CHAPTER VIII

COMBINATION OF CENTERED REFRACTIVE SYSTEMS

The general properties of centered systems in the linear approximation have been discussed in chapter V (§III). We have seen in particular how it is sufficient to know the positions of focal points and principal points to define completely a centered optical system: we can from there find all the other cardinal points and make all the constructions of rays; we can use the conjugation formulas and the transverse magnification with reference to the principal points (Descartes) and to the focal points (Newton); we can use the Lagrange-Helmholtz invariant and the relationship $g_y = g_x g_\alpha$ to find all the other magnifications.

In this chapter we will see how we can find rapidly the focal points and principal points of a centered system composed of individual systems of which the cardinal points are known. We will study in particular the case of afocal systems.

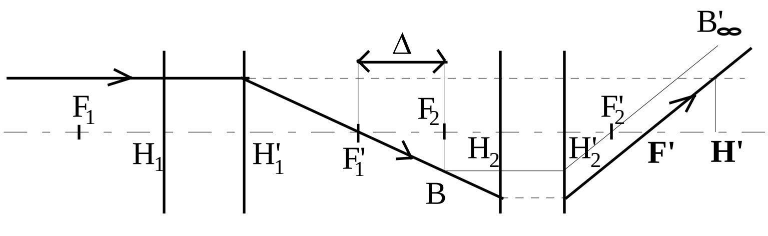
The catadioptric systems (combination of systems including at least one mirror) will be studied after the study of mirrors in chapter IX.

I. Combination of two centered refractive systems

We have already seen one such system when we combined two refractive surfaces to make a thick lens. We will use here a slightly different and more general demonstration, which will give us the position of the focal points and principal points of the whole system as a function of those of the individual elements.

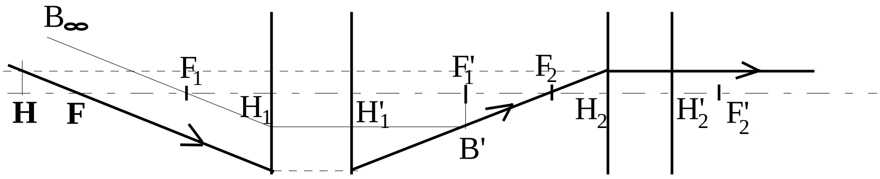
1) Determination of the focal points and of the focal length

We will represent the two centered systems through their principal planes and their focal points. Graphically we can find F' and H' using the construction of an incident ray parallel to the axis:



Note that the relative positions of H and H' for each subsystem can be anything, however the quantities $\overline{HF}$ and $\overline{H'F'}$ must be opposite in sign and in the ratio of the indices of the extreme media. We will characterize the distance between the two systems by the parameter $\Delta = \overline{F'_1 F_2}$, distance from the second focal point of the first system to the first focal point of the second system. This quantity, which is called the optical interval between the two systems, can be positive or negative.

We can follow the same procedure to find the first focal points:



Points F'_1 and F' are conjugate through the second system, so we can write Newton's formula:

$$\overline{F'_2\,F'} = -\frac{f_2 \cdot f'_2}{\Delta}$$

Similarly F and F'_2 are conjugate through the first system, so that:

$$\overline{F_1F} = +\frac{f_1 \cdot f'_1}{\Delta}$$

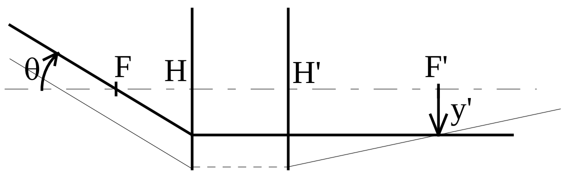
These formulas, also called "Gullstrand's", give us access to the positions of the focal points of the whole system with respect to the focal points of each individual system.

The principal points can be obtained using the construction, and we could use the geometrical properties of the figure to calculate them. We will rather calculate the focal length of the whole system as a function of the focal lengths of each individual system.

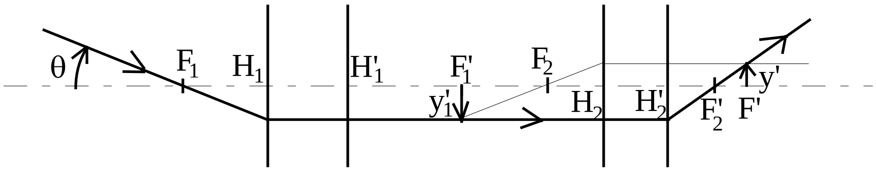
Let us consider an object at infinity with an angular diameter θ and let us calculate the size y' of its image through the whole system.

Using the first focal length f of the whole system, we can write:

$$y' = -f\,\theta$$



On the other hand, we can calculate the size of the final image with respect to the size of the intermediate image given by the first system:



The size of the intermediate image can be written $y'_1 = -f_1\,\theta$. Then we can write the final size $y' = y'_1 \cdot (g_y)_{2 \text{ for } F'_1F'}$, so that we get the following relationship between the focal length of the whole system and the characteristics of each subsystem:

$$f = f_1 \cdot (g_y)_{2 \text{ for } F'_1F'}$$

This relationship will often be used also in the study of telescopes, where the combination of two spherical mirrors leads to a system equivalent to one refractive system (see chapter IX). According to Newton's formulas, the transverse magnification of the second system for the couple of conjugate points F'_1, F' can be written:

$$(g_y)_{2 \text{ for } F'_1F'} = \frac{f_2}{\Delta}$$

Finally we get the first focal length of the whole system as:

$$f = \frac{f_1 f_2}{\Delta}$$

For the second focal length, we can either make a similar calculation for an object *y* giving an image at infinity with an angular diameter θ', or use the relationship between first and second focal lengths as a function of the indices *n* and *n*' of the incident and image media. We get:

$$f' = -\frac{f'_1\, f'_2}{\Delta}$$

Remark: We can generalize the above method to the case of a system consisting of many centered subsystems S_1 to S_n, if we know the positions of the intermediate images A_2 to A_n corresponding to an object A_1 at infinity on axis. For the system S_i, we note x_i the position of the object A_i with respect to its first principal point, x'_i the position of its image A'_i with respect to its second principal point and $(g_y)_i$ the corresponding transverse magnification. The first focal length of the whole system will then be written as:

$$f = f_1\,(g_y)_2 \cdots (g_y)_n$$

Using the expressions of the transverse magnifications with respect to the principal points $(g_y)_i = n_i x'_i / n'_i\, x_i$, knowing that $n'_i = n_{i+1}$, and that $x'_1 = f'_1 = -(n_2/n)\, f_1$ we get:

$$f = -\frac{n}{n'}\frac{x'_1\, x'_2 \cdots x'_n}{x_2 \cdots x_n}$$

2) Gullstrand's formula for the power of the whole system

We would like now to write the power *P* of the whole system to the powers P_1 and P_2 of each individual system. The expression of the first focal length calculated above allows us to write:

$$P = -\frac{n}{f} = -\frac{n\Delta}{f_1 f_2}$$

The convenient parameter to characterize the distance between the two systems is in this case the interspace $e = \overline{H'_1 H_2}$, distance between the principal planes that we can write as a function of the optical interval $\Delta = \overline{F'_1 F_2}$:

$$e = f'_1 + \Delta - f_2$$

We finally get Gullstrand's formula for the power:

$$P = P_1 + P_2 - \frac{e}{n_i} P_1 P_2$$

where n_i is the index of the <u>intermediate</u> medium and $e = \overline{H'_1 H_2}$.

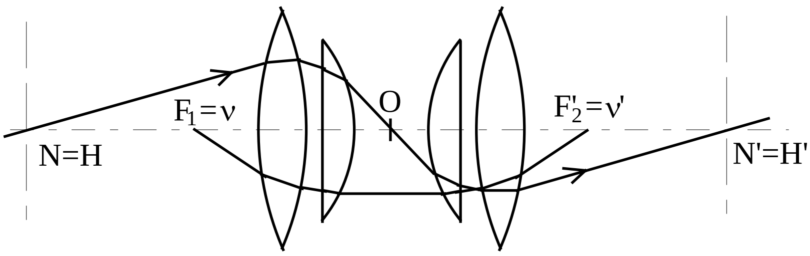
Remark: Gullstrand's formula can be applied to the case of systems composed of mirrors as long as we are careful with the sign conventions (as is always the case for mirrors!). One way to avoid sign errors consists in unfolding the system by replacing a mirror with curvature R by a thin lens with focal length R/2 (converging if the mirror is concave, diverging if it is convex). It is however necessary to make a construction to check the coherence of the result obtained in that way.

3) Case of symmetric centered systems

In addition to their symmetry around the optical axis, these systems are symmetric with respect to a plane orthogonal to the axis, including the indices of refraction (must thus be identical for the incident and emerging media). A biconvex or biconcave thick lens with the same radius of curvature for each refractive surface, a lens made of a total glass sphere, the combination of two thin lenses with the same focal length, are a few examples of such symmetric systems (provided the extreme media have the same index of refraction).

For these systems, there is a fast method to determine both principal and antiprincipal points.

If point O is the intersection of the plane of symmetry and the optical axis, it is also **the optical center of the system**. As we saw for thick lenses, the optical center is such that each ray passing through O will correspond in the object and image spaces to rays that are parallel to each other.



It thus suffices to calculate the image of O through the second half of the system to find the second nodal point, which is also the second principal point (since the extreme media must have the same index). The first nodal point (and first principal point) is symmetric from the second principal point with respect to O.

The antiprincipal points (magnification=-1) are also easily found for a symmetric system. Indeed let us consider a ray that is parallel to the optical axis in the intermediate space containing the plane of symmetry of the system: it corresponds to incident and emerging rays that make equal and opposite angles with respect to the axis. On the other hand, the incident ray must originate from the first focal point of the first half of the system, and the emerging ray must exit through the second focal point of the second half of the system. F_1 and F'_2 are thus the antiprincipal points of the whole system.

4) Examples of combination of centered systems

Such examples will be seen in problems, and when we will study optical instruments. For example they can be doublets (combination of two thin lenses glued together in order to compensate for geometric and chromatic aberrations), eyepieces (combination of two lenses for the visual observation of a small object at a finite distance), microscopes, telescopes, etc.

II. Afocal systems

These systems have a power equal to zero, or in other terms they make an image at infinity from an object at infinity. The simplest afocal systems are the plane refractive surface, the parallel plate and the plane mirror, which are studied in the chapters VI to IX. The splitting into two focal systems is not interesting for these very basic afocal systems.

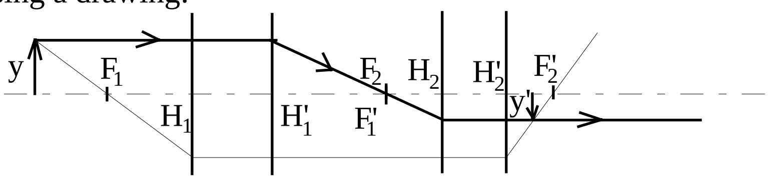
For all the non-trivial afocal systems, we will study them through decomposition into two focal subsystems, where the second focal point of the first system is superimposed with the first focal point of the second system. This decomposition is not unique; we can thus choose the one that is best suited.

We already considered afocal systems in chapter V relative to the linear approximation (see §III.4). We will recall here their properties with the addition of a few extra remarks.

1) Magnifications

Let us emphasize the fact that an afocal system can not only form the image at infinity of an object at infinity, but can also make the image of an object at a finite distance, which will be at a finite distance. The main property of an afocal system is that this imaging will be done **with a fixed transverse magnification.**

It is easy to prove this property, and also determine the value of that fixed magnification, using a drawing:



$$g_y = \frac{f_2}{f'_1}$$

We immediately find that the angular and axial magnification are also constant and are equal to:

$$g_\alpha = \frac{n}{n'} \frac{f'_1}{f_2} \qquad g_x = \frac{n'}{n} \left(\frac{f_2}{f'_1} \right)^2$$

2) Conjugation formulas

They can be found using different methods. For example if we us the fact that the axial magnification is constant and integrate this relationship:

$$\frac{dx'}{dx}=g_x=cst$$

$$x'=g_x \cdot x+cst$$

In this relationship, x and x' are the positions of the object A and its image A' respectively with respect to any origin (not necessarily the same one for A and A'). If we choose to refer A and A' to two conjugate points (H and H', or F_1 and F'_2, or any other couple), we then know that if x=0, x'=0.

We thus get a conjugation formula with origins at any couple of conjugate points:

$$\frac{x'}{x}=\frac{n'}{n}\left(\frac{f_2}{f'_1}\right)^2$$

3) Examples of afocal systems

All the visual instruments designed to observe very distant objects are afocal systems (astronomical telescope, binoculars); we will discuss those in the chapter devoted to optical instruments. Afocal systems are also used to increase the diameter of a parallel laser beam.