NONLINEAR ELECTROMAGNETISM
M2 OPTICS - OMP

Homewok Exercise - 2011

1 Parametric amplification in a BBO crystal

Our aim is to amplify a signal beam, with a wavelength $\lambda_1 = 808\,\text{nm}$, using a BBO crystal that is pumped by a $\lambda_3 = 355\,\text{nm}$ beam. The two beams propagate in the same direction, denoted $\mathbf{z}$. The wavevectors corresponding to the signal and pump beams will be respectively denoted $\mathbf{k}_1 = |k_1|\mathbf{z}$ et $\mathbf{k}_3 = |k_3|\mathbf{z}$. Let θ be the angle between $\mathbf{k}_{1,3}$ and the optical axis of the crystal.

BBO is a negative uniaxial crystal and the principal refractive index values are given in Table 1. It exhibits a second order nonlinearity and belongs to the crystal system $3m$. Kleinman's symmetry is satisfied in this medium. The non zero coefficients of the second order susceptibility tensor are given in Table 2.

λ	n_o	n_e
λ_3 =355 nm	1,705246	1,576383
λ_1 =808 nm	1,660043	1,544002
$\lambda_2 = \cdots$ nm	1,666977	1,549368

TAB. 1 – *BBO principal refractive index values for the considered wavelengths*

$d_{i\ell}$	valeur en pm/V
$d_{22} = -d_{21} = -d_{16}$	2,3
$d_{31} = d_{15} = d_{24} = d_{32}$	0,1

TAB. 2 – *Non-zero second order susceptibility tensor values in BBO.*

1. **Questions relative to the course**
   (a) What does Kleinman's symmetry mean? In what case is it satisfied?
   (b) Recall the meaning of the contracted notation[1] $d_{i\ell}$.
   (c) Show that Kleinman's symmetry implies that $d_{15} = d_{31}$ et $d_{21} = d_{16}$.
2. What is the wavelength of the third wave produced by the nonlinear interaction? It will be denoted λ_2.
   Subsequently, type I phase-matching is considered in this BBO crystal.
3. What are the polarisation states of the various waves in this case?
4. Write the phase-matching condition.

1. $d_{i\ell} = \frac{1}{2}\chi^{(2)}_{ijk}$; with the following correspondence between jk and the index ℓ :

jk	11	22	33	23 ou 32	13 ou 31	12 ou 21
ℓ	1	2	3	4	5	6

5. Show that the phase matching condition determines the value of a refractive index. Which one? Give its value.
6. Derive the angle θ value that must be used in order to satisfy the phase matching condition. Give its value inside the crystal in degrees .

   N.B. *Use the equation giving the extraordinary refractive index as a function of the angle between the wavevector of the considered beam and the optical axis in a uniaxial crystal*

$$\left(\frac{1}{n_\theta}\right)^2 = \left(\frac{\cos\theta}{n_o}\right)^2 + \left(\frac{\sin\theta}{n_e}\right)^2$$

7. The interacting waves can be expanded over the base of the extraordinary and ordinary propagation modes with respective polarisation directions $\mathbf{e}_o$ et $\mathbf{e}_\theta$. Their coordinates in the crystal coordinate system are:

$$\mathbf{e}_o = \left| \begin{array}{c} \sin\phi \\ -\cos\phi \\ 0 \end{array} \right. , \quad \mathbf{e}_\theta = \left| \begin{array}{c} -\cos\theta\cos\phi \\ -\cos\theta\sin\phi \\ \sin\theta \end{array} \right. ;$$

   (a) Give the expresion of the effective susceptibility $\chi^{(2)}_{eff}$ as a function of $\mathbf{e}_o$, $\mathbf{e}_\theta$ and $\underline{\underline{\underline{\chi}}}^{(2)}$.

   (b) Show that in this case, $\chi^{(2)}_{eff} = d_{31}\sin(\theta) - d_{22}\cos(\theta)\sin(3\phi)$.

   (c) For what ϕ value is the effective susceptibility maximal?

   (d) Give this maximum effective susceptibility value. It will be kept hereafter.

8. **Calculation of the parametric gain** The pump beam is a pulsed beam with 10 ps pulse duration and 300 μJ per pulse. The BBO crystal thickness ℓ_c is only 10 mm, so that diffraction effects can be neglected.

   The parametric gain in BBO can be calculated from the wave equation giving the evolution of the envelope amplitude $A_1(z)$ of the wave at ω_1 (derived in the case of perfect phase-matching):

$$\frac{d^2A_1}{dz^2} - |\gamma_0|^2 A_1 = 0, \qquad (1)$$

   where

$$|\gamma_0|^2 = \frac{\omega_1\omega_2}{2n_1n_2n_3c^3\epsilon_0}\left(\chi^{(2)}_{eff}\right)^2 I_3,$$

   is the parametric gain coefficient expressed as a function of refractive index values $n_{1,2,3}$ at frequencies $\omega_{1,2,3}$ and the pump beam intensity I_3.

   (a) Solve this equation with the following boundary conditions:

$$A_1(z=0) = A_{10} \text{ et } \left(\frac{dA_1}{dz}\right)_{z=0} = 0.$$

   (b) Calculate the peak intensity of the pump beam in MW/cm^2.

   (c) Derive the crystal parametric gain in m^{-1}.

   (d) Calculate the intensity gain $I_1(z=\ell_c)/I_1(z=0)$..

   (e) The considered input signal has the same pulse duration as the pump and 1 μJ energy per pulse. What is the energy per pulse of the amplified signal?

   (f) Estimate the light power generated at λ_2.

   (g) The output beam and pump are then sent into a second crystal, using the same configuration as in the first one. Is the amplification factor in this second crystal necessarily identical to the first one? Comment on this.