Entanglement and spontaneous parametric down conversion

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Hamiltonian

$$\hat{\mathcal{H}} = \sum_{j=p,s,i} \hbar\omega_j \left(\hat{n}_j + \frac{1}{2}\right) + \hbar g \left(\hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_p + cc\right) \quad (1)$$

$\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ photon number operator
g coupling constant, including the nonlinear susceptibility

Parametric down conversion: interaction hamiltonian

$$\hat{\mathcal{H}}(t) = \frac{\epsilon_0}{2}\int_V d^3 r \chi^{(2)} \hat{E}_p(r,t)\hat{E}_s(r,t)\hat{E}_i(r,t)$$

$$\hat{\mathbf{E}}^{(+)} = \iota \sum_\ell \sqrt{\frac{\hbar\omega_\ell}{2\epsilon_0 L^3}} \vec{\varepsilon}_\ell\, \hat{a}_\ell e^{\iota(\mathbf{k}_\ell\cdot\mathbf{r}-\omega_\ell t)}$$

$$\hat{\mathcal{H}}_{int}(t) \propto \frac{1}{L^3}\epsilon_0 \sum_\ell \sum_{\ell'} \mathcal{A}_p\, \vec{\varepsilon}_p \chi^{(2)}(\omega_p,\omega_\ell,\omega_{\ell'})\vec{\varepsilon}_\ell \vec{\varepsilon}_{\ell'} \int_V d^3 r\, \hat{a}_\ell \hat{a}_{\ell'}$$
$$\times\, e^{\iota(\mathbf{k}_p-\mathbf{k}_\ell-\mathbf{k}_{\ell'})\cdot\mathbf{r}} e^{\iota(\omega_\ell+\omega_{\ell'}-\omega_p)t} + H.c.$$

Time evolution

$$i\hbar \frac{d|\psi\rangle}{dt} = \hat{\mathcal{H}}_{int}(t)|\psi\rangle$$

$$|\psi(t)\rangle = \exp\left(\frac{1}{i\hbar}\int_0^t \hat{\mathcal{H}}_{int}(t')dt'\right)|\psi(0)\rangle$$

with $\quad |\psi(0)\rangle = |0\rangle_\ell |0\rangle_{\ell'}$

Spontaneous parametric down conversion photon pair state

$$|\psi(t)\rangle \simeq \left(1 + \frac{1}{i\hbar}\int_0^t \hat{\mathcal{H}}_{int}(t')dt'\right)|\psi(0)\rangle$$

$$|\psi(t)\rangle \simeq |\psi(0)\rangle + C\sum_{\ell}\sum_{\ell'}\Phi(\Delta k_{\ell,\ell'},\omega_\ell,\omega_{\ell'},\omega_p)\delta(\omega_\ell+\omega_{\ell'}-\omega_p)\,|1\rangle_\ell|1\rangle_{\ell'}$$

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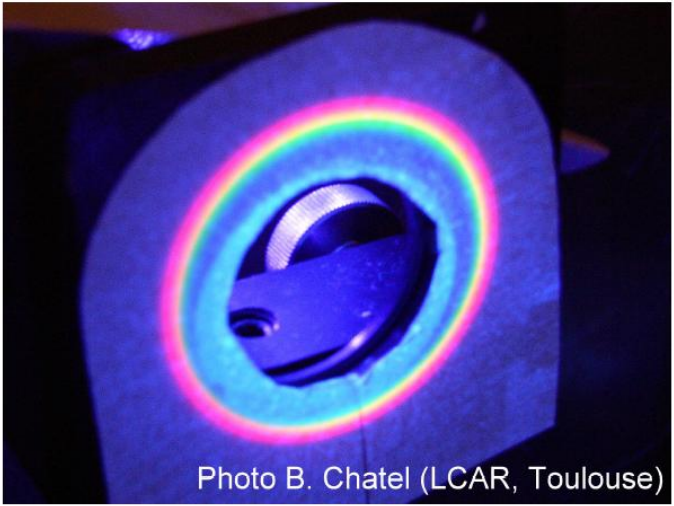
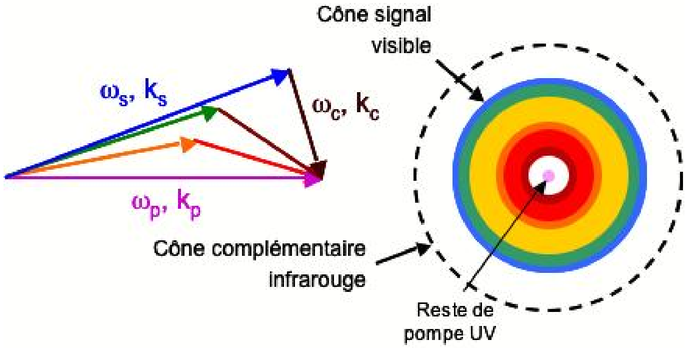
entangled state

Spontaneous parametric down conversion



$$\lambda_P = 400\text{nm}$$

Spontaneous parametric down conversion



Franson Bell experiment

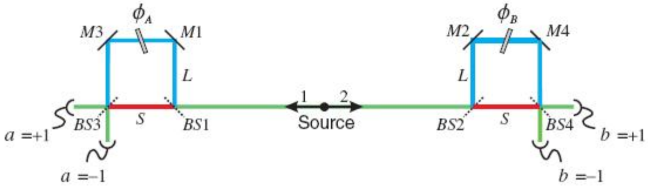


FIG. 1 (color online). Generic setup of the Franson Bell experiment.

Franson Bell experiment

source can be

- atomic cascade
  - first excited level : long lifetime τ_1
  - intermediate level: $\tau_2 \ll \tau_1$

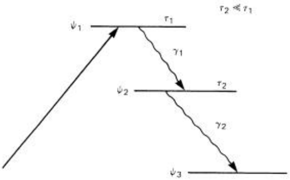


FIG. 1. Three-level atomic system with a relatively long lifetime τ_1 for the initial state and a much shorter lifetime τ_2 for the intermediate state.

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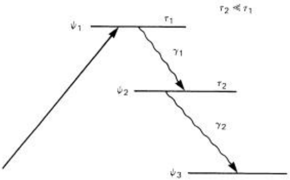
  - $\tau_1 \longleftrightarrow \mathcal{T}^c_{\text{pump}}$
  - τ_2 very short (virtual level)



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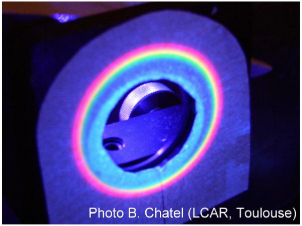

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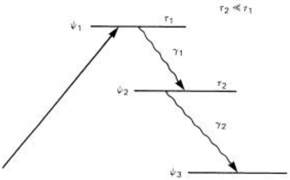
in both case,

- emission time
- time difference between the two photons



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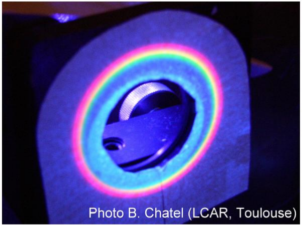

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  - $\tau_1 \longleftrightarrow \mathcal{T}^c_{\mathrm{pump}}$
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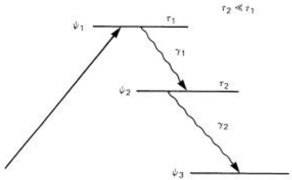
in both case,

- emission time unknown
- time difference between the two photons precisely known



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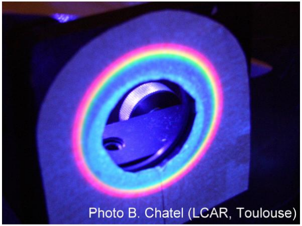

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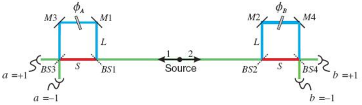
Franson Bell experiment: interferences

Indistinguishability between

$|S\rangle_A \otimes |S\rangle_B$ and $|L\rangle_A \otimes |L\rangle_B$



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Feynman and interferences in quantum physics

Indistinguishable paths $\longleftrightarrow$ interference

- A given path $(|s\rangle_A \otimes |s\rangle_B) \longrightarrow$ given result with a probability $|\alpha_A|^2$
- Another path $(|l\rangle_A \otimes |l\rangle_B) \longrightarrow$ same result with a probability $|\alpha_B|^2$
- If the paths are distinguishable , the probability of the result is $|\alpha_A|^2 + |\alpha_B|^2$ (incoherent sum)
- If the paths are indistinguishable, the probability of the result is $|\alpha_A + \alpha_B|^2$ (coherent sum) $\longrightarrow$ **the relative phase is important**

Franson Bell experiment: interferences

Coincidence rate

$$R_c = \frac{1}{4} R_0 \cos^2\left(\frac{\Delta E \Delta T/\hbar + \Phi_A + \Phi_B}{2}\right)$$

ΔT transit time difference between short and long arm of the interferometer (ΔL)

physical process	ΔT	ΔE
atomic cascade	$\tau_2 \ll \Delta T \ll \tau_1$	$E_3 - E_1$
SPDC	$\mathcal{T}^c_{\text{pump}} \gg \Delta T \gg \mathcal{T}^c_{\text{photons}}$	$\hbar\omega_{pump}$

Synchronization issue

Possible only in the case of spontaneous down conversion in a nonlinear crystal

[1] Tanzilli, λ=710 nm
[2] Tanzilli, λ=657 nm

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- OK with CW laser $\mathcal{L}^c_{\mathrm{pompe}} \sim 300$ m[1] ($\Delta L \sim$ qq cm)
- impossible with a pulsed laser : 400 ps $\rightarrow \mathcal{L}^c_{\mathrm{pompe}} \sim 140\,\mu\mathrm{m}$[2] ($\Delta L \sim \mu\mathrm{m}$)

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Synchronization issue

With a pulsed pump

$|S\rangle_A \otimes |S\rangle_B$ and $|L\rangle_A \otimes |L\rangle_B$
are no longer indistinguishable

because $\mathcal{T}^c_{\mathrm{pulsedpump}} \ll \Delta T$

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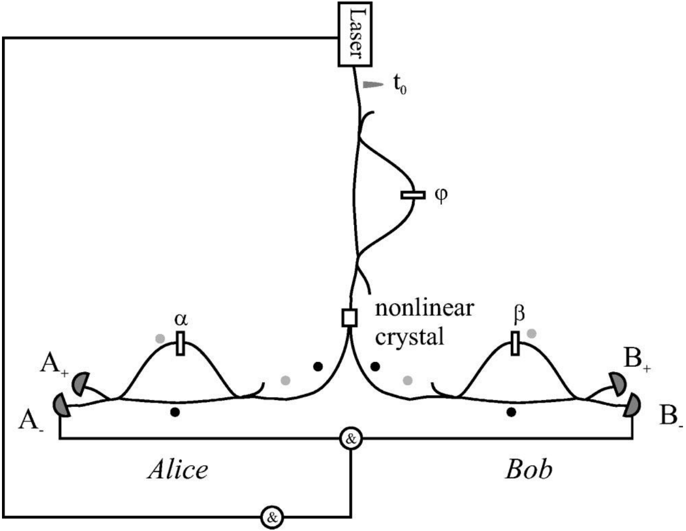
Restaure undetermination of the emission time

Synchronization issue : time-bin entanglement

Pump preparation



Time-bin entanglement

because

$\Delta L \gg \mathcal{L}_c(pulsed\ \ pump)$

The pump photon has passed

- **either** through the short path
- **or** through the long path

States that can be produced

$$\frac{1}{\sqrt{2}}\left(|S\rangle_P|S\rangle_P + e^{i\phi}|L\rangle_P|L\rangle_P\right)$$
$$\frac{1}{\sqrt{2}}\left(|S\rangle_P|L\rangle_P + e^{i\phi}|L\rangle_P|S\rangle_P\right)$$

Time-bin entanglement

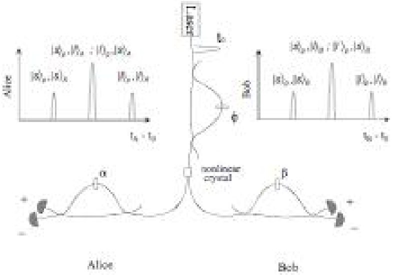


FIG. 1. Schematics of quantum key distribution using energy-time Bell states.

Two independent bases

- Time basis
  $|SS\rangle_A \otimes |SS\rangle_B$ (left satellite time-bin)
  $|LL\rangle_A \otimes |LL\rangle_B$ (right satellite time-bin)
  Time correlation

Time basis

$\longrightarrow$ "Particle like behavior"

- Energy basis
  $|SL\rangle_A \otimes |SL\rangle_B$ or $|LS\rangle_A \otimes |LS\rangle_B$ indistinguishable
  Interferences
  $P_{i,j} = \frac{1}{4}(1 + ij\cos(\alpha + \beta - \phi))$ where $i, j = \pm 1$ (detector labels)

Energy basis

$\longrightarrow$ "Wave like behavior"

Polarisation entanglement

$$\frac{1}{\sqrt{2}}\left(|H\rangle_A|H\rangle_B + e^{i\phi}|V\rangle_A|V\rangle_B\right)$$

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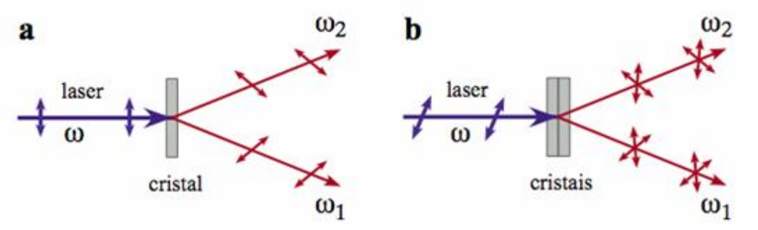

Type I phase matching

Polarisation entanglement

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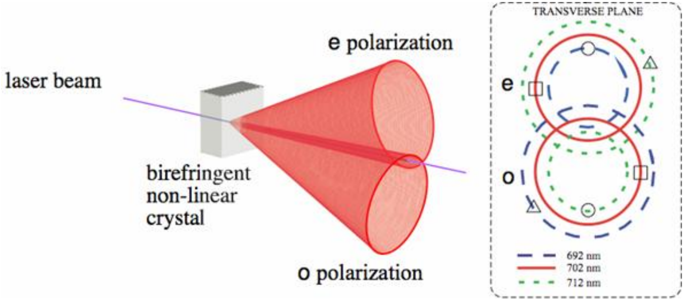
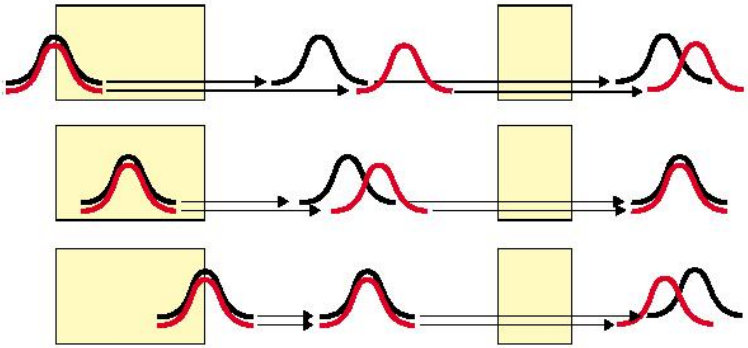


Type II phase matching

Temporal walk-off and compensation



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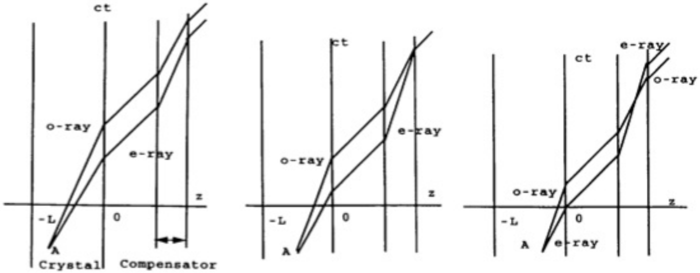
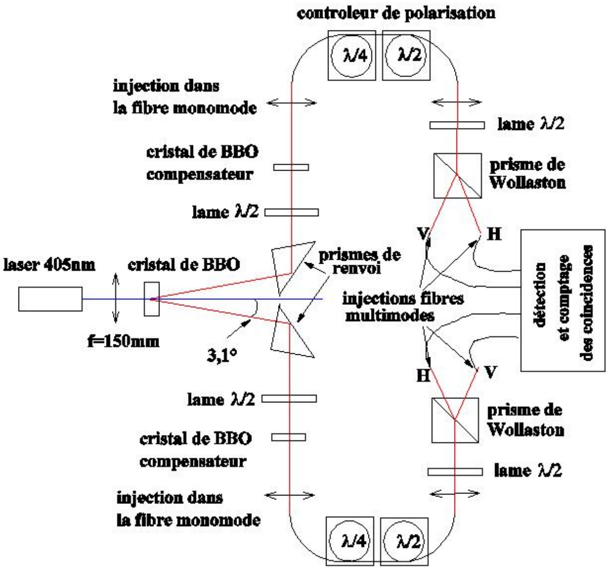


FIG. 6. These diagrams illustrate how the compensator effects pairs that are created at point A near the input, at the center, and near the output of the crystal.

Polarisation entanglement



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