NONLINEAR ELECTROMAGNETISM
M2 OPTICS - OMP

Homework Exercise - 2012

1 Three-wave Mixing in counter-propagating configuration

We consider the nonlinear interaction of 3 monochromatic plane waves, at the frequencies ω_p, ω_s and ω_i. We suppose that $\omega_p > \omega_s$ and $\omega_p > \omega_i$. The pump (ω_p) and the idler (ω_i) waves are propagating along the direction $z > 0$, while the signal (ω_s) wave is propagating along the direction $z < 0$ (counter-propagating configuration).

1. Neglecting the birefringent property of the material, derive the 3 nonlinear wave equations. Use the parameter $\Delta k = k_p + k_s - k_i$, where k_p, $-k_s$ and k_i denote respectively the pump, signal and idler wave vectors.
2. The frequencies ω_p, ω_s and ω_i differ significantly from any resonance frequency of the nonlinear material. Compare the effective nonlinear susceptibilities used in the nonlinear wave equations?
3. The nonlinear interaction takes place in a birefringent crystal with an extraordinary refractive index n_e higher than the ordinary refractive index n_o. The dispersion of the refractive index will be neglected. Each wave being polarized either along the ordinary or the extraordinary direction, give the relations to be satisfied between the pump and signal frequencies and the refractive indices (n_o and n_e) in order to maximize the transfer of energy from the pump to the signal. Calculate the two possible values for the signal wavelength in the following case: $n_e = 3{,}58$, $n_o = 2{,}78$ and $\lambda_p = 1{,}06\,\mu$m (pump wavelength).
4. The pump depletion is neglected and the condition that maximized the transfer of energy is satisfied. The incident amplitudes for the signal and idler waves are denoted $A_s(z = L)$ and $A_i(z = 0)$. Calculate their amplitudes at the output of the crystal. Use the parameter:

   $$g_0^2 = \frac{\omega_i \omega_s}{4 n_i n_s c^2} |\chi_{eff}^{(2)}|^2 |A_p|^2,$$

   with $\chi_{eff}^{(2)}$ the effective nonlinear susceptibility of the crystal, n_s and n_i the refractive indices experienced by the signal and the idler waves, and A_p the amplitude of the pump wave.
5. What happens when $g_0 L \to \frac{\pi}{2}$? How should we need to proceed to correctly describe this situation? Would it be possible to generate an intense infrared beam by means of a single intense beam at $1{,}06\,\mu$m? Why?

2 Reading

For the next course of November 6: Please, read the following sections of the Boyd's textbook "Nonlinear Optics": 3.1, 3.2 to 3.2.5, 3.3 to 3.6. These sections describe a microscopic theory of the nonlinear susceptibilities.