CHAPTER X

PROPERTIES OF OPTICAL INSTRUMENTS: Magnification, power and magnifying power

We will now use the basics of geometrical optics that we have detailed in the previous chapters to analyse and design optical instruments, that is to say optical systems designed for a specific use: observation of small objects (magnifying glass, eyepiece, microscope), photography, observation of very distant objects (binoculars, astronomical telescope, reflective telescope), etc.

In this first chapter of instrumental optics, we will define some properties, which will allow us to qualify and compare instruments, and then we will go into more detail about the properties relative to the size of the image formed by the instrument. Some of these definitions remain valid out of the framework of gaussian approximation, however we will always do the calculations here within that approximation.

I. Classification and qualification of instruments

We will consider here instruments designed to form an image. There are also instruments that are designed to carry or concentrate electromagnetic energy (for example lighting systems) that we will not consider here.

Among the image forming instruments, we usually distinguish two categories:

1) Projection instruments

Those are sometimes called objective instruments . They make **a real image** on a screen, or on a photoelectric detector (photodiode, photomultiplier, CCD matrix), or on photographic film. Cameras belong to this category, as well as movie projectors or telescopes (without eyepiece) combined with a CCD detector.

2) Visual instruments (or ocular or subjective)

Those are designed to be used with an eye, to facilitate the observation (in general increase the size of the image). They must make an image at infinity, or virtual, that the eye will turn into a real image on its retina. Their properties will thus be connected and compared to those of the eye itself. This is why we will recall a little further the main properties of the eye, which is itself a very sophisticated instrument.

Among these visual instruments, we distinguish two types of functions, depending on whether they are designed to observe small near objects (microscope type of instruments) or to observe very distant objects (telescope with eyepiece, type of instruments).

3) Qualities of an instrument

We will qualify an instrument by the following properties:

(i) the size of the image compared to that of the object, which will be defined differently depending on the type of object (near or distant) and on the instrument (projection or visual);

(ii) the field of the instrument, meaning the portion of space, which is seen "clearly" (notion to be defined) through the instrument: we will distinguish the field of view (surface in the plane of the object) and the depth of field (distance along the axis of the optical system, around the object);

(iii) the resolution power, which is the ability for the instrument to distinguish details of the object;

(iv) the clarity, and more generally, all the photometric properties of the instrument.

The first property will be studied in detail in the rest of this chapter. The field will be the subject of the next chapter. The resolution power is a more complex notion, which involves aberrations of the optical system, diffraction effects, as well as the properties of the detector, which is attached to the system (resolution power of the eye, pixel of the photographic film or of the CCD detector). On a geometrical point of view, it is the aperture of the optical system, which we will discuss in the chapter about fields, that can influence the resolution power of the instrument. Finally we will see in chapter XII an introduction to the photometric properties (those are studied in more detail in the radiometry course), which are also connected to the study of fields.

II. Basic properties of the eye

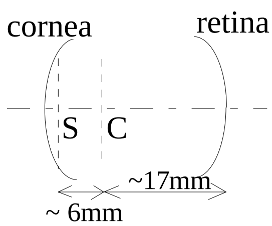
This paragraph is aimed at listing some properties of the eye (seen in more detail in a specific physiological optics course), which will be useful to qualify visual instruments. In addition the eye is also a complex optical instrument, and some notions discussed about the eye (resolution, pupil) will come again in the discussion of the properties of instruments.

1) The optical system of the eye

The converging properties of the eye are due to the cornea (air/water refractive surface with radius of curvature of 8mm) and to the eye lens (biconvex converging lens immersed in water, with an index of refraction slightly larger than water, and with a variable focal length, which allows for accommodation). This optical system is combined to a detector which is the retina, and the only part of interest to us for image formation is the fovea (slightly off center from the axis of the optical system).

a) equivalent spherical surface

When we calculate the cardinal points of the optical system of the eye, we find that the nodal points N and N’ are almost superimposed. Consequently principal points H and H’ are also superimposed, but different from N=N' since the indices of the extreme media are different. The system is thus equivalent to an air/glass spherical surface with center C=N=N' and vertex S=H=H'. The configuration is the following:



For a normal eye at rest, the second focal point of the system is located on the retina (clear vision at infinity); the power is 60 diopters, which corresponds to an equivalent refractive surface with a radius of curvature of 5.6mm. When the eye accommodates, its power increases, the second focal point of the system moves towards C, the radius of curvature of the equivalent surface decreases (about 5.2mm for an accommodation at 25cm). This variation is done by the variation of power of the eye lens, points S and C move by a few tenths of mm, which can be considered as almost zero compared to the shortest distance to the observed object.

b) accommodation

The "normal" eye, also called emmetropic, looks comfortably at infinity (ponctum remotum $D=\infty$) and can accommodate up to a typical distance d=25cm (ponctum proximum). When this range of accommodation is different, the eye is called ametropic. Different ametropies can appear such as:

- myopia or near-sightedness: the eye is too converging, it sees clearly between d<25cm and a finite distance D;
- hypermetropia or far-sightedness: the eye is not convergent enough, it sees clearly from d>25cm till D<0 (this means that it can see an object which is virtual with respect to its eye)
- presbytia: the eye loses its ability to accommodate, it sees clearly from d>25cm to infinity; its amplitude of accommodation $A=1/d-1/D$ goes to zero, when it is equal to 4 diopters (or m^{-1}) for a normal eye.

c) size of the image

We have seen that the center and the vertex of the equivalent refractive surface move very little when the eye accommodates. This implies that **the size *y'* of the image on the retina depends only on the angle θ from the edge of the object to the eye**. Indeed, if we consider for example that the center is fixed at a distance *a*=17mm from the retina, the size of the image formed on the retina is $y'=a\theta$ and depends only on the angle θ from the edge of the object to point C.

If the observed object is at infinity, this angle is the angular size of the object, independent on the position of the eye. For an object at a finite distance, with size *y*, we should place it at the ponctum proximum so that the angle θ is as large as possible ($\theta_{max}=y/25cm$ for a normal eye): the image on the retina will then be the largest possible. An instrument will be useful if it increases the angle θ further than this value θ_{max}.

d) resolution limit

There is no unique answer to the value of the resolution limit of the eye, because it depends on several parameters: shape of the details observed, color, luminance, contrast, radius of the eye pupil, fatigue of the observer, etc. We can however give an order of magnitude: θ_{min}=1 to 2'. This means that for an object at a finite distance the minimum size that can be distinguished by a naked eye is $\theta_{min} \times 25cm \approx 100\mu m$, so we will need instruments such as a magnifying glass or a microscope to see smaller objects.

2) The eye pupil

The eye contains an iris diaphragm with a variable diameter, located between the cornea and the eye lens. It limits the aperture of the rays entering the system: this is what will be called the aperture stop in the general framework of optical instruments. Its image through the cornea (what we see when we look at somebody else's eye) is the entrance pupil of the eye. The diameter of this entrance pupil varies between 1 and 4mm (typically 2mm in day vision).

This parameter is important to design a visual instrument: the eye's pupil should not in general limit the aperture of the combination instrument+eye, which means in other words that the exit pupil of the instrument (image of its entrance pupil) must be smaller than the entrance pupil of the eye.

3) Field of view of the eye

It is limited by the size of the fovea, part of the retina where clear images can be formed. When the eye is still, this field of view is small (angular diameter $2\omega=1°$), but thanks to the mobility of the eye in its orbit this field of view expands to $2\omega=40$ to $50°$. There is thus no advantage in increasing the field of view of an instrument (in its image space) above this value.

III. Size of the image: magnification, power, magnifying power.

We want to qualify the link between the size of the object and the size of its image formed by the instrument. Depending on the nature of the object (near or at infinity) and depending on the type of instrument (projection or visual), these dimensions will be given by a length or by an angle, leading to characteristics with different definitions and even different units.

1) (transverse) Magnification

This parameter characterizes projection instruments, for which the image is real, thus characterized by its size y'.

In the case of an object at a finite distance with a size y, this quantity will simply be the transverse magnification g_y of the optical system for the two conjugate points for which it will be used.

In the case of an object at infinity defined by its angular diameter θ, the magnification, ratio of the size of the image by the size of the object, is replaced by the quantity $y'/\theta = -f$, which is the first focal length of the system.

2) Power

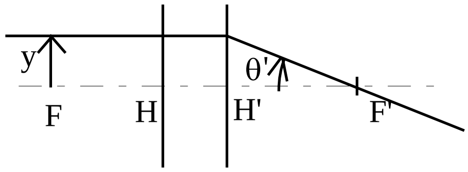
This parameter characterizes visual instruments designed for the observation of small near objects. The power is defined as the ratio:

$$P = \frac{\theta'}{y}$$

where θ' is the angular size of the image formed by the instrument (as seen by the eye) and y is the size of the object. This parameter can in principle depend on the adjustment of the instrument (position of the image) and on the position of the eye.

a) power in the case of an image at infinity

This is the right adjustment of the instrument for a normal eye that does not accommodate. In this case, the two quantities θ' and *y* are independent on the position of the eye.

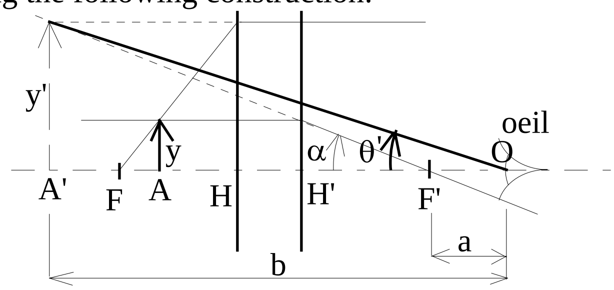


The power is then: $P = -\frac{1}{f'}$ where f' is the second focal length of the system. It is thus the same as the power defined for an optical system in the gaussian approximation.

For this adjustment of the instrument, the power depends only on the intrinsic properties of the instrument: it is sometimes called the **intrinsic power** P_i.

b) power in the case of an image at a finite distance

Since we are still discussing visual instruments, this image formed by the instrument must be placed between the ponctum proximum and the ponctum remotum of the eye. It will be at a finite distance for example when the observer is ametropic or if the emmetropic observer accommodated while adjusting the instrument. In this case, the angle θ' will depend on the adjustment and on the position of the eye. We can determine the expression of the power in this case using the following construction:



The position of the eye with respect to the optical system is characterized by the distance to its second focal point $a = \overline{\mathrm{F'O}}$, and the position of the image is characterized by the distance to the eye $b = \overline{\mathrm{A'O}}$.

Using the figure, we find:

$$\alpha = -\frac{y}{f'} = -\frac{y'}{b-a} \qquad \theta' = -\frac{y'}{b}$$

We deduce from it the expression of the power:

$$P = -\frac{1}{f'}(1-\frac{a}{b}) = P_i\,(1-\frac{a}{b})$$

We thus find that the power depends in general on the position of the eye and on the adjustment of the instrument. It is equal to the intrinsic power P_i in two cases:

- the image is at infinity (b=∞): this is the situation already discussed earlier

- **the eye is located in the second focal plane of the instrument** (a=0): the size of the image θ' is then independent on the adjustment of the instrument.

We should thus always try to design an instrument so that the observer places his eye in the second focal plane of the instrument.

The power will be given in diopters (or m^{-1}). We will often give the absolute value of the power and specify separately whether the system is converging or diverging (for example a plano convex lens with focal length 100mm is a converging system with a power of 10 diopters).

For a microscope type system composed of an objective (with a short focal length) and of an eyepiece, the power of the system can be written as a function of the characteristics of each of its elements in the following way: $P_{microscope} = \frac{\theta'}{y} = \frac{\theta'}{y_{\text{int}}} . \frac{y_{\text{int}}}{y}$

$$\boxed{P_{microscope}=P_{eyepiece}. (g_y)_{objective}}$$

3) Magnifying power

This parameter characterizes most logically the visual instruments designed for the observation of distant objects. We will see that it can also be extended to the case of microscope type instruments, for the observation of near objects.

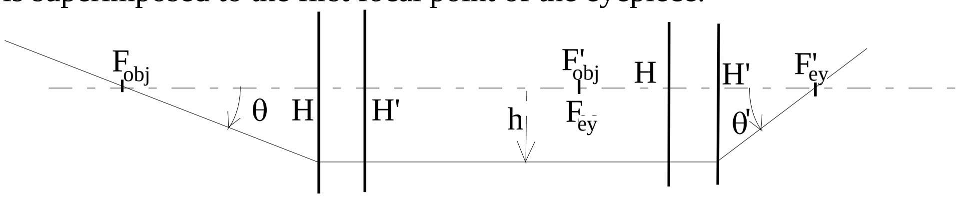
The magnifying power is defined as the ratio between the angular size θ' of the image seen by the eye through the instrument, and the angular size θ of the object as seen with the naked eye.

$$G = \frac{\theta'}{\theta}$$

a) object and image at infinity

The instrument is adjusted to be afocal, which is the comfortable observation situation for a normal eye. The angles θ and θ' are the angular diameter of the object and of the image. The magnifying power is simply the angular magnification of the afocal system. It is independent on the position of the eye of the observer: it is called the **intrinsic magnifying power G_i**.

Its expression can be determined by decomposing the afocal system into two focal systems, usually called objective and eyepiece, placed so that the second focal point of the objective is superimposed to the first focal point of the eyepiece.

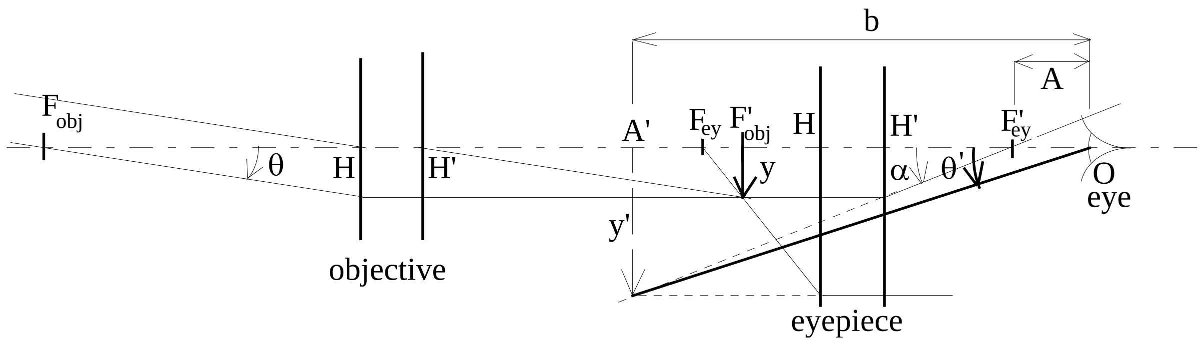


We can easily determine the intrinsic magnifying power using the construction of a ray coming from the object at infinity and passing through F_{obj}. We then get:

$$G_i=\frac{f_{objective}}{f'_{eyepiece}}$$

b) the object is at infinity but the image is at a finite distance

Since it is a visual instrument, the image must be between the ponctum remotum and the punctum proximum of the eye. In this case the angle θ' will depend on the position the eye and on the adjustment of the instrument. We can write the magnifying power using the following construction:



The figure resembles the one we made to determine the power in the case of an image at a finite distance, except here we note A the distance between the second focal point of the eyepiece (and not of the whole system) and the eye: $A=\overline{F'_{ey}O}$. We can for example write the angular magnification of the afocal system as a function of the power of the eyepiece:

$$G=\frac{\theta'}{\theta}=\frac{\theta'}{y}\cdot\frac{y}{\theta}=-P_{ey}\cdot f_{obj}$$

Using the expression of the power of the eyepiece when it is adjusted for an image at a finite distance, we get:

$$G=\frac{f_{obj}}{f'_{ey}}(1-\frac{A}{b})=G_i(1-\frac{A}{b})$$

We verify that the magnifying power is equal to the intrinsic magnifying power when the instrument is afocal (b=∞), but also when **the eye is located in the second focal plane of the eyepiece**. We should thus try to design the instrument so that this is possible, so that the magnifying power does not depend on the adjustment of the instrument.

c) magnifying power in the case of an object at a finite distance

Even though the power is the most logical parameter to characterize a visual instrument designed to observe near objects (microscope type), we can also define a magnifying power for such an instrument. In this case, the angular size θ of the object as seen by the naked eye will depend on the properties of the eye. It is logical to choose for θ the maximum value obtained in the situation when the object is placed at the ponctum proximum, at a distance *d* from the eye. We will thus get:

$$\theta=\frac{y}{d}$$

As for θ', we can take it from the expression of the power in the case of an arbitrary adjustment of the instrument (using notations of 2b):

$$\theta'=P\cdot y=-\frac{y}{f'}(1-\frac{a}{b})$$

The magnifying power can thus be written:

$$G=\frac{\theta'}{\theta}=-\frac{d}{f'}(1-\frac{a}{b})$$

We can define a so-called **commercial intrinsic magnifying power** (a=0 or b=∞) taking a normal emmetropic eye with a ponctum proximum at *d*=25cm:

$$G_{ic}=-\frac{250\text{mm}}{f'(\text{in mm})}=\frac{P_i(\text{in diopters})}{4}$$

For example a so-called « x10 » eyepiece is an eyepiece with a commercial intrinsic magnifying power of 10: its intrinsic power is thus 40 diopters and its focal length is 25mm.

For a microscope, a magnifying power is also given by:

$$G_{microscope}=\frac{P_{microscope}}{4}=\frac{P_{eyepiece}}{4}.(g_y)_{objective}$$

$$\boxed{G_{microscope}=G_{eyepiece}.(g_y)_{objective}}$$

So if we combine an objective with a magnification of 20 and a «x10» eyepiece, we get a microscope with a magnifying power of 200, a power of 800 δ and a focal length of 1.25mm.