

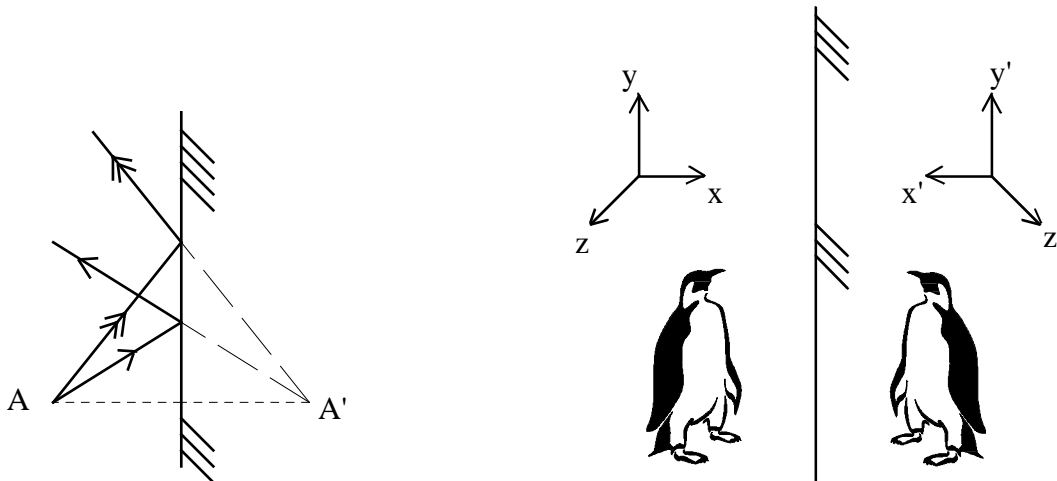
CHAPTER IX

MIRRORS, COMBINATION OF MIRRORS AND CATADIOPTRIC SYSTEMS

I. The plane mirror

1) Basic properties

A plane mirror is a plane surface that reflects light at least partially. The reflected ray is included in the plane of incidence and it is symmetrical of the incident ray with respect to the normal to the surface. The image of a point A is its symmetric with respect to the plane of the mirror, and this is true for any ray originating at A: the plane mirror is perfectly stigmatic for any object. If the object is real, its image is virtual and vice-versa.



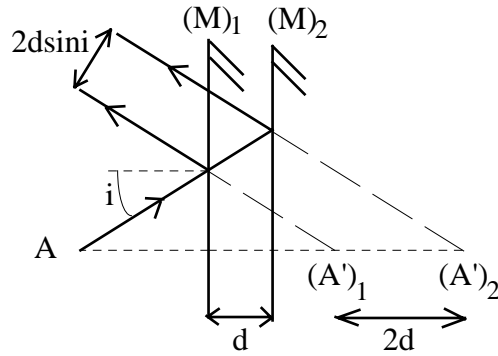
When we image a direct set of reference axes in a plane mirror, it becomes indirect; for an extended object, the image in the mirror cannot be superimposed to the object: a left-handed penguin becomes a right-handed penguin, for example. We also say that the transformation of the object into its image is not congruent (cannot be decomposed into translations and rotations). This is an important property when mirrors are used for imaging.

2) Displacement of a mirror

We consider a fixed object point A and we look for its image A' when we move the plane mirror, in translation or rotation. We will also study the effect of these movements on the emerging ray for a fixed incident ray.

a) translation of the mirror

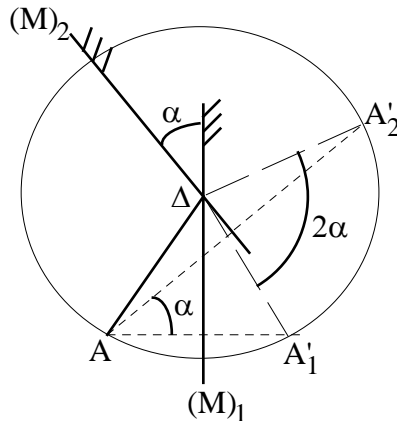
For any direction of translation of the mirror, the relevant quantity is the displacement d perpendicular to the plane of the mirror. The image A' will be displaced by twice this value, $2d$, perpendicularly to the plane of the mirror.



For a fixed incident ray with incidence angle i , the emerging ray translates parallel to itself by $2d \sin i$.

b) rotation of the mirror

We rotate the plane mirror M by an angle α around an axis parallel to its plane. The new position $(M)_2$, after the rotation, cuts the initial plane $(M)_1$ along a line Δ parallel to the axis of rotation. Let us represent the reflection in a plane orthogonal to Δ that includes the object A . The lines of intersection of the mirrors with this plane make an angle α . The images A'_1 and A'_2 of A corresponding to the two positions $(M)_1$ and $(M)_2$ of the mirror are both in this plane according to Snell-Descartes's law



Distances from A to Δ , from A'_1 to Δ and from A'_2 to Δ are equal. The image has moved on a circle centered in Δ passing through A , by an angle 2α .

To determine the effect of the rotation on the reflected ray, we will use the previous result on two objects A and B , which define the direction of the incident ray. The emerging ray passes through the images A' and B' in the mirror. When the mirror rotates by an angle α , A' and B' rotate by an angle 2α around Δ : the emerging ray thus rotates also by an angle 2α .

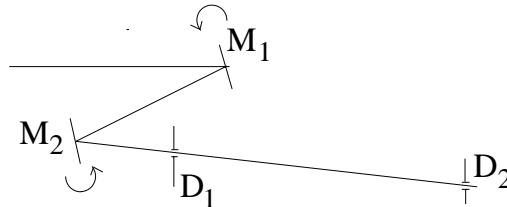
c) alignment of the direction of a ray using plane mirrors

Plane mirrors are often used in optical set-ups to modify the direction of a beam, for example to superimpose two beams, or to center a beam on the optical axis of a system. In the most general case when the initial and final directions are arbitrary, we need 4 adjustable

parameters (a straight line is defined by 4 parameters in a 3D space: for example the coordinates of its intersection point with one of the reference planes Oxy, Oyz or Oxz and two additional coordinates of a unit vector in the direction of the line).

This cannot be adjusted using only one plane mirror, since the two directions are not, most generally, included in the same plane. We thus need two plane mirrors with two adjustable orientations each, which are in general only rotations since it is mechanically difficult to create pure translations.

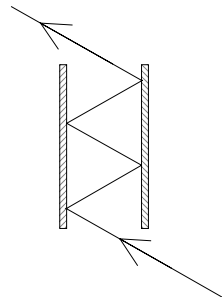
Let us choose for example to mount each mirror on a mount with two adjustable rotations. The rotation axes are chosen horizontal and vertical, which allows us to decouple the adjustments in the two planes, and they are included in the plane of the mirror so that the point of impact of the incident ray almost does not move when we rotate the mirror. The following figure shows what happens in one of the two planes with the two corresponding rotations.



The alignment procedure to make the beam go through the two diaphragms D_1 and D_2 consists in adjusting M_1 to make the beam go through D_1 then adjusting M_2 to make it go through D_2 , and after a few iterations of this procedure, it converges rapidly (more so if M_2 is close to D_1). Note that, the mirrors having a finite size, it is better in practice to place them approximately in the right position before trying to align the system.

3) Combination of plane mirrors

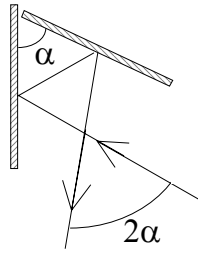
a) Two parallel plane mirrors



The normals to the two mirrors are identical. The ray stays in a plane and is reflected on both mirrors with the same angle of incidence. It comes back parallel to itself after an even number of reflections. This type of system is used for example as a Fabry-Perot cavity.

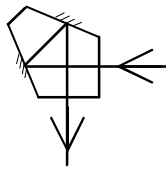
b) two plane mirrors making an angle α

Starting from the situation above, we rotate one of the mirrors by an angle α :



The emerging ray thus rotates by 2α compared to the previous situation: it thus makes an angle 2α with the incident ray.

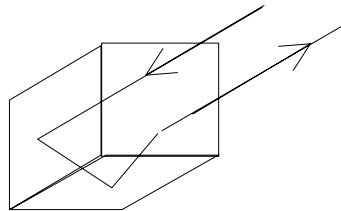
In particular if the two mirrors make an angle of 45° , the emerging ray will be orthogonal to the incident ray, for any direction of the incident ray. This is the basis for the optical set square, often made of a pentaprism with reflective coating on its surfaces:



This setsquare can be used to give a horizontal reference through the reflection of the out coming beam on the surface of a liquid. It also has the advantage of not inverting nor rotating images (you can prove this as an exercise), so that it can be used in the viewfinders of SLR cameras.

c) three plane mirrors making a corner cube

This system consists of three mirrors orthogonal to each other:



We can show easily, using Snell-Descartes's laws, that the emerging ray always comes out parallel to the incident ray. Such a system is often used instead of a single plane mirror when you cannot adjust its direction. For example, such retro reflectors have been set on the Moon to measure the Moon-to-Earth distance, through a measurement of the time it takes for light to travel back and forth; they are also used in the arms of a Michelson interferometer for interferometric measurement of displacements, so that the alignment of the interferometer is maintained throughout the motion.

The corner cubes, and more generally many combinations of plane mirrors, can be either made « hollow », using actual mirrors, or in the shape of glass prisms such as those we already described in chapter III (§II). We had shown in that chapter the possibility for these prisms to be achromatic like mirrors, in spite of refraction. The choice between these two designs is made as a function of weight constraints (lighter system when using mirrors) or rigidity constraints (usually better with prisms).

4) Reflective coatings

We will review briefly here the different ways to modify the amount of light reflected by a surface, in order to make mirrors. These techniques will be studied in more details in other courses.

a) Fresnel coefficients. Total internal reflection.

We already mentioned in chapter I the dependence of the reflection coefficient between two homogenous media on the angle of incidence, the polarisation and the indices of the two media. Let us just remind here that for a glass-air interface, there will be total internal reflection for an incidence angle larger than about 42° . In particular in the case of the pent prism described above, the angles of about 22.5° did not allow the use of total internal reflection.

b) metallic mirrors

Many mirrors are obtained by deposition of a thin metallic layer (of a few nanometers) on the surface of a piece of glass. It is very often made of aluminium ($R \approx 90\%$ in the visible), but other metals are used such as silver (better in the visible and even more in the near infrared $R \approx 95\%$) or gold (especially in the infrared, $R > 98\%$ above 700 nm). The reflectivity is due to the fact that the electric field is zero in a perfect conductor, so all the incident field is reflected.

The advantage of these mirrors lie in their good reflectivity over a large range of incidence angle and their relatively low cost.

c) dielectric mirrors

Dielectric mirrors are made of several thin dielectric layers (thickness of order of the wavelength of light). The reflectivity is based on interference effects. The reflection coefficients can be much higher than those of metallic mirrors, up to 99.95%, but they depend much more on wavelength, incidence angle and polarisation.

This kind of mirror is always used for laser cavities, as well as for any application where high intensities are involved (metallic mirrors burn due to their residual absorption).

Using all the adjustable parameters (thickness and indices of the layers, number of layers that can be as high as several dozens), we can get intermediate reflection coefficients, to make partially reflecting plates for example.

5) Achromaticity of mirrors compared to refractive systems

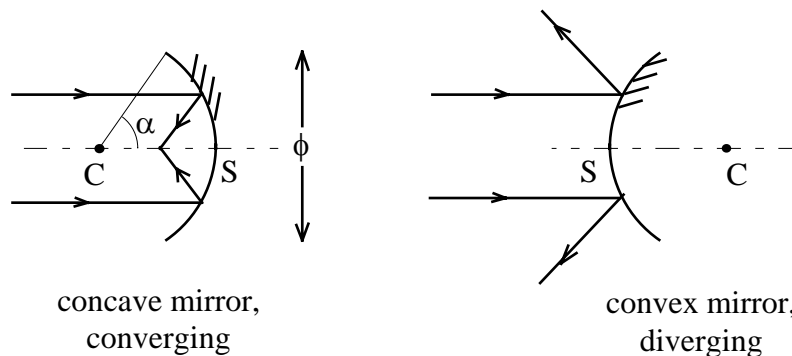
We emphasized in the previous chapters the difficulty to make an optical system that can be perfectly stigmatic, in particular if we want to use this system for a large aperture. We have seen that some systems fulfilled these conditions (parabolic mirrors for example), while others could be stigmatic if they were used in the conditions of the gaussian approximation. However, there is an additional aberration that is still present even for small angles of incidence: it is the chromatic aberration, due to the fact that the index of refraction depends on the wavelength. This effect will be discussed in more detail later, but it is obvious that

systems composed only of mirrors will be achromatic, since the reflection law does not involve indices of materials. An optical system that must be efficient for a very large range of wavelengths (from the far UV to the far infrared, for example an astronomical telescope) will thus preferentially be designed using mirrors with coatings that have a wide spectral range.

II. The spherical mirrors

1) Description

A spherical mirror is a portion of reflective sphere, characterized by its radius of curvature R , its diameter ϕ and its concavity (concave or convex).



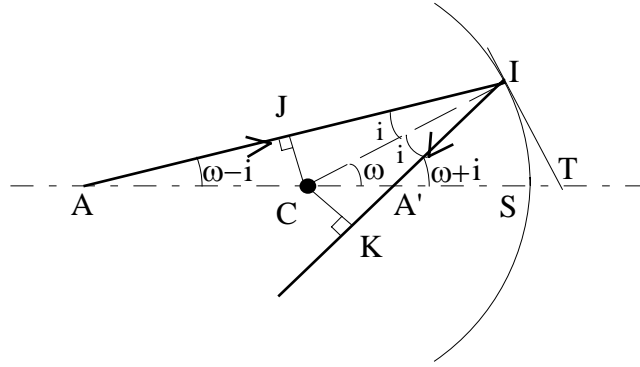
The center C of the spherical mirror is the center of the sphere. The vertex S is the point on the surface of the sphere such that SC is the axis of symmetry of the mirror; SC is called the principal axis of the mirror, the other axes passing through C are called secondary axes.

To distinguish concave mirrors from convex mirrors, we give a sign to their radius of curvature. Our convention is chosen such that $R = \overline{SC}$ is measured in the direction of the incident light. A concave mirror thus has a negative radius of curvature and it is also converging. A convex mirror has a positive radius of curvature and it is always diverging.

2) Perfect stigmatism and aplanatism.

We had already seen in chapter IV that the spherical mirror was not perfectly stigmatic for an object at infinity (§II), but that it was stigmatic for its center and the points on its surface. We have also seen (§IV.4) that the spherical mirror was aplanetic for its center.

We will consider here the case of an arbitrary object point.



For an incident ray originating at the object A on the axis, defined by its point of impact I on the surface of the mirror, we want to calculate the position of point A', intersection of the emerging ray with the axis. We will use for that purpose the point T, intersection of the plane tangent to the sphere at point I with the axis. According to the law of reflection, segments CI and IT are the bisectors of the angles formed by the segments AI and IA'. It can be shown that the points A, A', C and T thus satisfy the following relationships:

$$\frac{1}{\overline{CA}} + \frac{1}{\overline{CA'}} = \frac{2}{\overline{CT}} \quad \frac{1}{\overline{TA}} + \frac{1}{\overline{TA'}} = \frac{2}{\overline{TC}}$$

We can prove rapidly the first relationship for example :

- from the right triangle CIT we get that $\overline{CT} = -R/\cos\omega$ (here $R = \overline{SC} < 0$);
 - from triangles IJC and IKC, we get that $CJ = CK = -R \sin i$ (CJ and CK positive distances);
 - from triangle AJC, we get that $\overline{CA} = -CJ/\sin(\omega - i)$;
 - similarly from A'KC, we get that $\overline{CA'} = CK/\sin(\omega + i)$;
- Combining these four equations, we get the relationship above.

We find that in general point T depends on the incident ray, so does point A' : there is no perfect stigmatism. We can write this relationship as a function of the angle ω between CS and CI in the following way:

$$\overline{CA'} = -\frac{\overline{CA}}{1 + 2\frac{\overline{CA}}{R}\cos\omega}$$

Note that the relationships that we just wrote are still valid for an object off axis, if we replace the principal axis SC by a secondary axis passing through A and C. When the object A is very close to C on a scale of the radius of the mirror, we can make a linear expansion of $\overline{CA'}$:

$$\overline{CA'} \cong -\overline{CA} \left(1 - 2\frac{\overline{CA}}{R}\cos\omega\right) \quad \text{to the 2}^{\text{nd}} \text{ order}$$

$$\overline{CA'} \cong -\overline{CA} \quad \text{to the 1}^{\text{st}} \text{ order}$$

The image A' is thus symmetric of A with respect to the center C, and this is valid for any aperture ω of the rays: indeed the second order terme is negligible for any ω if \overline{CA} is small compared to R.

3) The spherical mirror in the gaussian approximation

The gaussian approximation for the spherical mirror consists in limiting the aperture α or the ratio of the size of the mirror to its radius of curvature, so that the incident rays all make a small angle with respect to the axis, and have a small angle of incidence on the mirror.

a) note about sign conventions

In previous chapters, we chose as positive along x the direction of propagation of light, from left to right. In the case of mirrors the difficulty comes from the change in the direction of propagation of light, and it is thus important to define precisely the sign convention, both for the positions along the x axis and for the positive angles.

If we want to keep the same expressions as in the general theory of the gaussian approximation in chapter V, it is necessary to inverse the axes and direction of rotation after reflection. This amounts in fact to unfolding the system, replacing the spherical mirrors by thin lenses with the same focal lengths (converging for a concave mirror, diverging for a convex mirror). This is necessary when we deal with complex systems including many reflective and refractive surfaces on which we want to make calculations using matrices.

In the following paragraphs of this chapter, we will discuss single mirrors, and choose only one positive direction, in the direction of the incident light, with its corresponding positive direction of rotation, whether we discuss objects or images. This is more natural when all the points are on the same axis. It is in particular the implicit convention used in the relationships above between points A, A', C and T. Be careful that this implies that formulas that were demonstrated in the general frame of the gaussian approximation in chapter V will appear with different signs.

Note: there is a simple « trick » that allows us to change from one sign convention to the other : it consists in using for all that relates to the rays after reflection an index n_2 that is negative, opposite to that of the real medium. Yet another possibility consists in choosing the direction of the emerging light as positive, with $n_1 < 0$ and $n_2 > 0$.

b) conjugation formulas

We can get it for example from the general relationship seen in the previous paragraph §2: the tangent plane in I is superimposed to the tangent plane in S and point T is identical to S. We thus get:

$$\frac{1}{CA} + \frac{1}{CA'} = \frac{2}{CS}$$

This formula is called the conjugation formula with the origin at the center.

Knowing that, in the formula of §2, points C and T play the same role, we can also write the relationship after exchanging their roles. In the case of the gaussian approximation where T and S are identical, we get:

$$\frac{1}{SA} + \frac{1}{SA'} = \frac{2}{SC}$$

This formula is the conjugation formula with origin at the vertex S.

Note : the formula with origin at the vertex S can easily be found using a construction in the gaussian approximation (try it as an exercise). This is probably the best way to remember it. The formula with the origin at the center can then be deduced from it knowing that the two formulas are analogous by an exchange of S and C; or it can also be found using a construction.

c) Cardinal points and focal points

Principal planes are both on the surface itself, the nodal points are at the vertex S, and the antinodal points at the center (with the sign convention corresponding to the general gaussian approximation, which amounts to unfolding the mirror as a thin lens). In the simple case of a single mirror, we will not use these cardinal points, and we will only use the center and the vertex as characteristics points.

Focal points are useful too, there are superimposed ($F=F'$) and located in the middle of SC. One way to prove this is to choose A or A' at infinity in the above conjugation formulas. We thus get:

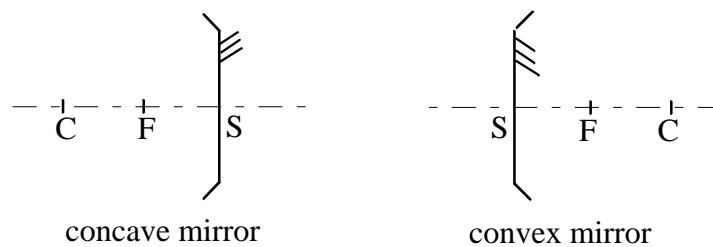
$$\overline{CF} = \frac{\overline{CS}}{2} = -\overline{SF}$$

The focal point is real for a concave mirror, virtual for a convex one. Since principal points are on the surface of the mirror, the focal length of the mirror is defined as:

$$f = \overline{SF} = f' = \overline{SF'} = \frac{R}{2}$$

d) constructions

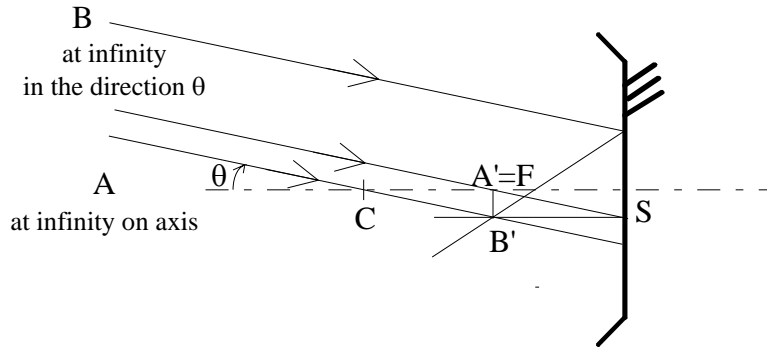
In a similar way as what was done for the refractive surfaces, in order to make accurate constructions in the gaussian approximation, we will represent the spherical mirror by its tangent plane, indicating its concavity using the orientation of lines on the edge of the mirrors.



The specific rays that are easy to construct are : the incident ray parallel to the axis that comes out through the focal point, the incident ray passing through the focal point that comes out parallel to the axis, the incident ray passing through the center that is not deviated, the incident ray passing through the vertex s that comes out symmetrically with respect to the axis. In addition we recall that in the gaussian approximation the image of a plane orthogonal to the axis is also a plane orthogonal to the axis.

** Object at infinity with angular diameter θ*

The angular diameter of an object at infinity is the angle between rays coming from the two edges of the object. For the figure we chose one edge on the axis of the mirror, and a concave mirror.

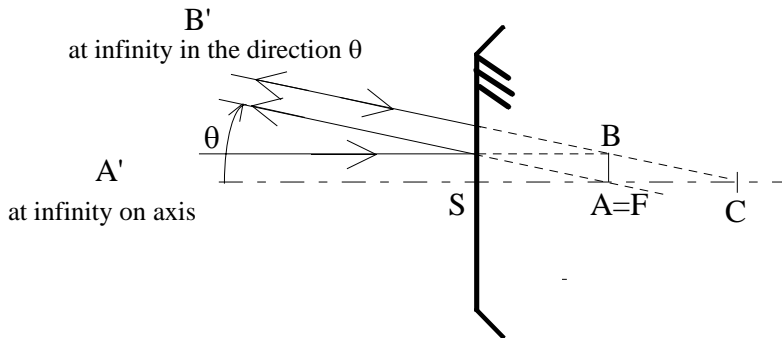


The image A'B' of AB is located in the focal plane. Its size is, according to the figure:

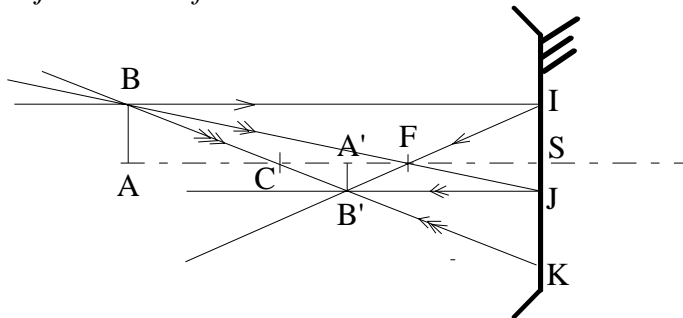
$$y' = \overline{A'B'} = -f \cdot \tan \theta \cong -f \cdot \theta$$

For example the image of the Moon, which has an angular diameter of $30'$, at the focus of a mirror with a radius of curvature of one meter is a circle with a diameter $0.5 \times 30 \times 3.10^{-4} = 4.5\text{mm}$ and it is inverted.

** object AB in the focal plane (case of a convex mirror)*



** object AB at a finite distance*



e) magnifications

We can, using the ray originating at B and passing through the center C (see the construction above), calculate the transverse magnification g_y . Using the ray passing through the center, we get:

$$g_y = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{CA'}}{\overline{CA}}$$

We can also draw a ray originating at B and passing through S: the reflected ray is symmetric with respect to the axis and passes through B'. We get in that way another expression for the transverse magnification:

$$g_y = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

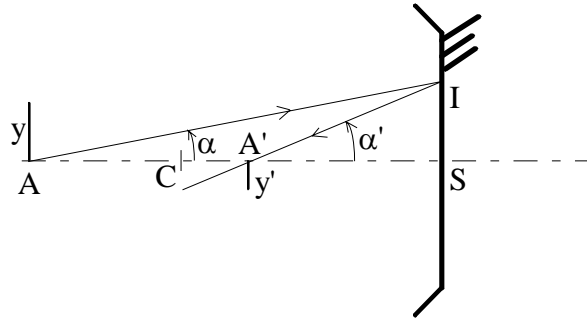
It is also interesting to determine the lateral magnification relating the size $dx = d(\overline{SA})$ of the object along the axis to the size $dx' = d(\overline{SA'})$ of its image. We get it by differentiating one of the two conjugation formulas:

$$-\frac{dx'}{x'^2} \frac{dx}{x^2} = 0 \quad \text{thus} \quad g_x = \frac{dx'}{dx} = -\frac{x'^2}{x^2} = -g_y^2$$

The sign of g_x is always negative, indicating that the image always move in the opposite direction compared to the object.

f) Lagrange-Helmholtz's invariant

We have discussed this relationship earlier for refractive systems, and it was written $ny\alpha = n'y'\alpha'$. We will see here how it appears in the case of a mirror.



As we said in the paragraph a), our sign convention for the angles is the same for the incident and the emerging ray. On the figure above, α' is positive as well as α . According to the figure, we can write:

$$\frac{\alpha'}{\alpha} = \frac{\overline{SA'}}{\overline{SA}}$$

Thus using the expression of the transverse magnification with the origin at the vertex:

$$y\alpha = -y'\alpha'$$

We find the Lagrange-Helmholtz's invariant in its general form if we take $n' = -n$.

g) relationship between magnifications

Let us summarize all the expressions for the three magnifications as a function of $x = \overline{SA}$ and $x' = \overline{SA'}$:

$$g_y = -\frac{x'}{x} \quad g_\alpha = \frac{x}{x'} \quad g_x = -\frac{x'^2}{x^2}$$

These three magnifications are interdependent, it is sufficient to calculate one to know the two others. In general, we will calculate the transverse magnification g_y and find the other two from:

- Lagrange-Helmholtz's invariant, that we can memorize in its general form $ny\alpha = n'y'\alpha'$ taking $n = -n'$ in the case of a mirror;
- the relationship:

$$g_y = g_x \cdot g_\alpha$$

that, since it does not involve indices of refraction, is valid for refractive and reflective systems.

h) Newton's formulas

We find them for example through a construction of the image of an object AB using the rays passing through the focal point (see figure on page 102):

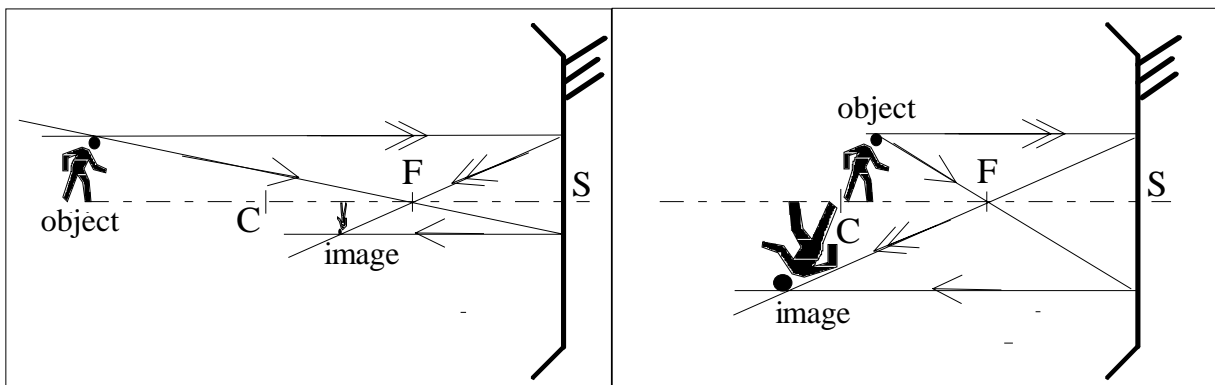
$$\frac{\overline{AB}}{\overline{FA}} = \frac{\overline{SJ}}{\overline{FS}} = -\frac{\overline{A'B'}}{f} \qquad \frac{\overline{A'B'}}{\overline{FA'}} = \frac{\overline{SI}}{\overline{FS}} = -\frac{\overline{AB}}{f}$$

We get from these two relationships:

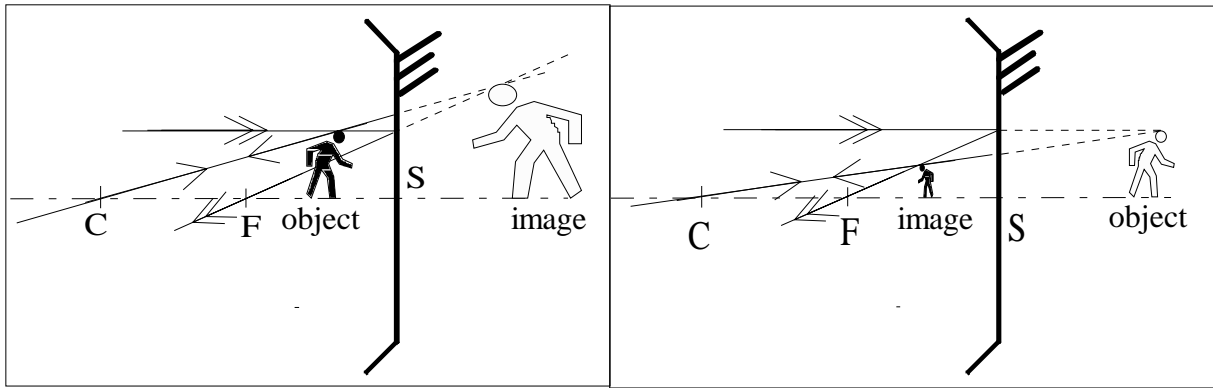
$$\overline{FA} \cdot \overline{FA'} = f^2 \qquad g_y = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{f}{\overline{FA}} = -\frac{\overline{FA'}}{f}$$

i) displacement of the image when the object is moved along the axis

Using geometrical constructions, we can visualize the evolution of the position of the image, in particular see whether it is real or virtual, as a function of the position of the object. We did it below in the case of a concave mirror, the case of a convex mirror should be done as an exercise.



When the object, real at first, moves from infinity to the left of the focal plane, its image moves from the focal plane towards infinity in the opposite direction. The image is real and inverted. It is first smaller than the object until the object and its image meet at the center of the mirror; then the image becomes larger than the object.



When the object moves further, to the right of the focal plane, the image comes back from infinity from the right and keeps moving to the left as the object moves to the right. The image is virtual as long as the object is real, then it becomes real when the object turns virtual as it crosses the plane of the mirror. The image is upright, like the object. It is larger than the object when it is virtual, is superimposed to the object at S, then is smaller than the object when it becomes real.

4) matrix of the spherical mirror in the gaussian approximation

We will here make the connection with the matrix formalism that we saw in chapter V in the general frame of the gaussian approximation. This will be useful mostly to study optical cavities, such as laser cavities, when matrices are the most convenient tool.

The whole point of the matrix formalism is to dispose of a general formalism, able to consider in a similar fashion refractive or reflective surfaces. As a consequence, for matrices, we will go back to the sign convention used for refractive surfaces, taking the direction of propagation of light as the positive direction. We will thus invert the positive direction on the axis, as well as the positive orientation of angles, when we consider reflected rays. This amounts to, as was already said earlier, unfolding the system by replacing the spherical mirror by a thin lens with the same focal length.

As a reference frame to write the matrix, we will most often use, for the simple elements that can be assimilated to their tangent plane, the origin of the reference frame in that plane, at the vertex S of the mirror. We can then calculate the matrix of a spherical mirror with radius of curvature R for example going back to the definition of each term of the matrix:

$$\mathbf{M} = \begin{bmatrix} g_y & 0 \\ -C & \frac{n'}{n} g_\alpha \end{bmatrix}$$

At S, g_y is equal to 1; $g_\alpha = 1$ and $n = n'$ with the sign convention that we consider. We could have also found this matrix using the transformation of specific rays.

Thus the matrix of a spherical mirror with radius R can be written:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \quad \text{with } |P| = 2n/R$$

$P > 0$ for a concave mirror and $P < 0$ for a convex mirror

Note: If you are very uncomfortable with the signs, remember the rule given at the beginning of this chapter. If we take the incident direction of the light to be the positive one, then the index of refraction of the incident medium is positive; if we choose the reflected light to be the positive direction, then it is the index n' which is positive. Thus a concave mirror is also converging: if we choose $n > 0$ then $R = SC < 0$, and if we choose $n' > 0$ then $n < 0$ but $R = SC > 0$, and in both cases we get a positive value for the power P .

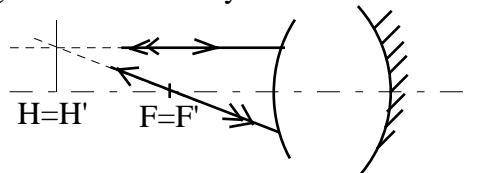
III. Catadioptric systems

Those are made of both refractive and reflective surfaces.

1) Equivalent optical system

a) Catadioptric system with one mirror

Let us consider the general case of a system limited on its right by a spherical mirror:



An incident ray parallel to the axis will go through the refractive part of the system, then reflect on the mirror and go a second time through the refractive part. The corresponding emerging ray will intersect the axis at a point that is the second focal point F' of the catadioptric system.

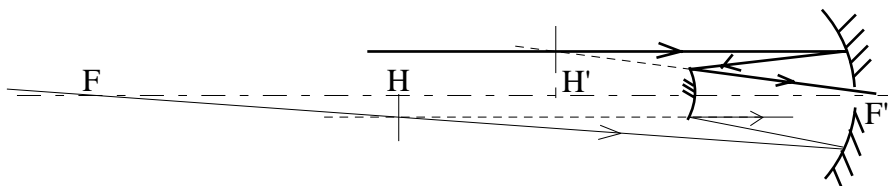
Let us follow this same ray in the opposite direction. Because of the reversibility of the light path, it will come out parallel to the axis. The point F' is thus also the first focal point of the catadioptric system.

We can do the same reasoning for the principal planes (see figure): the first and second principal planes are also superimposed.

As a result a catadioptric system with one mirror is equivalent to a single spherical mirror with a vertex $S = H = H'$ and focal point $F = F'$.

b) catadioptric system with two mirrors

This is the case for example of a telescope, where the secondary mirror is pierced to make way for the light to come out of the system. Take for example the case of a Cassegrain telescope:



The emerging light has the same direction of the incident light. We can determine the principal planes and the focal points of the system, which is thus equivalent to a refractive system. In particular the image through such a system can be superimposed to the object (it has the same helicity).

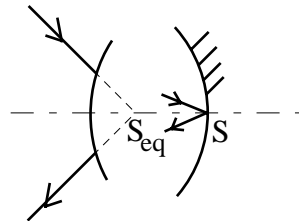
More generally, a system including an odd number of mirrors will be equivalent to a single mirror, while a system including an even number of mirrors will be equivalent to a refractive system.

2) Determination of the equivalent mirror for a catadioptric system with one mirror

a) finding the vertex of the equivalent mirror

We have seen that one way to find the vertex of the equivalent mirror is to find the second principal plane of the catadioptric system using the path of a ray initially parallel to the axis. We will see here a different method, usually more efficient.

Let us consider an incident ray such that after going through the refractive part of the system it comes out through the vertex S of the mirror. It will be reflected symmetrically to the axis.



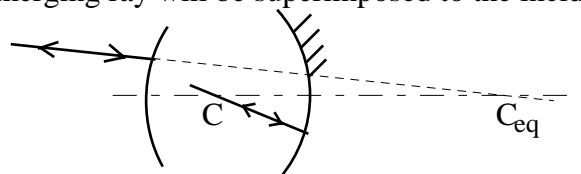
Taking into account the symmetry of revolution of the system, the emerging ray will necessarily be symmetric to the incident ray with respect to the axis. These two rays intersect at a point S_{eq} on the axis. This point S_{eq} , superimposed with its image through the catadioptric system, is such that an incident ray passing through S_{eq} comes out symmetrically with respect to the optical axis: it is the vertex of the equivalent mirror. On the other hand, according to the way we constructed S_{eq} , it is the point that has S as its image through the refractive part of the system, for the light travelling from left to right. Thus :

The vertex S_{eq} of the equivalent mirror can be found by saying that its image through the refractive part of the system is in S , vertex of the real mirror, for the light travelling from left to right (direction of the incident light).

b) finding the center of the equivalent mirror

We have seen a graphical method to determine the focal point of the equivalent mirror. We could use it to determine its center since we know how to find its vertex. We will see here a faster method.

Let us consider an incident ray such that after passing through the refractive part of the system it goes through the center C of the mirror. It will be reflected on itself by the mirror, so that the emerging ray will be superimposed to the incident ray.



The point C_{eq} where this incident ray intersects the axis of the system thus verifies the following property : it comes back on itself after going through the whole catadioptric system. It is thus the center of the equivalent mirror.

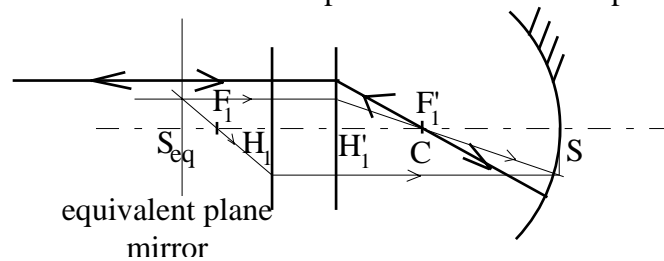
The center C_{eq} of the equivalent mirror is such that its image through the refractive part of the system is in C, center of the actual mirror, for light propagating from left to right (in the direction of the incident light).

3) Afocal catadioptric systems

We will study now the specific cases when the vertex or the center of the equivalent mirror are at infinity.

a) systems equivalent to a plane mirror

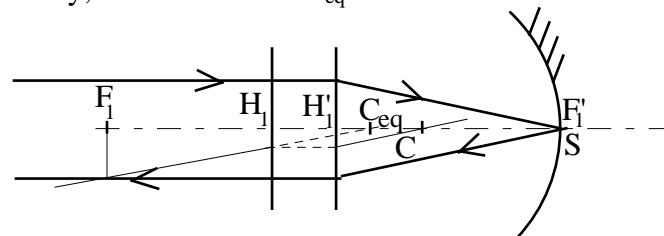
When the center of the equivalent mirror is at infinity, the catadioptric system is equivalent to a plane mirror. It is an afocal system. This happens when the center C of the real mirror is superimposed with the second focal point of the refractive part of the system:



For this afocal system, the transverse magnification is equal to +1 for any conjugate points.

b) systems equivalent to a spherical mirror located at infinity

Let us consider the case when the second focal point of the refractive part of the system is superimposed with the vertex S of the real mirror. The vertex S_{eq} of the equivalent mirror is located at infinity, while its center C_{eq} is at a finite distance.



For this afocal system, the transverse magnification is equal to -1.

This type of system cannot be made with a single mirror. It is also called **cat's eye**. It has the following interesting properties:

- (i) the image by this system of an arbitrary point B is a point B', symmetric of B with respect to the center C_{eq} of the equivalent mirror;
- (ii) any incident ray comes out parallel to itself.

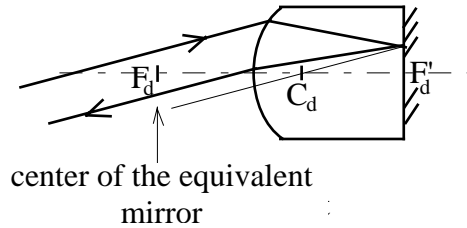
We can prove property (i) using the conjugation formula of the spherical mirror with SC infinite:

$$\frac{1}{C_{eq}A} + \frac{1}{C_{eq}A'} = 0 \quad \text{so that: } \overline{C_{eq}A'} = -\overline{C_{eq}A}$$

To prove property (ii), we can always consider that an incident ray making an angle α with the optical axis comes from a point at infinity S_α in the direction α , this point S_α being

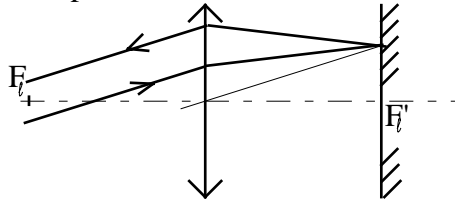
on the surface of the mirror. Since a point on the surface is its own image, the emerging ray will thus also pass through S_{α} , meaning that it will have the same direction as the incident ray.

As an example of such a system, we can consider a thick plano convex lens such that the second focal point of the spherical refractive surface is located on the plane surface, which is itself coated to make a plane mirror:



The fact that light is always reflected in the incident direction makes such a system very useful as a retroreflector (for car lighting or road signs for example).

Another configuration, that is often used, consists in a converging lens and a plane mirror located in the second focal plane of the lens:



APPENDIX ABOUT THE USE OF MATRICES

This method can be useful to study complex systems that do not have specific properties of symmetry or when we want to vary several parameters (radii of the refractive surfaces, distance between surfaces, indices, etc.) in a systematic way. It is typically the method of choice in an optical design software where the user enters the parameters of his optical system and wants to get the cardinal points, make a ray tracing, determine the position and the size of the image, etc.

This method is based on the matrices that we discussed in chapter V relative to the gaussian approximation. Here we want to determine the characteristics of a centered system in the paraxial domain. We will not justify the method here (see chapter V).

The basic principle is as follows:

1) we calculate the matrix \mathbf{M}_{ES} of the system between its entrance surface and its output surface, by multiplying (in the right order !) the matrices of each of the individual elements that constitutes it:

$$\mathbf{M}_{ES} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \mathbf{M}_n \cdot \mathbf{M}_{n-1} \cdots \mathbf{M}_1$$

list of all the different matrices of simple elements:

- free space with length d in a medium of index n : $\begin{bmatrix} 1 & d/n \\ 0 & 1 \end{bmatrix}$
- refractive surface n/n' with radius of curvature R $\begin{bmatrix} 1 & 0 \\ -(n'-n)/R & 1 \end{bmatrix}$
- lens in air with second focal length f $\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$
- mirror with radius of curvature R thus with power $|P|=2n/R$, where $P>0$ for a concave mirror and $P<0$ for a convex mirror $\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix}$

2) We immediately get the power (thus the focal lengths since we know the indices), which is the opposite of the term a_3 of the matrix

3) We get the positions of the principal points with respect to the entrance and output surfaces of the system by writing the relationship between the matrix \mathbf{M}_{ES} and the matrix $\mathbf{M}_{HH'}$, which has a very simple form:

$$\mathbf{M}_{ES} = \begin{bmatrix} 1 & \overline{HS}/n' \\ 0 & 1 \end{bmatrix} \cdot \mathbf{M}_{HH'} = \begin{bmatrix} 1 & \overline{EH}/n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - P\overline{HS}/n' & * \\ -P & 1 - P\overline{EH}/n \end{bmatrix}$$

By identification of the terms a_1 and a_4 , we get \overline{HS} and \overline{EH} (n, n' : indices of the extreme media).

4) We can construct the path of rays using matrix \mathbf{M}_{ES} : we only need to multiply the coordinates of the incident ray in the reference frame with origin at E with the matrix to get the emerging ray in the reference frame with origin at S. If we want the intermediate rays, we have to decompose into the intermediate matrices.

5) We can calculate the position and the size of the image A'B' of an object AB through the system by writing the matrix $\mathbf{M}_{AA'}$ from the matrix \mathbf{M}_{ES} : we then write that the coefficient a_2 of $\mathbf{M}_{AA'}$ must be zero (which give the conjugation formula) and its coefficient a_1 gives the transverse magnification.