Manley-Rowe relations

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Introduction

Let us consider 3 waves at ω_1, ω_2, ω_3

$$\mathcal{E}_j(z,t) = \hat{\mathrm{e}}_j A_j(z,\omega_j) e^{i(k_j z - \omega_j t)} + C.C \qquad \text{with } j = 1, 2, 3$$

$$\omega_1 + \omega_2 = \omega_3$$

Assumption: isotropic medium

$\hat{\mathrm{s}}$ is parallel to the wave vector direction $\hat{\mathbf{k}}$

Energy exchanges

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with $\Delta k = k_3 - k_1 - k_2$

Energy exchanges

Energy density in the nonlinear medium $N\hbar\omega = Nh\frac{c}{\lambda} = |S| = 2\epsilon_0 cn|A|^2$

$$\frac{d|A_j|^2}{dz} = \frac{dA_j}{dz}A_j^* + \frac{dA_j^*}{dz}A_j = \frac{hc}{\lambda}\frac{1}{2\epsilon_0 cn}\frac{dN_j}{dz} \quad (1)$$

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Photon density evolution at ω_3

$$\frac{dN_3}{dz} = \frac{2\epsilon_0 n_3\lambda}{h}\frac{i\omega_3}{n_3 c}\hat{\mathbf{e}}_3\underline{\underline{\chi}}^{(2)}(\omega_1,\omega_2)\hat{\mathbf{e}}_1\hat{\mathbf{e}}_2 A_1 A_2 A_3^* e^{-i\Delta kz} + C.C.$$

$$\frac{dN_3}{dz} = \frac{2\epsilon_0}{\hbar}\hat{\mathbf{e}}_3\underline{\underline{\chi}}^{(2)}(\omega_1,\omega_2)\hat{\mathbf{e}}_1\hat{\mathbf{e}}_2\left(iA_1A_2A_3^*e^{-i\Delta kz} - iA_1^*A_2^*A_3e^{i\Delta kz}\right)$$

Photon density evolutions

$$\frac{dN_3}{dz} = \frac{-4\epsilon_0}{\hbar} \hat{\mathbf{e}}_3 \underline{\underline{\chi}}^{(2)}(\omega_1, \omega_2) \hat{\mathbf{e}}_1 \hat{\mathbf{e}}_2 \mathcal{I}\mathrm{m} \left(A_1 A_2 A_3^* e^{-i\Delta kz} \right)$$

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Using full permutation symmetry,

$$\chi_{eff}^{(2)} = \hat{\mathbf{e}}_3\underline{\underline{\underline{\chi}}}^{(2)}(\omega_1,\omega_2)\hat{\mathbf{e}}_1\hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_2\underline{\underline{\underline{\chi}}}^{(2)}(\omega_3,-\omega_1)\hat{\mathbf{e}}_3\hat{\mathbf{e}}_1 = \hat{\mathbf{e}}_1\underline{\underline{\underline{\chi}}}^{(2)}(\omega_3,-\omega_2)\hat{\mathbf{e}}_3\hat{\mathbf{e}}_2$$

Total energy density

$\frac{dN_2}{dz} = -\frac{dN_3}{dz}$ and $\frac{dN_1}{dz} = -\frac{dN_3}{dz}$

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Energy conservation

Manley-Rowe relations

$$\frac{d(N_1 - N_2)}{dz} = \frac{d(N_2 + N_3)}{dz} = \frac{d(N_1 + N_3)}{dz} = 0 \qquad (2)$$

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The annihilation (resp. creation) of a photon at ω_3 is automatically associated with the creation (resp. annihilation) of one photon at ω_1 and one photon at ω_2.