Nonlinear ELectromagnetism
Tutorial n°4
Optical Parametric Oscillator

A pump beam at frequency ω_3, propagating in a $\chi^{(2)}$ nonlinear crystal can amplify waves at frequencies ω_1 and ω_2, that have been initially generated in the crystal through spontaneaous parametric down conversion. These two waves are respectively called signal and idler and their frequencies satisfy the relation $\omega_3 = \omega_1 + \omega_2$.

When the pumped crystal, is placed in an optical cavity, with one of the frequencies ω_1,ω_2 (or both) resonant in the cavity, parametric amplification can, above a pump intensity threshold, generate an oscillation at ω_1 and ω_2. The system is then an optical parametric oscillator, in which losses of the cavity modes are compensated by the parametric gain. A simple parametric oscillator is considered (see Fig. 1). The whole cavity space between the mirrors M and M' is filled with the nonlinear crystal of length L. These mirrors with amplitude reflexion coefficients r_i and r'_i ($i = 1$,2) at the signal ($i = 1$) and idler ($i = 2$) frequencies are transparent at ω_3. For simplicity, r_i et r'_i are assumed real.

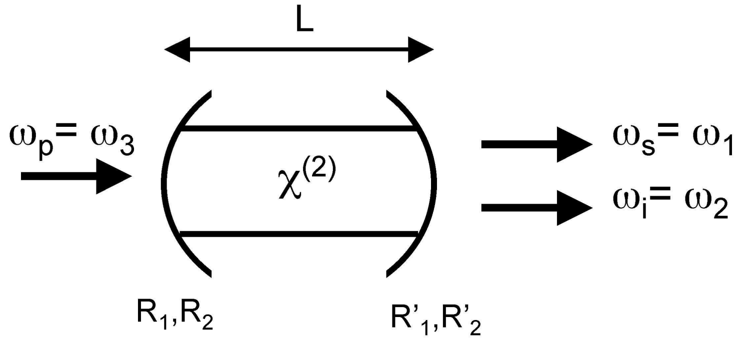

FIG. 1 – *Optical parametric oscillator*

When the parametric oscillator is pumped by the wave $\boldsymbol{\mathcal{E}}_3(z,t)$, at frequency ω_3, signal and idler waves propagating forward (they are denoted $\boldsymbol{\mathcal{E}}_1^F(z,t)$ and $\boldsymbol{\mathcal{E}}_2^F(z,t)$) and backward (they are denoted $\boldsymbol{\mathcal{E}}_1^B(z,t)$ and $\boldsymbol{\mathcal{E}}_2^B(z,t)$) are generated in the cavity from spontaneaous parametric down conversion. They can be written

$$\boldsymbol{\mathcal{E}}_{1,2}^F(z,t) = \boldsymbol{E}_{1,2}^F(z) \exp -i(\omega_{1,2}t) + C.C,$$
$$\boldsymbol{\mathcal{E}}_{1,2}^B(z,t) = \boldsymbol{E}_{1,2}^B(z) \exp -i(\omega_{1,2}t) + C.C,$$

with

$$\boldsymbol{E}_{1,2}^F(z) = \boldsymbol{e}_{1,2} A_{1,2}^F(z) \exp i(k_{1,2}z),$$
$$\boldsymbol{E}_{1,2}^B(z) = \boldsymbol{e}_{1,2} A_{1,2}^B(z) \exp -i(k_{1,2}z).$$

The phase mismatch is denoted $\Delta k = k_3 - k_1 - k_2$.

1. Taking parametric amplification into account, give the conditions relating the amplitudes $A_{1,2}^F(z = 0)$ and $A_{1,2}^F(z = L)$.
2. What can be said about the amplification of the waves $\boldsymbol{\mathcal{E}}_{1,2}^B(z,t)$? In the same way, give the conditions relating their amplitudes at $z = L$ and $z = 0$.

3. Use the boundary conditions on the mirrors to give the two relations satisfied by $A_1^F(L)$ et $A_2^{F*}(L)$.
4. Derive the threshold condition of the oscillator.
5. Give this condition in the specific case of perfect phase matching or very small phase mismatch. ($\Delta k \simeq 0$).
6. **Singly resonant oscillator** The cavity is only resonant at ω_1 ($r_2 = r_2' = 0$). Write the threshold condition in the case of a very small phase mismatch and a low loss resonator.
7. **Doubly resonant oscillator** The cavity is resonant at both signal and idler frequencies. As for the singly resonant oscillator, write the threshold condition in the case of a very small phase mismatch and a low loss resonator. Compare the two cases and comment.