CHAPTER XI

PROPERTIES OF OPTICAL INSTRUMENTS:
Field of view and depth of field

We have said in the previous chapter that the field of an instrument is the portion of space that is seen "clearly" through the instrument. We will now be more precise about what limits this portion of space, and this will lead us to characterize:

- the field of view, area perpendicular to the axis of the instrument, which will be limited by the different diaphragms inside the instrument (finite size of the lenses and mirrors, iris diaphragms), or in the plane of the image itself (size of the film, size of the detector);

- the depth of field, distance along the axis, which depends on the detector used.

Consequently, we see that the field is well defined for an instrument designed for a specific function (in particular a given position of the object observed), with specified diaphragms and detector.

I. Field of view

Our goal is now to take into account the finite size of the optical elements (and the mechanical diaphragms) that form an instrument, to determine the rays originating at the object plane, and that can get out of the instrument to form an image. This will lead us to define the notions of aperture and of field of view. We will consider here only centered systems, which will be supposed to be stigmatic and aplanetic for the couple of conjugate points used (this implies that the aberrations will be corrected for the aperture and the field that we will have determined).

We have chosen here to treat the problem first on an example, then to generalize the intuitive method to an arbitrary system, with two or more diaphragms.

1) Study on one example

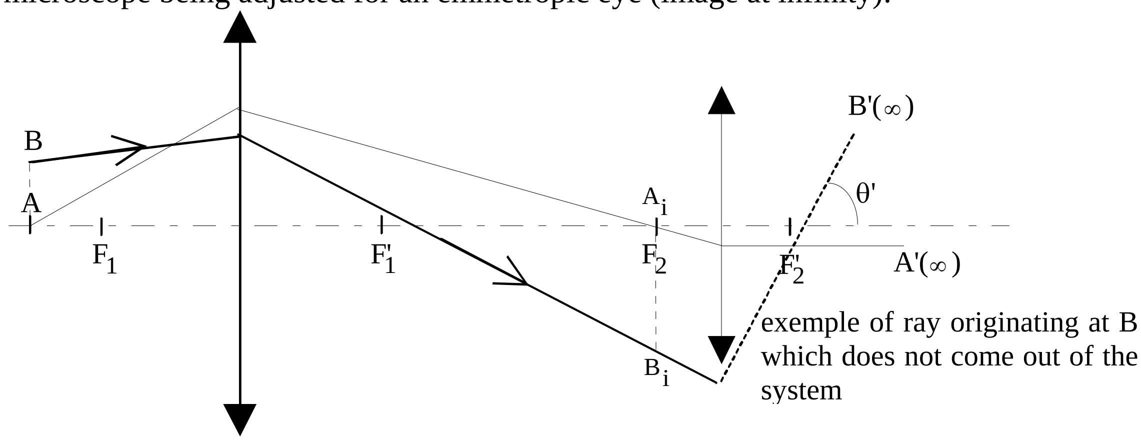
We consider a low magnification microscope composed of two thin lenses:

- an objective with magnification $g_y=-2$, focal length f'_1 =40mm and diameter $Ø_1$ = 6mm;

- a « x12.5» eyepiece with diameter $Ø_2$ = 4mm.

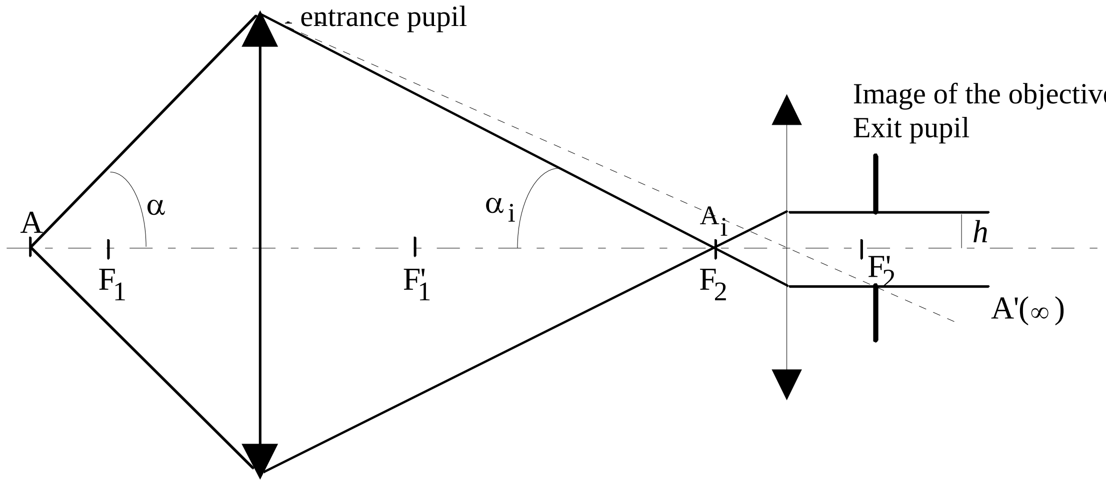
Note: this system has a negative focal length (-10mm), like most microscopes.

The microscope being adjusted for an emmetropic eye (image at infinity):



We have drawn on the figure one example of a ray originating at B in the field of A, which is blocked by the eyepiece mechanical mount.

Let us first determine the limits of a bundle of rays originating at the object A on axis and that can get out of the system. Graphically, we easily find this bundle of rays:

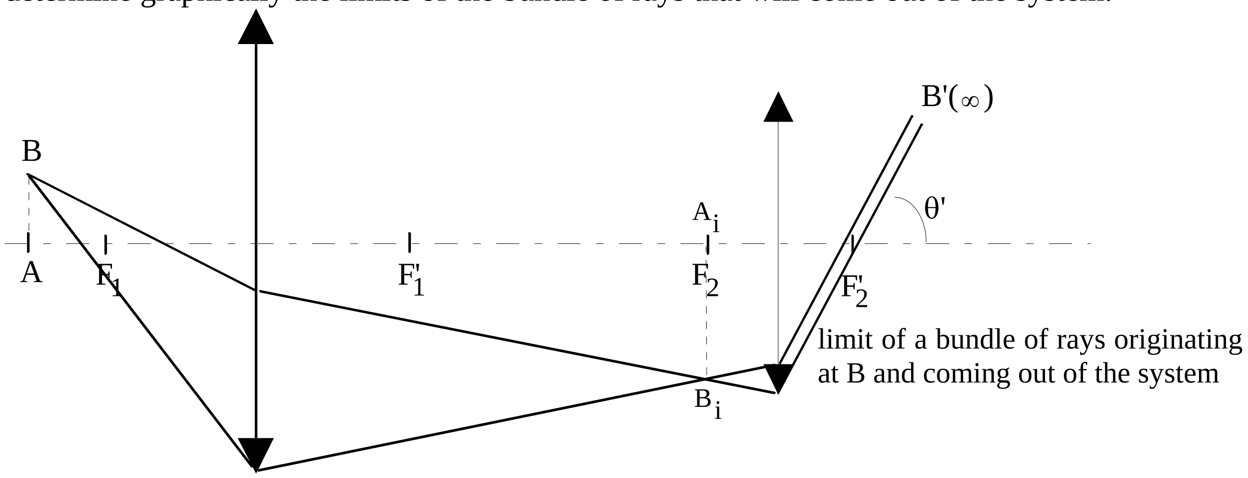


We have thus determined the aperture angle α of the microscope in the object space:

$$\alpha = \frac{\varnothing_1}{2O_1A} = 0.05 rad$$

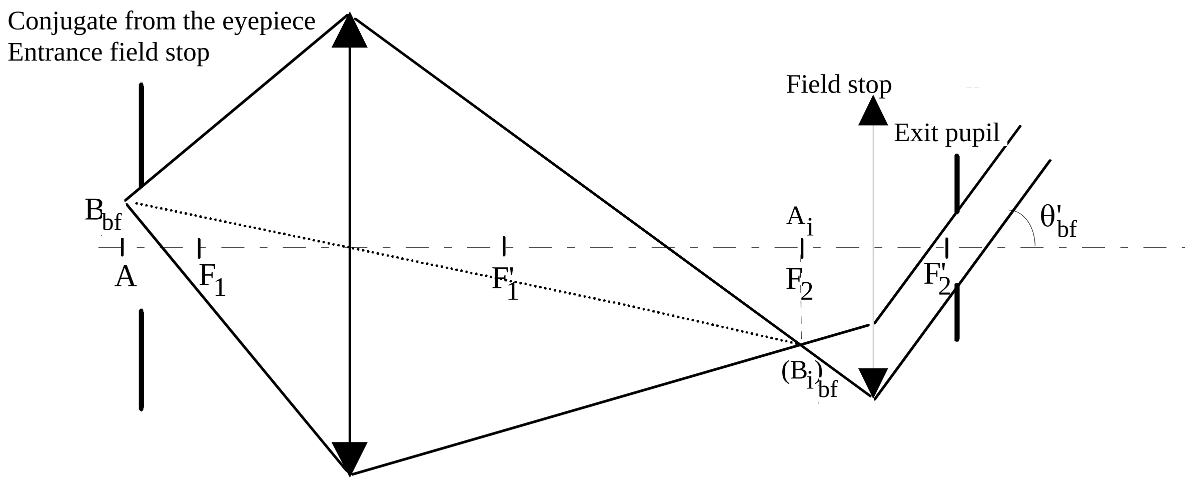
It is limited by the size of the objective lens, which is consequently called **the aperture stop of the instrument.** In this example, it is also **the entrance pupil** since this aperture stop is located in the object space. It is in the intermediate space, where the aperture angle is $\alpha_i = 0.025 rad$, that it is easier to make sure the objective is limiting the aperture, since it is the one which is seen under the smallest angle from the intermediate image A_i. We can however verify that the same reasoning applies to the object space or in the image space, if we conjugate the diaphragms in those spaces. For example in the image space, we can construct the image of the edges of the objective: we get a diaphragm with a radius $h = 1mm$, which is smaller than the radius of the eyepiece. We recover the fact that it is the image of the objective, which is seen under the smallest “angle” from the image (here the image is at infinity, the angle is replaced by a height with respect to the axis). The image of the objective, which limits the aperture in the image space, is called **the exit pupil** of the microscope.

Let us now look at an object point B off axis. For an arbitrary point B, we can determine graphically the limits of the bundle of rays that will come out of the system:



We can see on this example that the bundle of rays has a smaller aperture than in the case of point A, because the eyepiece now also limits it: the eyepiece is thus called **a field stop** of the instrument. The image of point B will not be as bright as the image of A.

Let us try to find at which distance from point A the limitation due to the eyepiece starts being effective. We can determine it graphically:

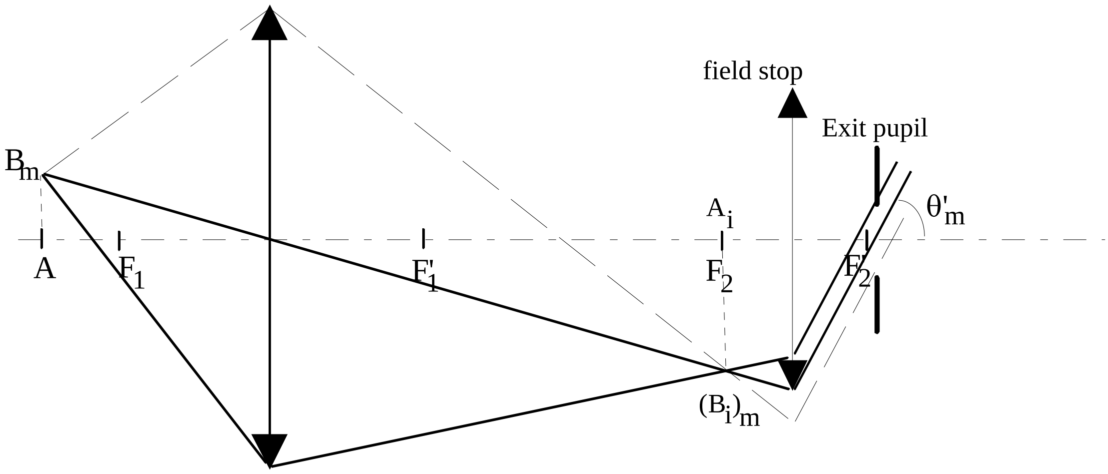


The point B_{bf} that we determined is called the edge of **the bright field of view**. Note that, because of the axial symmetry, the bright field of view is a circle with radius $y_{bf}=\overline{AB_{bf}}$.
Once again we can see that the construction is easier in the intermediate space, but it can be done in an equivalent way in the image space or in the object space, by imaging the objective and eyepiece in these spaces. In the intermediate space, the radius of the bright field of view $(y_i)_{bf}$ is given by:

$$\frac{3+(y_i)_{bf}}{120}=\frac{2-(y_i)_{bf}}{20} \qquad \text{so that } (y_i)_{bf}=1.29mm$$

Applying the conjugation formulas, we get in the object space: $y_{bf}=0.64mm$, and in the image space: $\theta'_{bf}=0.064rad$.

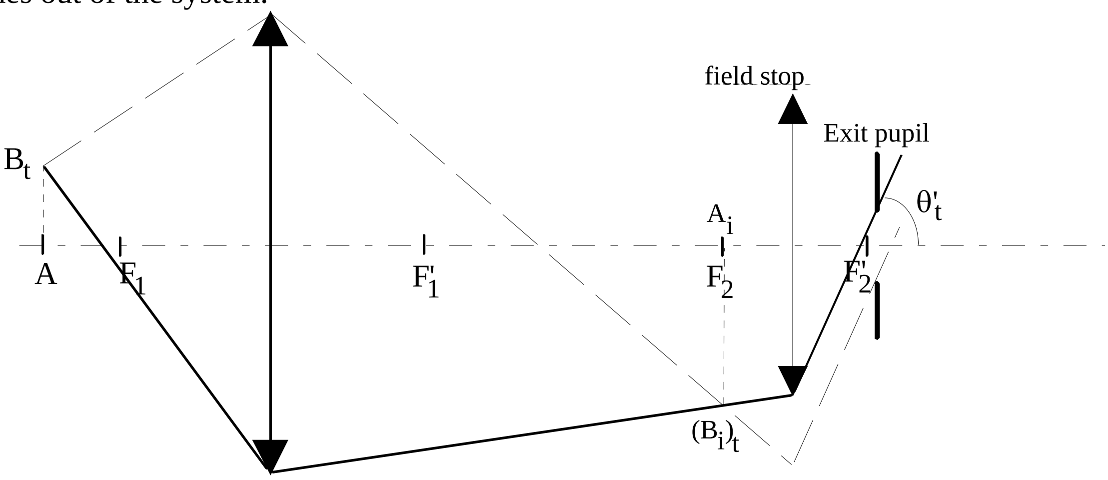
If we continue moving away from the axis further than the bright field of view, the aperture of the rays that come out of the system decreases. We go through an intermediate situation when approximately half of the beam is blocked compared to the case of the bright field:



This position is called the medium field of the instrument. We determined it in the intermediate space through a line joining the center of the pupil to the edge of the field stop, and we get:

$$(y_i)_m = 1.71mm \qquad y_m = 0.86mm \qquad \theta'_m = 0.086rad$$

Finally when we move even further from the axis, we reach a situation when no ray comes out of the system:



This is the **edge of the total field**: further than that, the instrument does not make any image. The value of its radius is here: $(y_i)_t = 2.14mm \qquad y_t = 1.07mm \qquad \theta'_t = 0.107rad$

2) General method

Along the lines of the example above, we can derive a general method to analyze the field of view of an instrument, for a system composed of two or more diaphragms. We will proceed in three steps:

- we **choose a « work space »** (object or image space, or one of the intermediate spaces) in which we will determine the position and size of the conjugates of all the diaphragms of the system, as well as the position of the intermediate image plane. We should of course choose the work space in order to minimize the number of conjugates that have to be calculated (such was the intermediate space in the previous example).

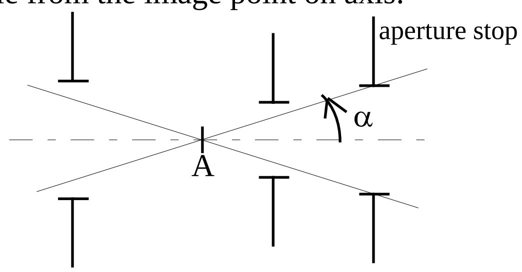
- we **solve the problem in this work space**, which means that we determine the aperture and the field of view in this space: we find the rays that go through all the diaphragms with the largest possible aperture, first from the image point on axis (determination of the aperture), then from an image point off axis (determination of the field of view). We will see in more detail in the next two paragraphs how to find these limit rays in practice.

- we know from imaging rules that the rays that were not blocked in the work space will not be blocked either in any of the other spaces. The only thing left to do is thus to calculate, using conjugation formulas, the aperture and the field of view in the object and the image spaces. We should finally **verify the calculations by a construction** of the path of a bundle of rays for each case (aperture, bright field, total field) through the whole system.

Note: in all the following paragraphs, we will use the term image instead of conjugate, independently of the real direction of propagation of light. One must then be careful about the way conjugation formulas are written, and must also identify clearly the space in which this image is located especially when it is a virtual image.

3) Aperture of an instrument. Pupils.

If we generalize the method used for the example of the microscope, we can see that the aperture stop is found as the diaphragm (real or conjugate of a real diaphragm), which is seen under the smallest angle from the image point on axis:



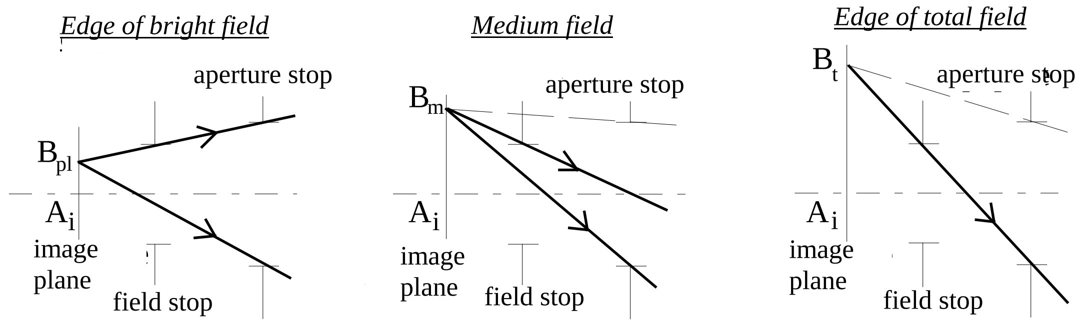
We get the aperture angle α of the system in this space, and then through imaging we get the aperture angle in the object space and the entrance pupil of the system, as well as the aperture angle in the image space and the exit pupil of the system.

Let us mention here a convention relative to the aperture of objectives (such as camera objective for example) that is often characterized by an **aperture number N or f-number**, sometimes called geometrical aperture number. This number represents the ratio between the focal length of the objective and the diameter of its entrance pupil: the objective is then said to be open at f/N (be careful, this quantity is not equal to the angle α since the distance from the object to the entrance pupil is not usually equal to f'), or that its f-number is equal to N. We will encounter again this parameter in the chapter about photometry (chapXII).

4) Field of view of an instrument. Field stops.

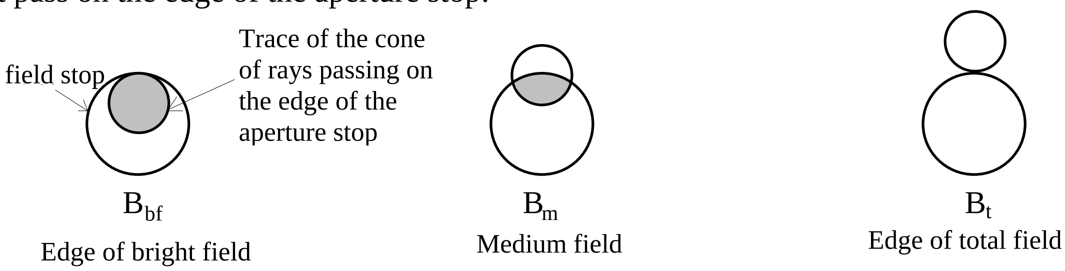
a) System with two diaphragms

This was the case of the microscope seen earlier as an example. One of the diaphragms was the aperture stop, so the other one has to be the one that limits the field, it is thus the field stop. Similarly as what we saw for the example of the microscope, we can determine the edges of the bright field of view, of the medium field and of the total field. In the work space where we brought through imaging the two diaphragms and the image space, we have the following situations:



We have deliberately chosen here a configuration where the relative positions of the diaphragms and of the image plane are different from those of the microscope example.

To illustrate the transitions between the different parts of the field of view, it is interesting to represent in the plane of the field stop the trace of the cone of rays originating at B that pass on the edge of the aperture stop:



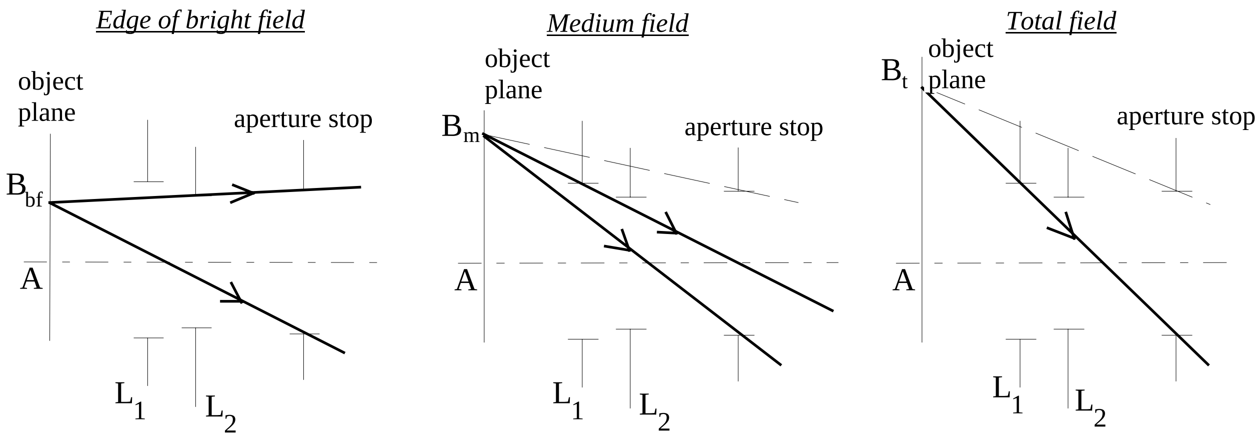
Using this drawing, we can connect the geometrical fields to their photometric properties. Indeed, we can see that the amount of light transmitted through the system will be approximately constant over the bright field (we will see later that it still varies slightly), then it decreases progressively and finally goes to zero on the edge of the total field. This intermediate area is also called **contour field.** We can understand that in general we will want a bright field as large as possible; in addition we will often prefer to avoid this contour field. We will see a method to suppress it a little further.

Note : it can happen that the two diaphragms are seen under the same angle from the object point on axis. In this case, there is no bright field. This situation should usually be avoided.

b) instrument with more than two diaphragms

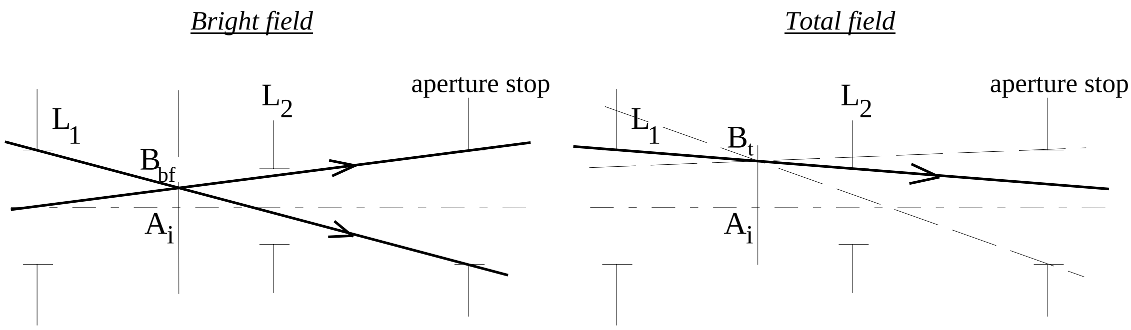
When the system includes more than two diaphragms, the problem is more complex because it can happen that several diaphragms contribute to the limitation of the field of view. A drawing is the best way to determine the fields in this case. To give an idea of the different possibilities, we have drawn below two examples:

example 1



In this first example, the bright field is limited by the field stop L_2, while the medium and the total fields are limited by the other field stop L_1.

example 2



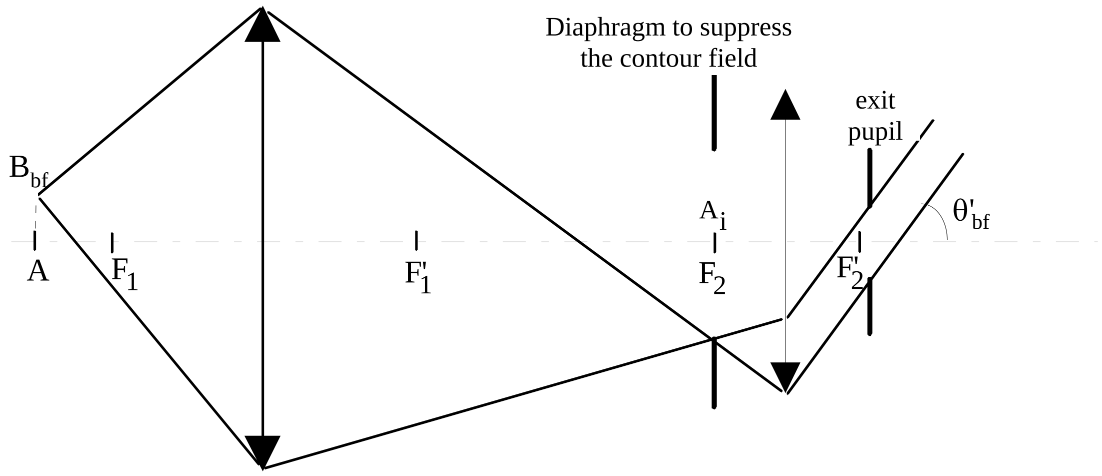
In this example, the bright field is limited by L_1, then L_2 will start blocking the other side of the beam, so that there is no clear medium field. The total field is finally limited by the two edges of L_1 and L_2 and does not involve the diameter of the aperture stop.

5) Suppression of the contour field

When we move out of the bright field, the illumination in the image plane decreases progressively until it reaches zero on the edge of the total field. We call “contour field” this area between bright field and total field.

It is often prefered to suppress this contour field, so that the illumination is abruptly zero on the edge of the bright field of view. For example in the case of a picture taken by a camera, an additional diaphragm is placed in the plane of the final image (24x36 frame in the case of standard film for example). We can also place this diaphragm in an intermediate space, provided the image is real in this space.

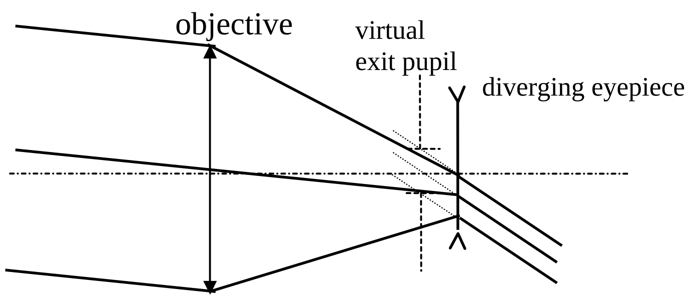
If we get back to the initial example of the microscope, we could suppress the contour field in the intermediate space, since the object space is usually not accessible and the image is at infinity:



6) Position of the eye for a visual instrument

The best location for the eye pupil behind a visual instrument is on the exit pupil of the instrument. Indeed, if it is not placed there, the eye pupil will act as an extra diaphragm in the system and might reduce the bright field of view. On the contrary, if the eye pupil is on the exit pupil of the instrument, and if we choose a diameter of the exit pupil of the instrument smaller than the eye pupil (which is approximately 2mm in diameter in day vision), the fields of view of the instrument will not be modified.

We should not that this configuration is only possible if the exit pupil of the instrument is real (and even not too close to the last lens of the instrument). One example of an instrument with a virtual exit pupil is Galileo’s telescope:

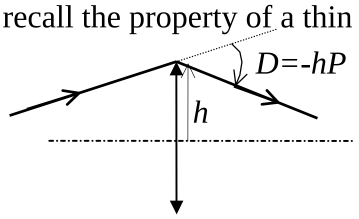


7) Role of the field lens in an eyepiece

Visual instruments are often composed of an objective and an eyepiece. The microscope-type instruments, such as the one we studied as an example, consists of an objective that makes a highly magnified image of an object at a finite distance (near), image that is then sent at infinity by the eyepiece, so that it can be seen comfortably by the eye (the intermediate image is then at the first focal point of the eyepiece. In a telescope-type instrument, the focal points of the objective and of the eyepiece are superimposed in order to constitute an afocal system.

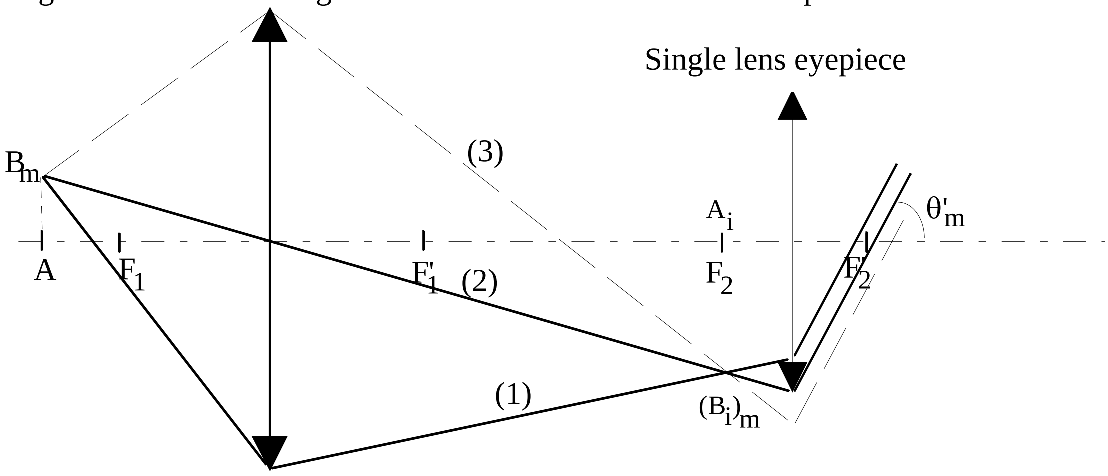
We shall see, on our microscope example, the advantage, in terms of field of view, of using an eyepiece with two lenses, the closest one to the eye being called **eye lens** and the other one **field lens**.

Let us first recall the property of a thin converging as a deflector of ray

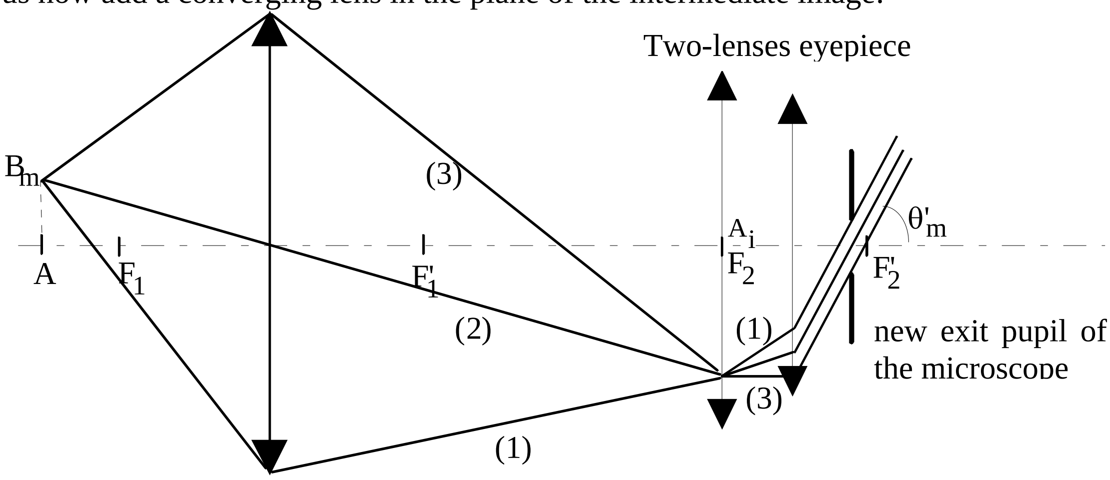


We see that a converging lens always deflects the rays towards the axis.

Let us go back to the drawing of the medium field of our microscope:



Let us now add a converging lens in the plane of the intermediate image:



Since it is located in the plane of the intermediate image, the additional lens does not change the position nor the size of the image, which remains at infinity with angular radius θ'_m. On the contrary, the field of view has increased since the ray (3), blocked previously, can now be deflected towards the axis and pass through the second lens of the eyepiece.

We thus see qualitatively that the additional lens increase the field of view of the microscope: it is the **field lens** of the eyepiece, while the lens closest to the eye is called **eye lens**.

Thanks to the previous demonstration and to the expression of the deflection of the lens L_2 ($D = -hP$), we see that the more converging the field lens (keeping its diameter constant), the larger the field of view. However this will bring the exit pupil of the instrument closer to the eye lens (see figure): when the focal length of the field lens is equal to that of the eye lens, the second focal point F' of the microscope is on the eye lens, and the exit pupil is very close to F'. It will then become virtual and we will not be able to place the eye pupil on it. This limits the increase in field of view that can be reached with the field lens.

In practice, it is preferable not to place a lens in the plane of the intermediate image, first because we want to place a reticule there, second because any dust or scratch on the surface of the lens will appear on the image itself. We thus keep the general idea of an eyepiece with two lenses but we move slightly the field lens with respect to the first focal point of the eye lens. The most common eyepieces are the Ramsden eyepiece 3-2-3, which has a real first focal point (positive eyepiece), slightly in front of the field lens, and the Huyghens eyepiece 3-2-1, which has a virtual first focal point (negative eyepiece) located between the two lenses.

Note: *when using a negative eyepiece, such as the Huyghens', the first focal point is virtual thus not accessible. If we want to use a reticule, it must be placed in the intermediate image plane between the two lenses of the eyepiece, that is, at the first focal point of the eye lens.*

II. Depth of field

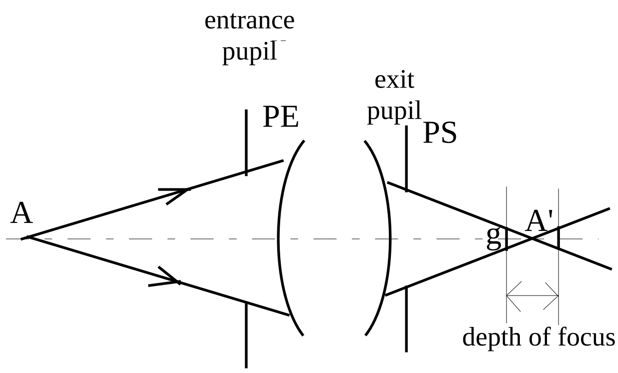
Depth of field is well known by photographers: it is the distance, along the direction of shooting, along which the image stays in focus. We know by experience that on a camera, this depth of field increase when the diaphragm is closed (larger f-number). It depends also on the type of detector used, more specifically on its pixel size (for recording film, this "grain" is larger when the sensitivity is higher).

More generally, depth of field is characteristic of the combination of an instrument and a detector.

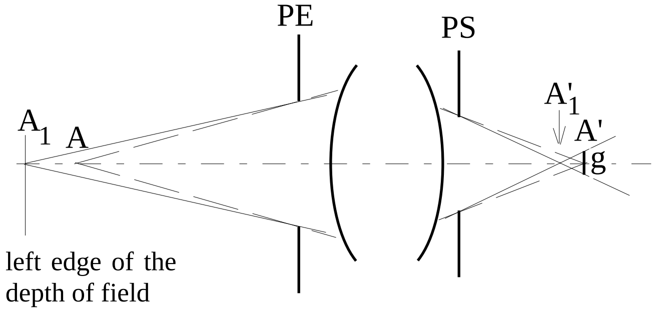
1) Depth of field for projection instruments

Let us continue with the example of a camera, that we will assume to be without aberrations and with a large enough aperture so that diffraction is negligible compared to the pixel size g of the detector.

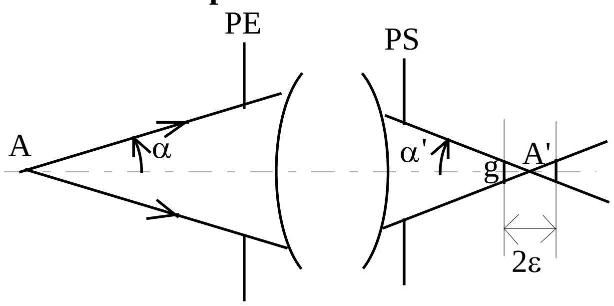
A punctual object A gives through the camera a punctual image A'. But, because of the detector, we cannot distinguish it from an image with size g. This implies that in the image space, there is a certain tolerance on the position of the detector, which is called **depth of focus:**



In the object space, for a given position of the detector, we will also have a certain tolerance on the position of the object A so that its image covers one pixel of the detector; this is called the **depth of field:**



a) Calculation of the depth of focus



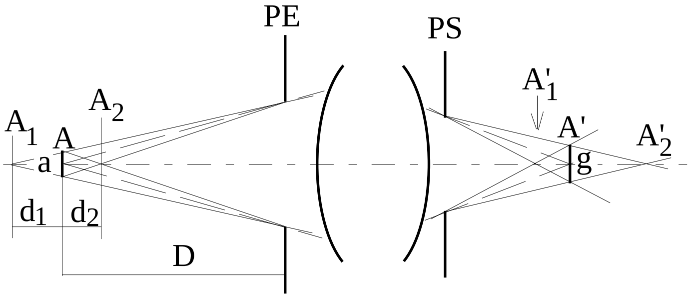
The depth of focus is symmetrical with respect to the position of the image A'. If we call α' the aperture angle in the image space, we can write:

$$2\varepsilon = g/\alpha'$$

The depth of focus is larger when the aperture of the instrument is smaller. To make sure the positioning of the detector is correct, we should use the maximum aperture of the system.

Notice finally that the sharpness of the image does not fall abruptly at ±ε but decreases progressively: the depth of focus 2ε gives an approximate value of this effect.

b) Calculation of the depth of field



The detector is now fixed in the plane of the image A', and we look for the extreme positions A_1 and A_2 in the object space such that the image spot in the plane of the detector has a diameter g. Through imaging, they correspond to an "object spot" in the plane of the object A with a diameter:

$$a = g/g_y$$

where g_y is the transverse magnification of the instrument for the conjugate points A and A'.

We name ∅ the diameter of the entrance pupil and D the distance from the object A to the entrance pupil. We get:

$$\frac{d_1}{D+d_1}=\frac{a}{\varnothing} \quad \text{so that:} \quad d_1=\frac{aD}{\varnothing-a}$$

$$\frac{d_2}{D-d_2}=\frac{a}{\varnothing} \quad \text{so that:} \quad d_2=\frac{aD}{\varnothing+a}$$

Note that **the depth of field is in general not symmetric around the object plane** ($d_1>d_2$). Indeed when we take a picture of a parade for example, we should focus approximately on the first third of the line. We can in fact be more precise and calculate the distance D of the best focus point knowing the extreme positions D_1 and D_2 of the parade (we suppose here that the aperture is small enough to allow for a large enough depth of field). You can show as an exercise that:

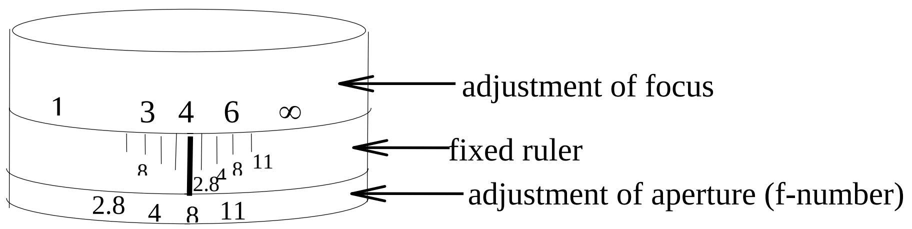
$$D = \frac{2D_1D_2}{D_1 + D_2}$$

For example for a depth of field between 3 and 6 meters, we should focus at 4 meters; or for a depth of field between 1 and 3 meters, we should focus at 1.5m (which is not far from the approximate rule of a third).

It is also interesting to determine what aperture is required for a given depth of field. We then try to connect the aperture number $N=f/\varnothing$, where f is the focal length of the camera, to the extent d of the field depth around the focus distance D. Making a few approximations (distance D large compared to the focal length of the system and diameter a small compared to the diameter of the entrance pupil), we get the following relationship (to prove as an exercise):

$$N = \frac{d\, f^2}{2g\, D^2}$$

For example, with a standard SLR camera objective with focal length 50mm, taking g=30µm as a pixel size, we need an aperture N≈8 to get a depth of field between 3 and 6m (focus distance at D=4m). This characteristic is often written on the objective mount:



One thing to remember is that **depth of field increases with the f-number and with focusing distance, and decreases with focal length**.

Note: for an instrument designed for an object at infinity, point A_2 is the only one that is defined. The depth of field is then characterized by the distance of A_2 from the instrument, with is called *hyperfocal distance*.

2) Depth of field for visual instruments

For a visual instrument, two phenomenon contribute to the depth of field:

- for a fixed accommodation of the eye, the combination instrument+eye+retina constitutes a projection system: the size of the individual receptors of the retina (or of the diffraction spot if it is larger) leads to a certain depth of field;

- in addition the eye accommodates almost automatically: it sees clearly any point along its accommodation length. We can thus define **a depth of accommodation** that corresponds to the distance in the object space of the instrument, which gives an image covering the accommodation length of the eye.

The depth of field of a visual instrument is the sum of these two contributions, where the depth of accommodation usually dominates for a normal eye (without presbytia).

Calculation of the depth of accommodation

Let us consider a microscope as an example. We place the eye in the second focal plane of the instrument (we saw the advantage of this configuration in terms of power, equal to the intrinsic power for any adjustment of the instrument, as well as for fields, the exit pupil of the instrument being often located in the second focal plane).

We note A'_P and A'_R the ponctum proximum and ponctum remotum of the eye. We look for the corresponding object points A_P and A_R.

$$\overline{FA_P}.\overline{F'A'_P} = -f^2 \qquad \overline{FA_R}.\overline{F'A'_R} = -f^2$$

$$\overline{FA_P} = -\frac{f^2}{d} \qquad \overline{FA_R} = -\frac{f^2}{D}$$

where *d* and *D* are the minimum and maximum distances for clear vision.

We get for there the expression of the depth of accommodation:

$$l = \overline{A_P A_R} = f^2\left(\frac{1}{d} - \frac{1}{D}\right) = f^2 A$$

where *A* is the range of accommodation in diopters (4 diopters for a standard eye). For example for a microscope with a power of 400 diopters (f=2.5mm), the depth of accommodation is equal to 25µm.

Note that the depth of accommodation is one of the main sources of uncertainty for longitudinal measurements along the axis. For a fixed object, the accommodating eye gets a clear image for several positions of the microscope, over a distance *l*. However, we can reduce this range of accommodation *A* using an eyepiece with a reticule, giving certain precautions in its adjustment (see optics labs).