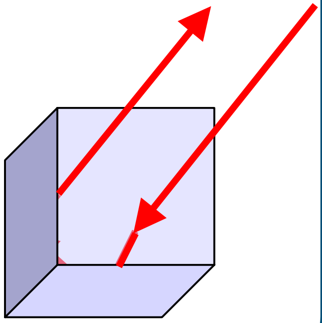
- **2 adjustable plane mirrors**

  ➔ Adjusting a beam in any direction



- **3 mirrors in a corner cube**

  ➔ The emerging ray is always parallel to the incident ray



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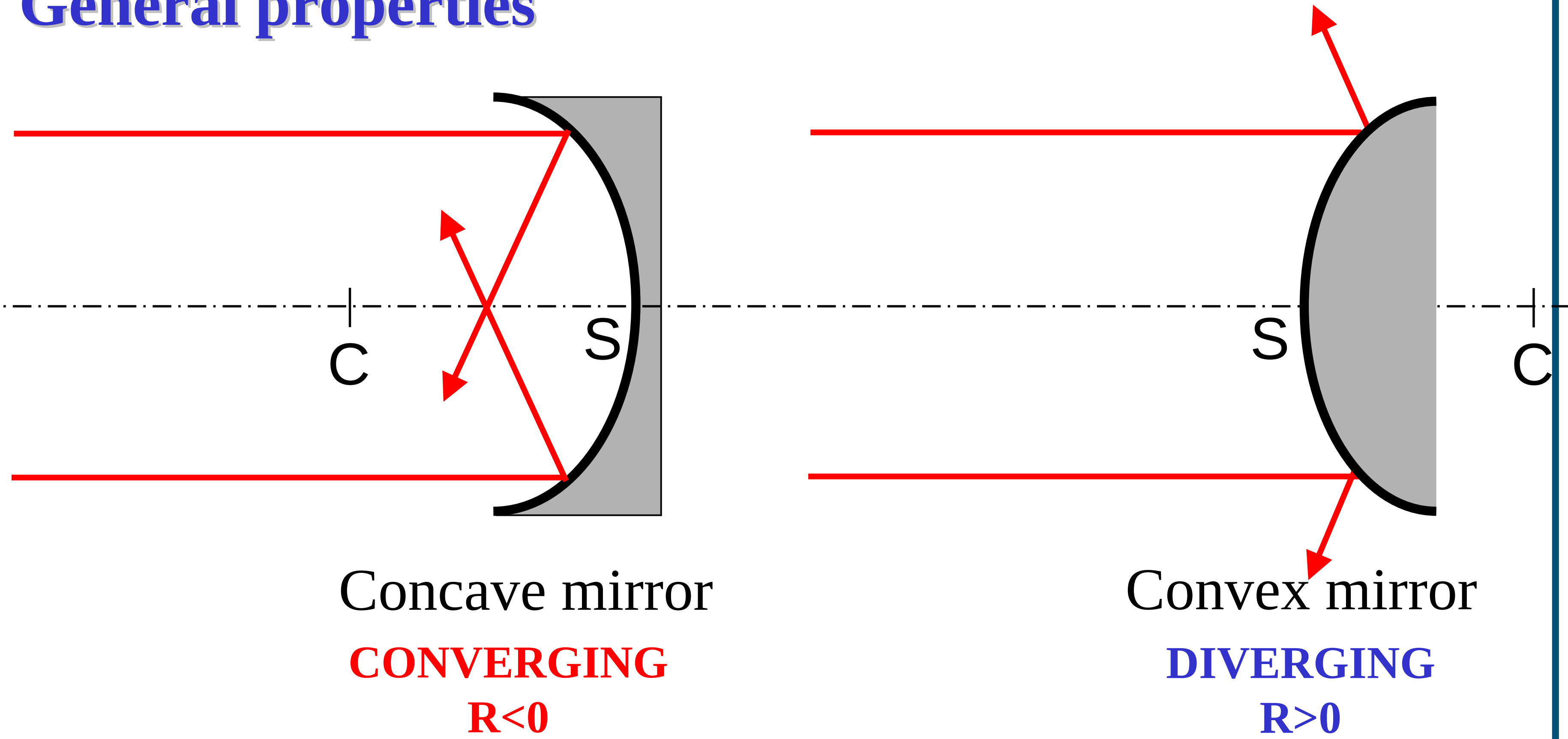
Spherical mirrors

General properties

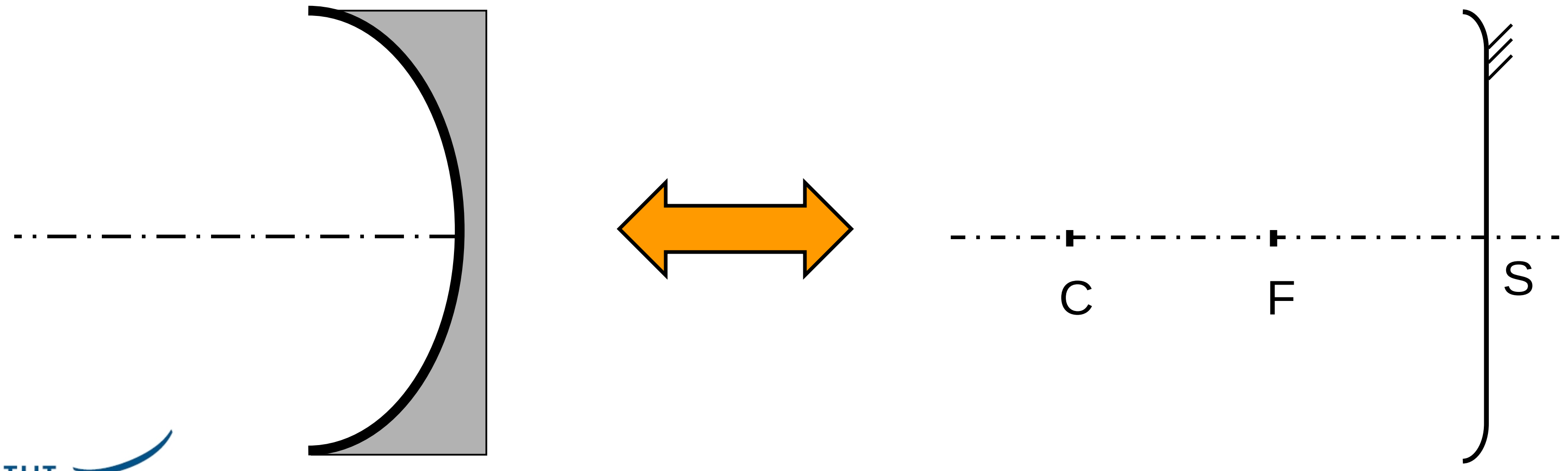


- Notations : *algebraic* radius of curvature $R = \overline{SC}$

- Positive sign for the incident direction of propagation

Paraxial approximation

Spherical mirror ⇔ tangential plane + vertex S + center C



Gaussian formula for imaging

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$$

Newton formula for imaging

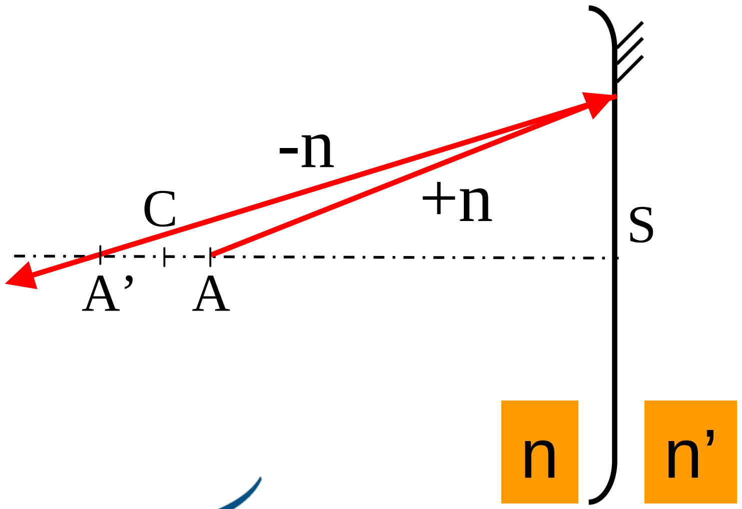
$$\overline{FA} \, . \, \overline{F'A'} = + f^2$$

Beware of the signs!

One way to figure out the signs: an analogy with an air/glass surface

For a mirror, to take into account the change of direction of the reflected ray

➔ ***negative*** index **-n** for the reflected ray



$$\frac{n'}{\overline{SA'}} - \frac{n}{\overline{SA}} = \frac{n'-n}{\overline{SC}}$$

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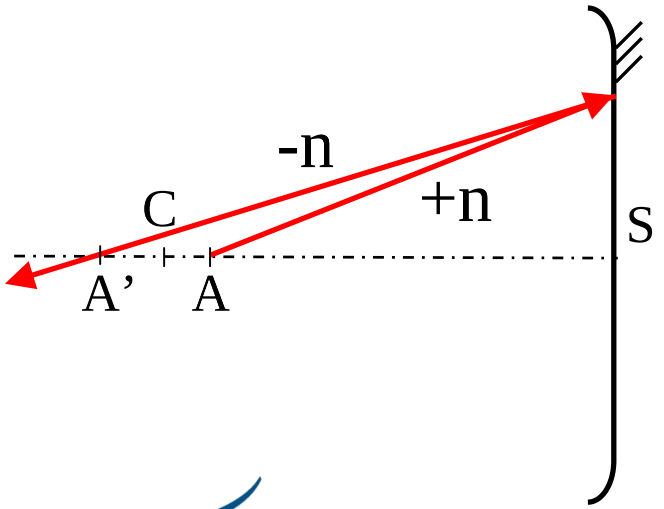

$$\frac{-n}{\overline{SA'}} - \frac{n}{\overline{SA}} = \frac{-n-n}{\overline{SC}}$$

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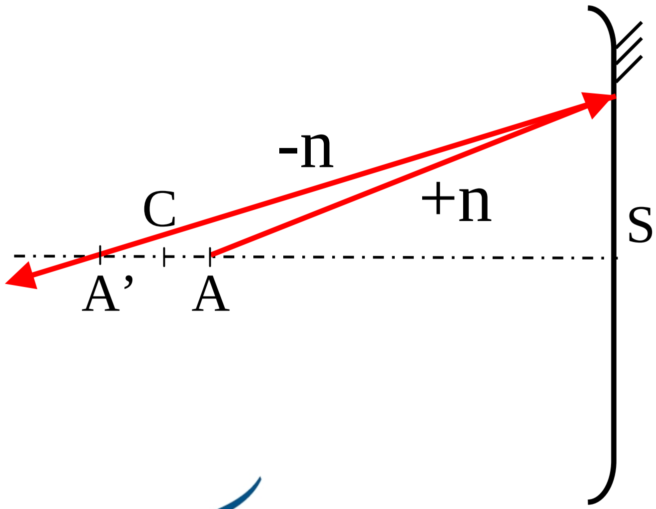

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$$

Formula valid for any sign convention (as long as you always keep the same one…)

Cardinal points

Principal points : $g_y = 1$ ➔ H = H' = S

Focal points ➔ F = F' = middle of [SC]

$\overline{SA} = \infty$ $\quad$ $\overline{SF'} = \overline{SC}/2$

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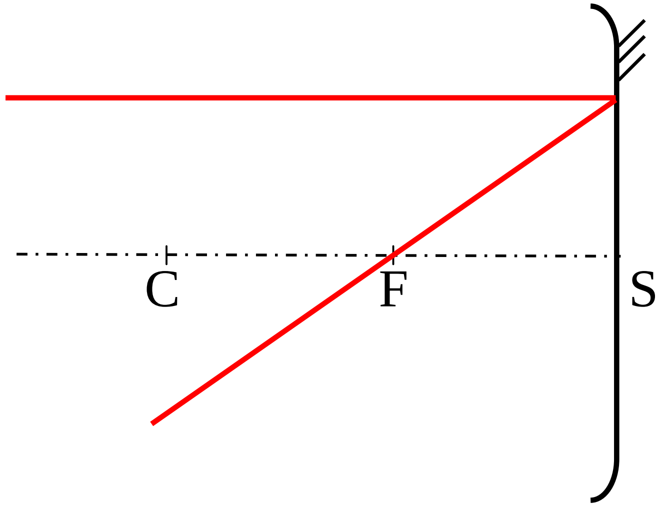


Ray tracing

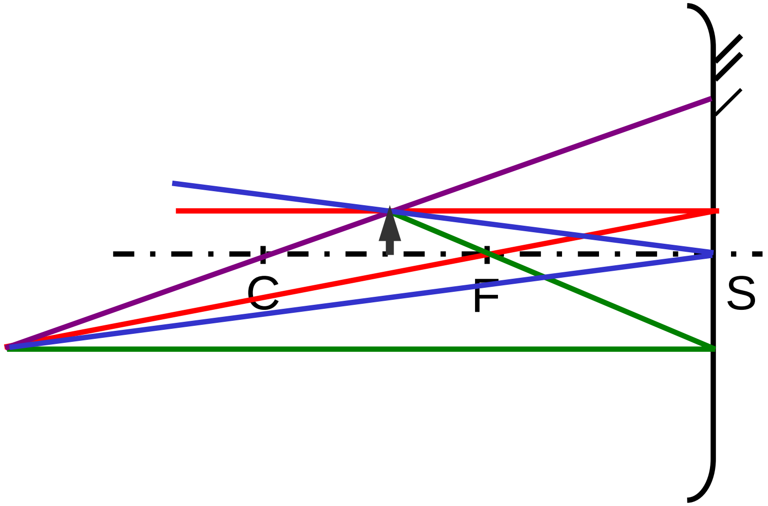
From:

the center ➔ undeflected

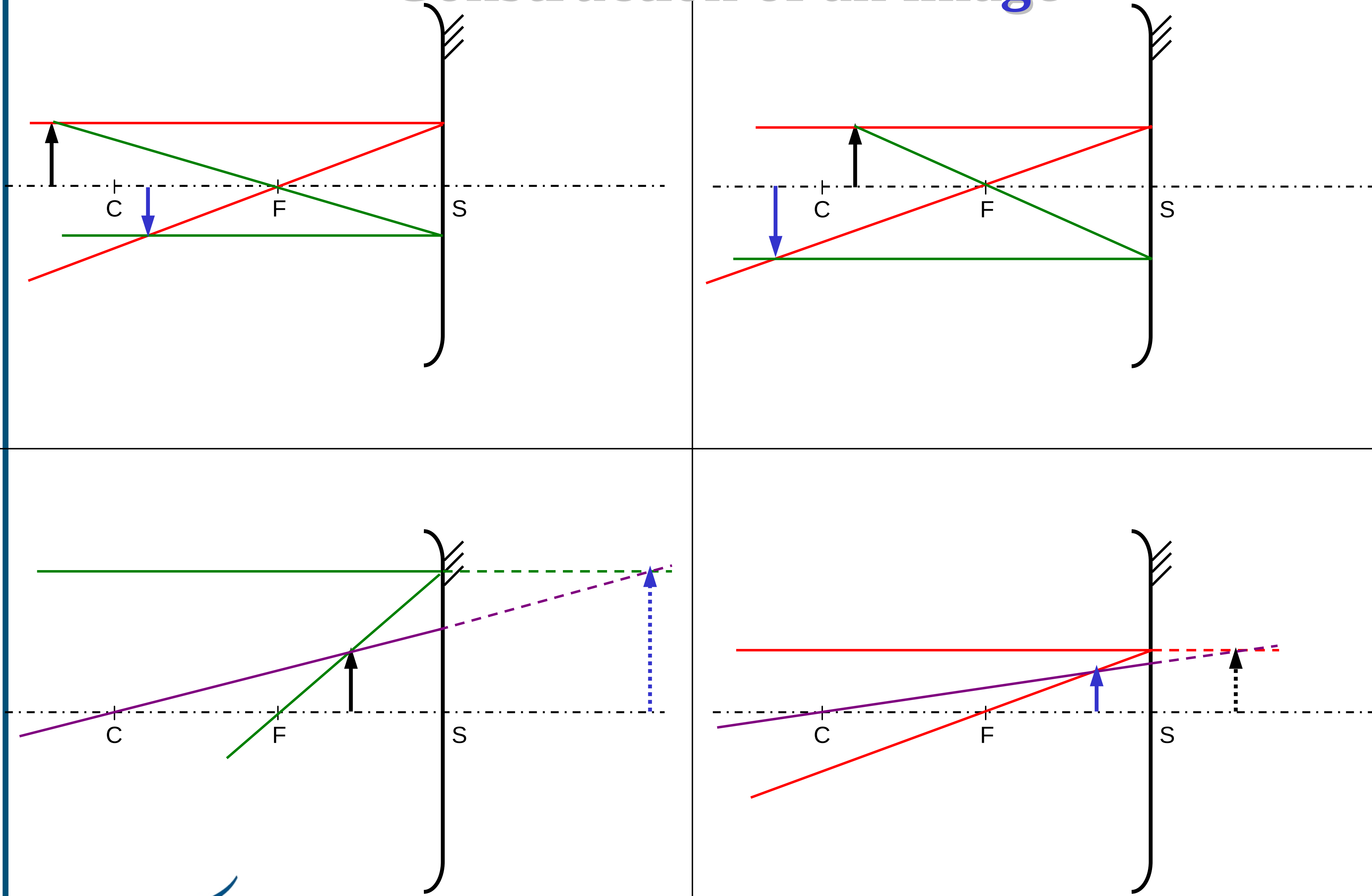
the foci ➔ infinity

infinity ➔ foci

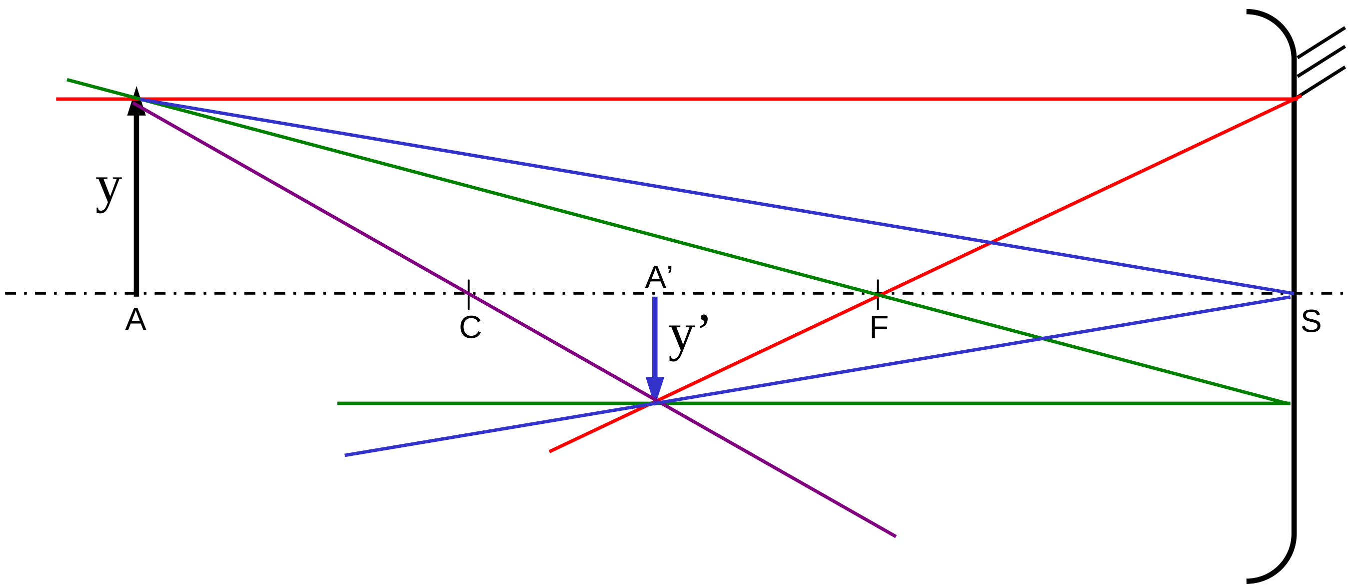
the vertex S ➔ symmetric with respect to the optical axis



Construction of an image



Transverse magnification



$$g_y = \frac{y'}{y}$$

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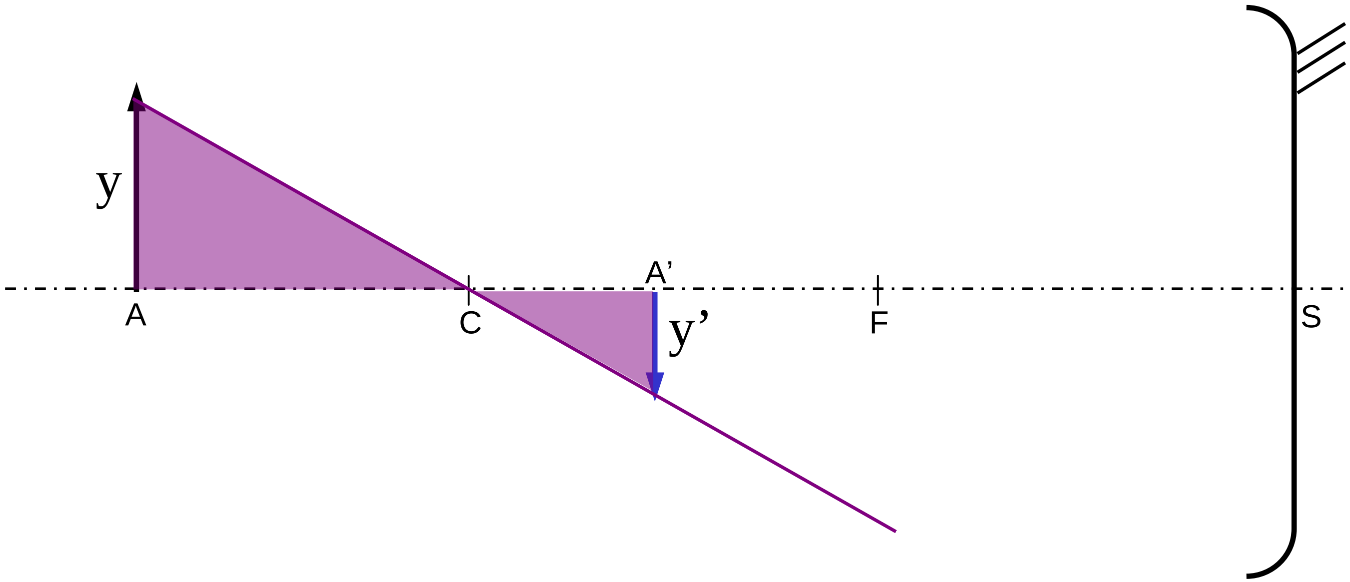
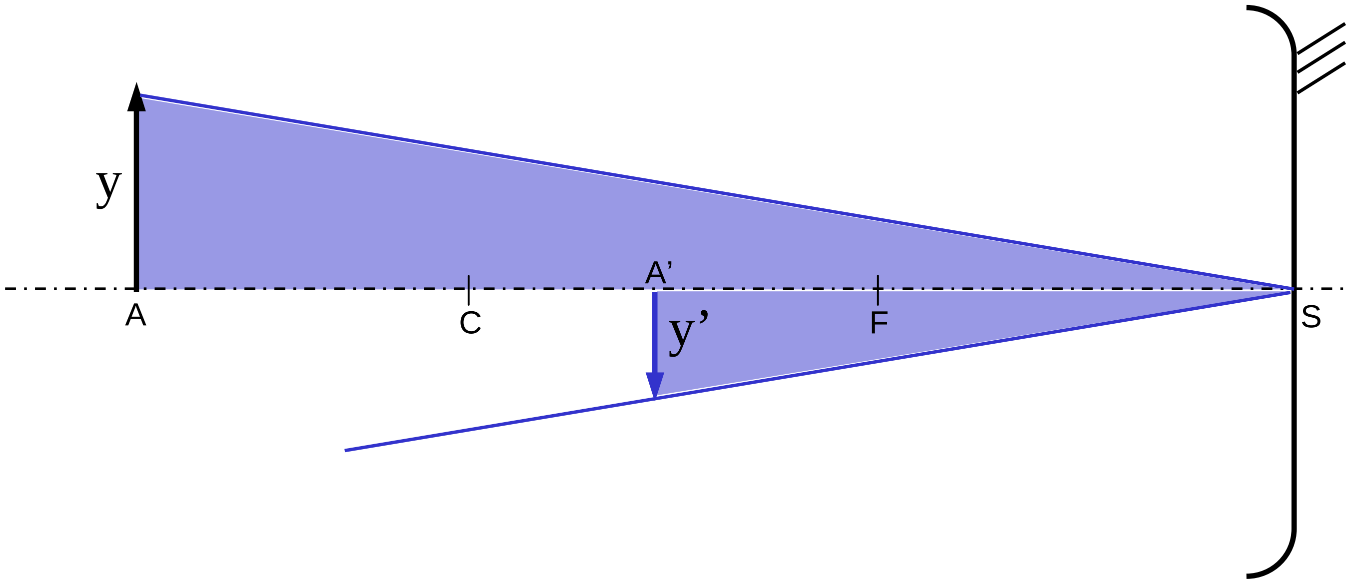
Transverse magnification refered to C



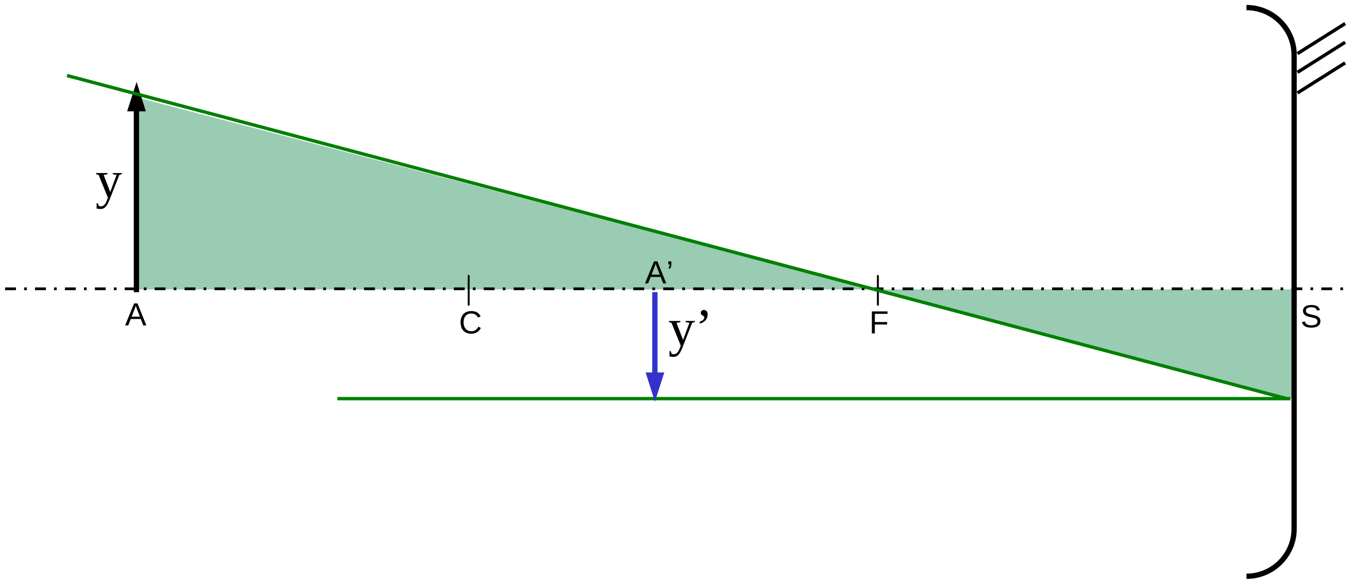
$$g_y = \frac{y'}{y} = \frac{\overline{CA'}}{\overline{CA}}$$

Transverse magnification refered to S



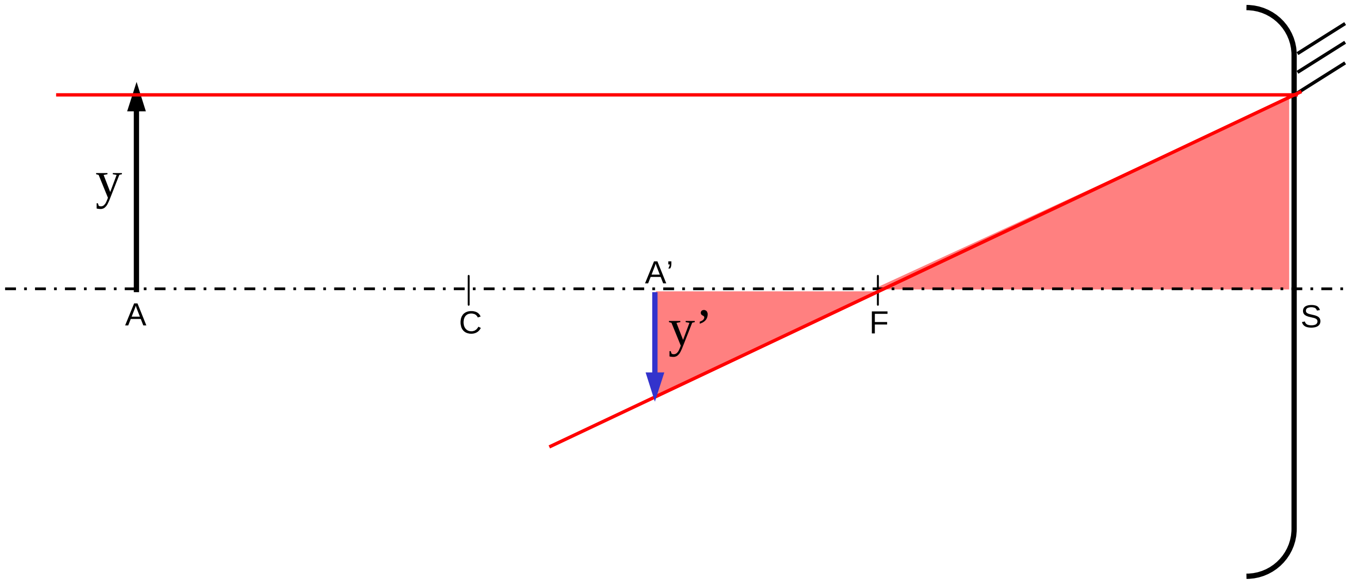
$$g_y = \frac{y'}{y} = \frac{\overline{CA'}}{\overline{CA}} = \frac{-\overline{SA'}}{\overline{SA}}$$

Newton formula for transverse magnification



$$g_y = \frac{y'}{y} = \frac{\overline{CA'}}{\overline{CA}} = \frac{-\overline{SA'}}{\overline{SA}} = \frac{-f}{\overline{FA}}$$

Newton formula for transverse magnification



$$g_y = \frac{y'}{y} = \frac{\overline{CA'}}{\overline{CA}} = \frac{-\overline{SA'}}{\overline{SA}} = \frac{-f}{\overline{FA}} = \frac{-\overline{FA'}}{f}$$

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Angular magnification

- Either through similar construction as for g_y
- Or using the relationship between g_y and g_α

➔ **Lagrange invariant** (~Abbe)

$$y\,\alpha = -\,y'\,\alpha'$$ ⟵ $n' = -n$!

➔ $g_y = -\,1 / g_\alpha$

Longitudinal magnification

Obtained by differentiating the Gaussian formula

$$g_x = -g_y^2 < 0$$

Image and object move in opposite direction !

Optical instruments with mirrors

part 2: reflective telescopes and catadioptric systems

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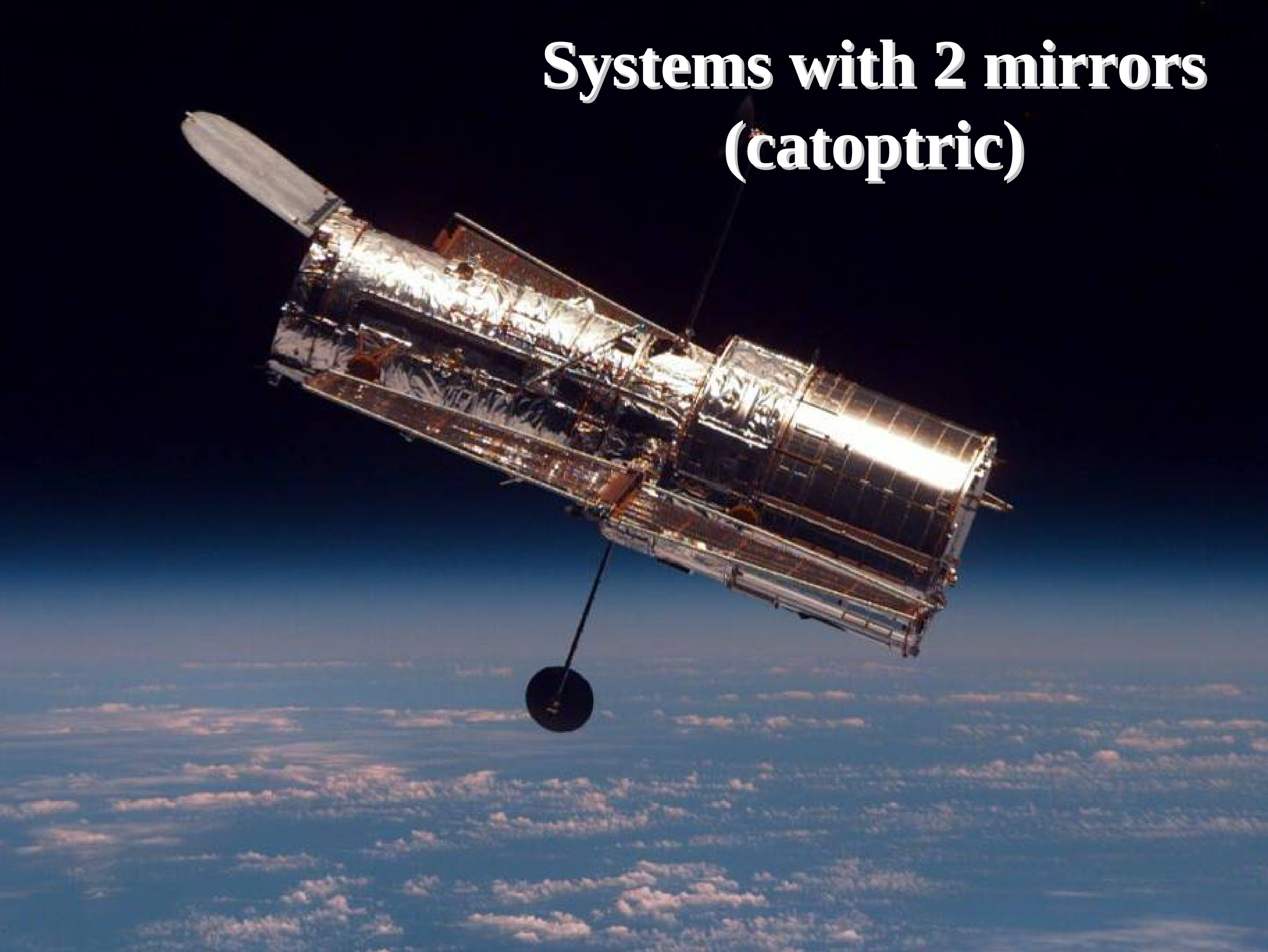
Glossary

For an optical system consisting of …

- only lenses → dioptric
- only mirrors → catoptric
- mirrors and lenses → catadioptric

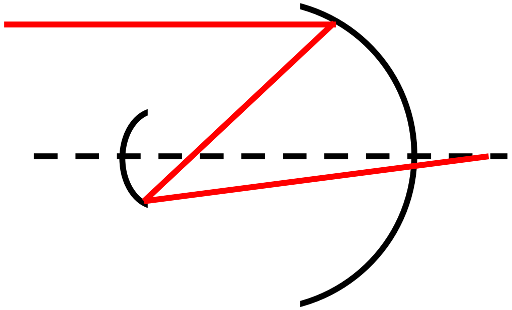
Systems with 2 mirrors (catoptric)

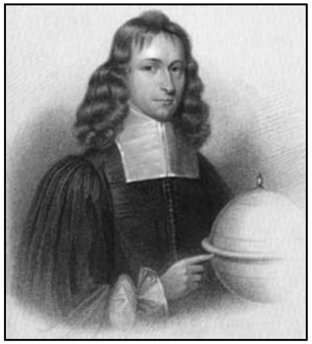


Classical telescopes

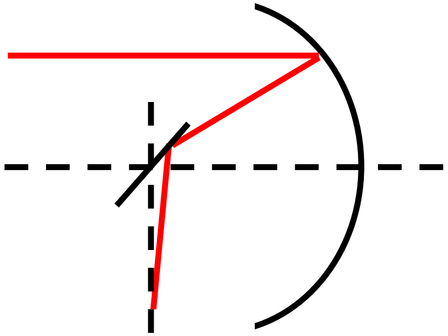
Object at infinity

Grégory (1663)
concave + concave
parabola + ellipse

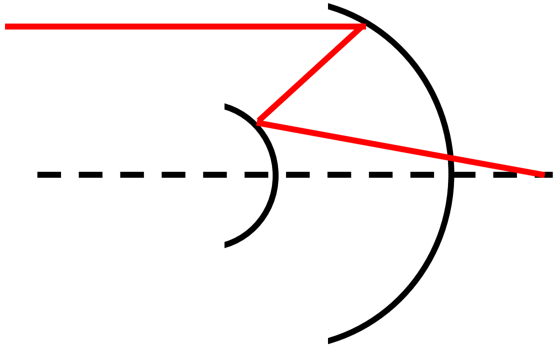




Newton (1672)
concave + plane
parabola + plane

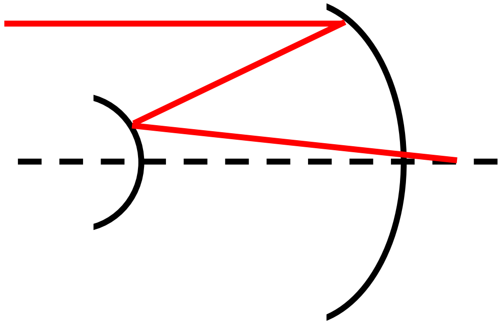


Cassegrain (1672)
concave + convex
parabola + hyperbola

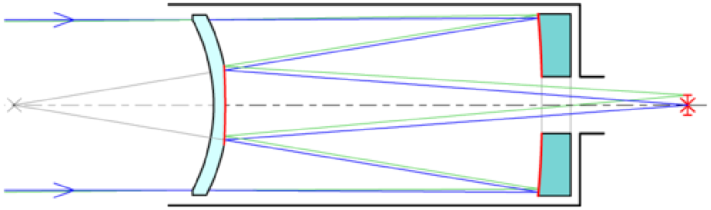


Classical telescopes

Ritchey-Chrétien (1910)
concave + convex
hyperbola + hyperbola



Maksutov (1944)
1 concave mirror only
meniscus + spherical + plane



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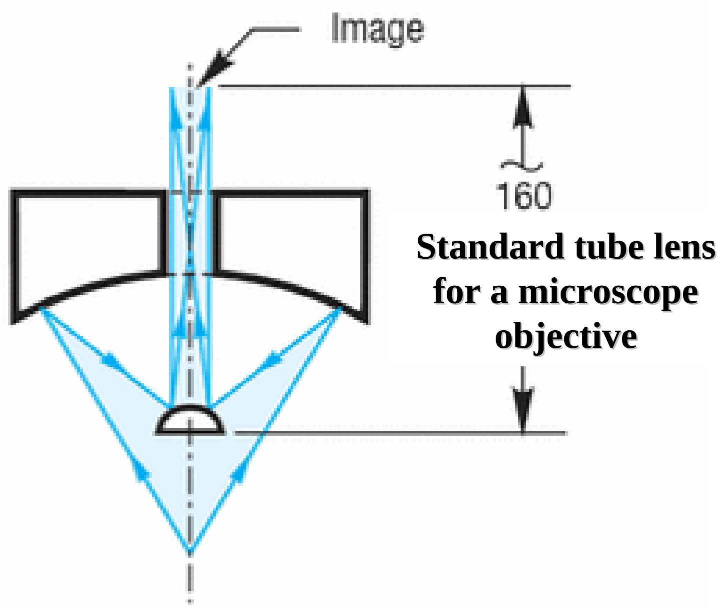
Microscope objectives using mirrors

Object and image at finite distance



Inversed Cassegrain configuration due to K. Schwarzschild

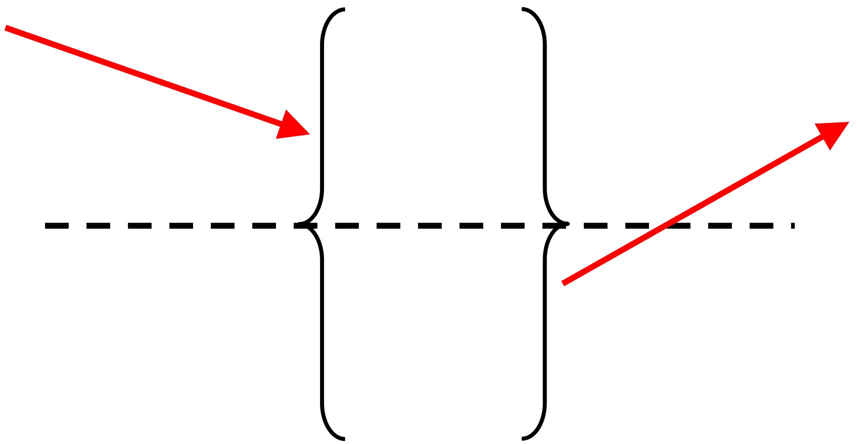
Concentric spherical mirrors

Converging system equivalent to a « thin lens »

Equivalent dioptric systems

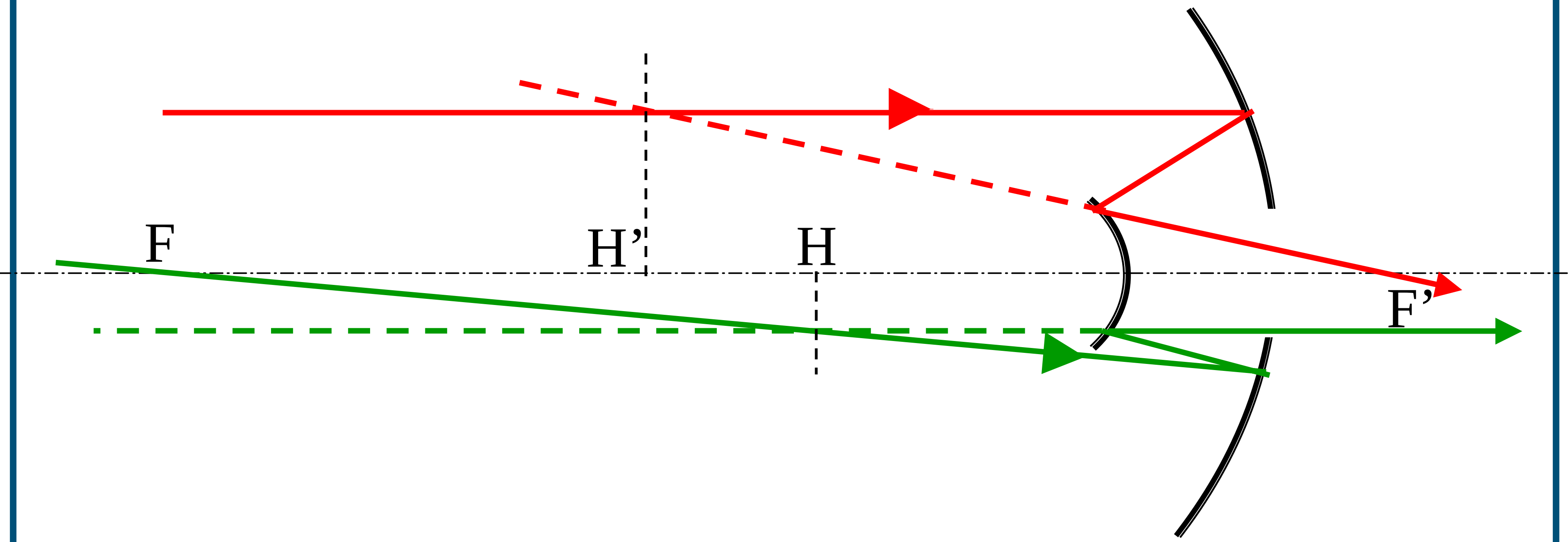
System with 2n mirrors ≡ dioptric system…

… but which one ?



Find the principal points of the equivalent system

Principal points of a Cassegrain telescope

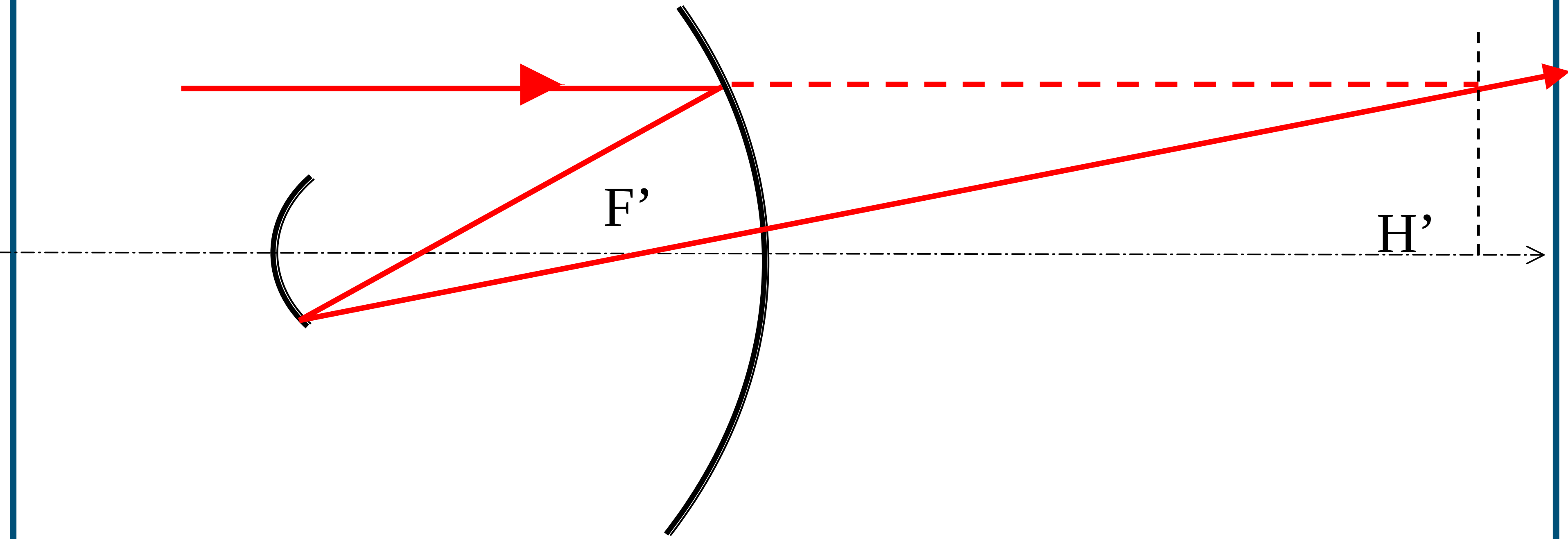


The Cassegrain telescope
is equivalent to a dioptric system…

CONVERGING (H'F'>0)

- with principal points H and H'
- with foci F and F'

Principal points of a Gregorian telescope



The Gregorian configuration

Is equivalent to a dioptric system…

DIVERGING (H’F’<0)

- with principal points H and H’
- with foci F and F’