NANOPHOTONICS (Photonic crystals and Emitters)

- Nov 8th : Waveguide - Confinement, representations
- Nov 22nd : Waveguide periodic array - the various regimes (3h)
  Defect and light line along a non periodic dimension.
- Nov 25th : Photonic band gap in >1D - mainly in 2D (1h30)
- Dec 6th : Defects in 2D photonic band gap - guides, cavities ; membranes (1h30)
- Dec 9th : Nano-emitters (quantum dots & nanoparticles), Density-of-States
  Emission issues → Purcell effect, light extraction (3h)
- Dec 13th : Combinations - from emitters at interfaces (3h)
  to emitters in nanocavities
- Dec 16th : Written exam in two parts
  (H B part will use ~4 Appl Phys Lett papers (available in Nov.) as a basis
- *?Dec xxth extra training session on the papers ? (if we can arrange an oral exercise)*

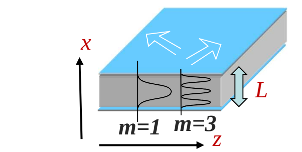
Waveguide - Confinement, representations

- Confinement = boundary conditions
  - ⇨ Example - Perfect conductor E=0
    Magnetic wall H=0
  - ⇨ Realistic - Real metal, dielectric interfaces
- Confinement shape
  - ⇨ 0 or 1or 2 remaining invariant dimensions (translation or rotation)
  - ⇨ Dictates coordinate system and choice of field basis
    Plane waves or cylindrical waves or spherical waves

A quick look to materials at hand

Oxides	Conductors	Semiconductors	organics
- glass SiO2+some others n=1.5 quartz - CaCO3? (calcite) n=1.6 to 1.8 - PbO, Ta2O5,... - TiO2 n=2.2 to 2.8 Al203=alumina sapphire n=1.7 LiNbO3 n=2.2 ... MgO n<1.5 ——— MgF2 n=1.38	- Au, Ag, Cu, Al,... Drude model index is complex-valued - Transparent conductor Indium-tin-oxide (n~2.0) ?- organics (low conductivity) ? -Graphene ?	Si n=3.5 + more dispersive Ge n=3.8 ? GaAs n=3.5 GaN (blue LEDs) n=2.5 Carbon : Graphene ? Nanotubes ? (Graphene is a semi-metal)	water n=1.33 alcohol n=1.4 PVC n=1.45-1.55 PMMA poly methyl metacrylate "Plexiglas" C=C chemistry n=1.7-1.8

Slab waveguide with perfect conductor boundary



$\varepsilon(x) = n(x)^2$

- Field takes the form

$$\vec{E} = E_o \; f(x) \; \exp(i\beta z - \omega t) \; \vec{e}_y \; ;$$

- Helmholtz eqn becomes 1D eqn.

$$-\frac{\partial^2}{\partial x^2}E_y - \frac{\partial^2}{\partial z^2}E_y = \varepsilon(x)\frac{\omega^2}{c^2}E_y$$

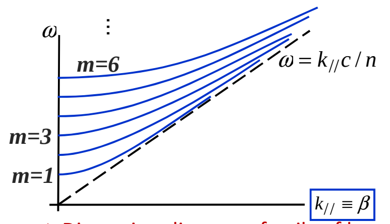
$$-\frac{\partial^2}{\partial x^2}f(x) + \beta^2 f(x) = \varepsilon(x)\frac{\omega^2}{c^2}f(x)$$

- Present case $\varepsilon(x) = \text{constant} = n^2$

$$f(x) = \sin(\frac{m\pi}{L}x) \equiv (\tfrac{1}{2i})\left[\exp(ik_x x) - \exp(-ik_x x)\right]$$

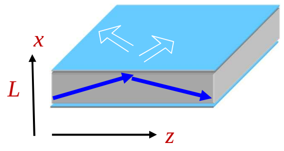
$$k_x^2 + \beta^2 = n^2\frac{\omega^2}{c^2} \quad = \frac{m^2\pi^2}{L^2} + \beta^2$$

$$\frac{\omega}{c} = \frac{1}{n}\sqrt{\beta^2 + m^2\frac{\pi^2}{L^2}}$$



- Dispersion diagram : family of hyperbola

Slab waveguide (perfect) ; group velocity



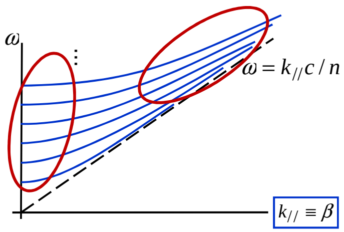
- Group velocity

$v_g = \frac{d\omega}{dk_{//}} = \frac{d\omega}{d\beta}$ in general $v_g = \nabla_{\mathbf{k}}(\omega)$

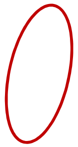
- Here : group velocity inverse of phase velocity

$v_g = c\frac{1}{n}2\beta\frac{1}{2}\left(\beta^2+m^2\frac{\pi^2}{L^2}\right)^{-1/2}$

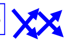
$v_g = c\frac{1}{n}\beta\frac{c}{n\omega} = \frac{\beta}{\omega}\frac{c^2}{n^2}$ $v_g\frac{\omega}{\beta} = \left(\frac{c}{n}\right)^2$ $v_g v_\varphi = \left(\frac{c}{n}\right)^2$

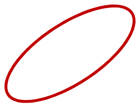


- Dispersion diagram

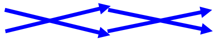


- Low, v_g High v_φ



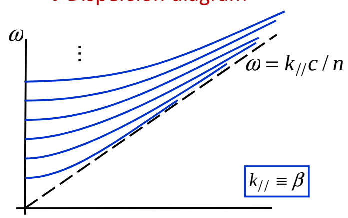


- Both v_g and v_φ → c/n

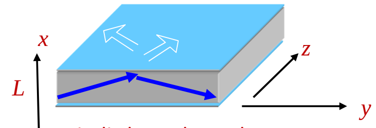


Density of states

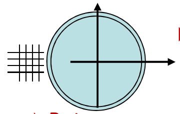
→Dispersion diagram



- There are two in-plane directions (z and y for us)
- $k_{//}$ has to be discretized in a big box $L_y \times L_z$



Born-van Karman periodic boundary cdts



- number of states in ring and per branch

$dN = \frac{L_yL_z}{(2\pi)^2}2\pi k\,dk$

- So

$dN = \frac{L_yL_z}{(2\pi)^2}2\pi k\frac{dk}{d\omega}d\omega$

- But

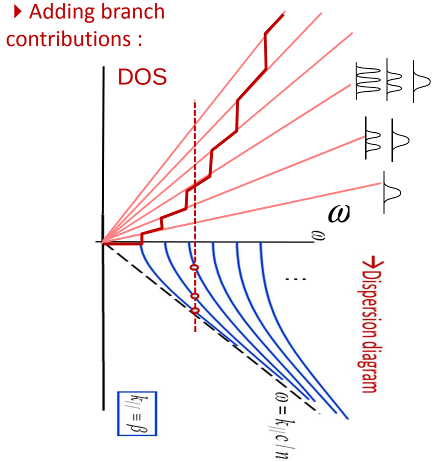
$k\frac{dk}{d\omega} = \frac{k}{v_g} \equiv \frac{\beta}{v_g} = \frac{n}{c}\sqrt{\beta^2+m^2\frac{\pi^2}{L^2}}$

- Hence

$\frac{dN}{d\omega} = \frac{S}{(2\pi)}\frac{n}{c}\sqrt{\dots} = \frac{S}{(2\pi)}\frac{n^2}{c^2}\omega$ PER BRANCH (m=1,2,...)

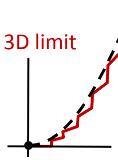
Density of states

- Adding branch contributions :



- Envelope of DOS is a parabola

$\frac{dN}{d\omega} \sim V\omega^2$ in 3D limit



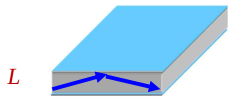
- There are cut-off frequencies

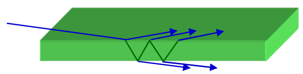
simply $\frac{\omega_m}{c} = \frac{m\pi}{nL}$ $\nu_m = \frac{c}{2nL}$

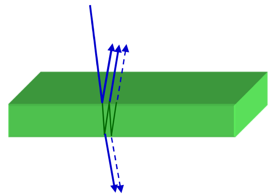
- These are relevant to Fabry-Perot physics

More general boundaries

- Reflectivity can be introduced
- R=1 : perfect
- 1-R<<1 : Fabry-Perot
- R<<1 : “usual” weak R

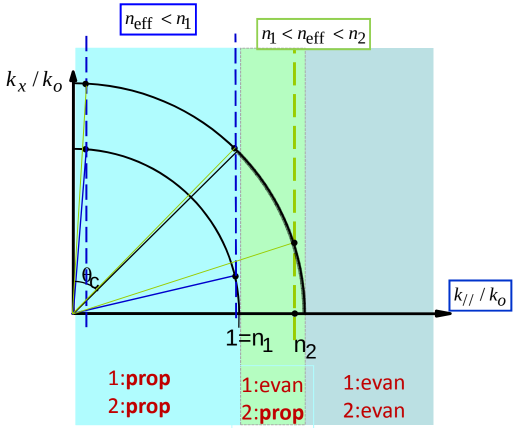






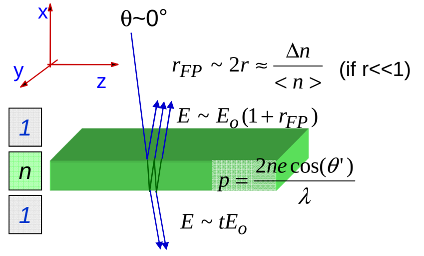
- Less singularity ? guided and non-guided (Fabry-Perot) modes...
- Outside medium now exists, so overall R,T can be defined (denoted T_{FP}, R_{FP})

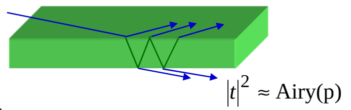
General representation for slab with two dielectric media

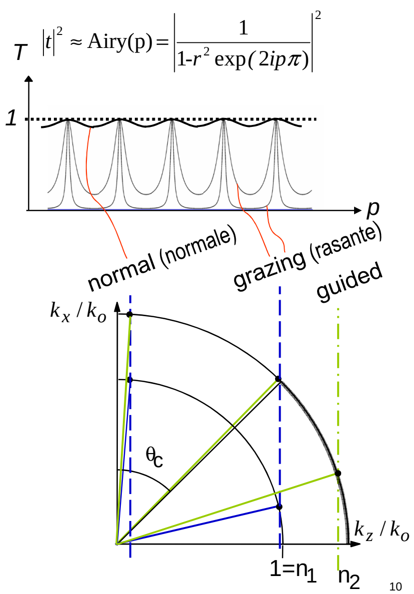


- In each medium, local solution is propagative or is evanescent
- Critical angle for the more refringent medium
- Snell-Descartes ≡ conservation of k//
- Definition of EFFECTIVE INDEX $n_{eff} = \frac{k_{//}}{(\omega / c)} = \frac{k_{//}}{k_{vacuum}}$

slab (plaque) = thin film ....reflectivity r=r(θ)

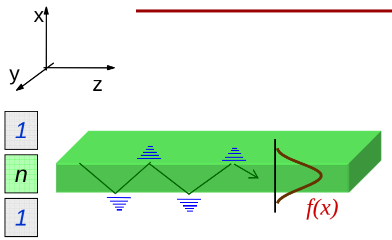




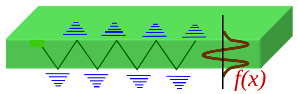


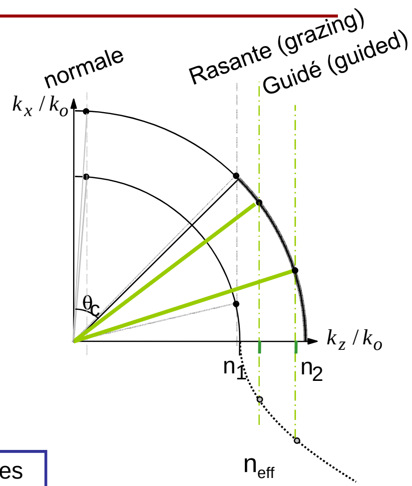
$$\vec{E} = E_o \; f(x) \; \exp(i\beta z - \omega t) \; \vec{e}_y \; ; \quad (\beta \equiv k_z)$$

slab (plaque) ... guided modes ...effective index



$$\vec{E} = E_o \; f(x) \; \exp(i\beta z - \omega t) \; \vec{e}_y \; ; \quad (\beta \equiv k_z)$$





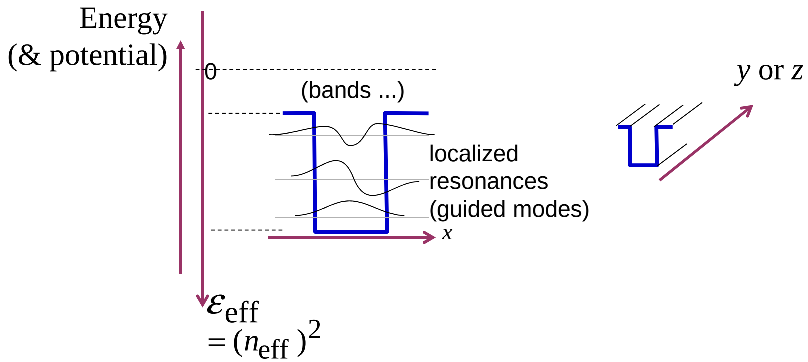
Guided modes

DEFINITION OF EFFECTIVE INDEX $k_z = n_{eff} k_o$

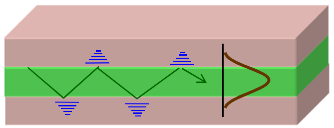
independent of the choice of a reference medium (≠angles)

FUNDAMENTAL MODE ⇔ highest ε_{eff} & n_{eff} (bottom of well)

Equivalence with potential (quantum) well



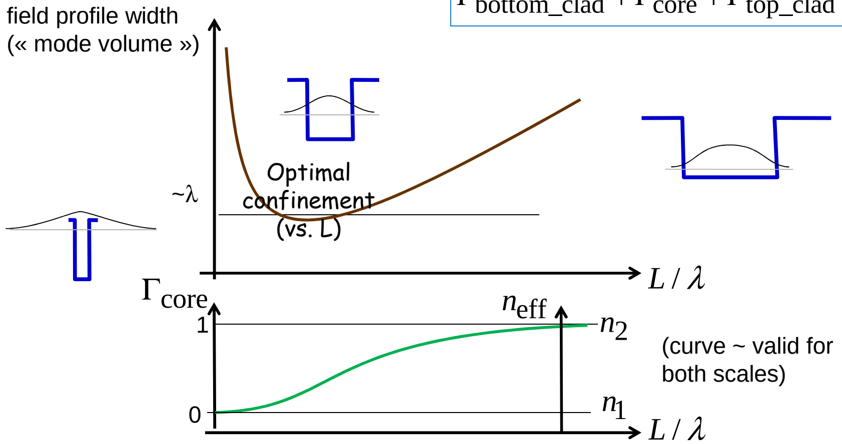
OVERLAP : CONFINEMENT FACTOR Γ



$$\Gamma_{layer} = \frac{\int_{x_{min}}^{x_{max}} |E|^2 dx}{\int_{-\infty}^{+\infty} |E|^2 dx}$$

Γ_{core}

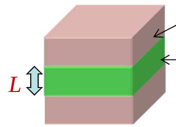
$$\Gamma_{bottom\_clad} + \Gamma_{core} + \Gamma_{top\_clad} = 1$$



INDEX CONTRAST ←→ size

- Field profile $-\frac{\partial^2 f(x)}{\partial x^2} + n_{eff}{}^2 f(x) = \varepsilon(x) f(x)$

- Best confinement vs. index contrast ? ?



$-K^2 / k_{vacuum}^2 + n_{eff}{}^2 = \varepsilon_{clad}$

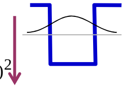
$k_x{}^2 / k_{vacuum}^2 + n_{eff}{}^2 = \varepsilon_{core}$

- For a typical « mid-well »case

$$\frac{k_x{}^2}{k_{vacuum}^2} = \frac{\varepsilon_{core} - \varepsilon_{clad}}{2} = \frac{K^2}{k_{vacuum}^2}$$

$$n_{eff}{}^2 = \frac{\varepsilon_{core} + \varepsilon_{clad}}{2}$$

$(n_{eff})^2$



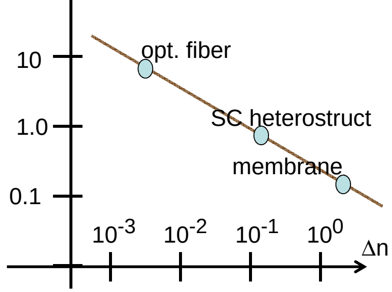
- « half » sine wave $k_x L \sim \frac{\pi}{2}$

$$\left(\frac{\pi}{2L}\right)^2 \left(\frac{\lambda}{2\pi}\right)^2 \sim \frac{\varepsilon_{core} - \varepsilon_{clad}}{2}$$

- So the typical size L of core associated to tightest mode

$$L \sim \frac{\lambda}{4} \left(\frac{\varepsilon_{core} - \varepsilon_{clad}}{2}\right)^{-1/2}$$

$$L \equiv \frac{\lambda}{4} (n_{core} - n_{clad})^{-1/2} \left(\frac{(n_{core} + n_{clad})}{2}\right)^{-1/2}$$



POLARISATION

- TE and TM cases (Ey,Hx,Hz) &(Ez,Ex,Hy)
  (also called « s » and « p » or « E » and « H »)
- TE and TM modes, their dispersion are interspersed

  TE symmetric solution easiest to get :
  same graphics solution as quantum well
  [tan(k_x*L/2) = some function of k_x]

- symmetric waveguide : no cutoff for fdtl modes TE or TM
- asymmetric waveguide : cutoff is possible also for fdtl mode, but different for TE and TM

GENERALISATION

- Channel guiding

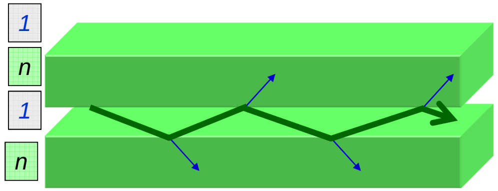
- Rectangular, Circular, Elliptic,...
- Field has all six em components of E and H !

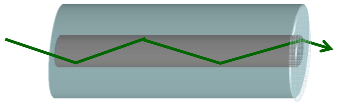
  → Approximate solutions for rectangular « wire »

  → Bessel/Hankel basis for circular : case of optical fibers,
  V-number : should be =2.4 for monomode criterion ...

- Still the concepts of overlap and optimal confinement of fundamtl mode hold
- But cutoff conditions may be widely different

Special : « Capillary » guiding : low-index core !





Never Total internal reflection, but still high enough to get
tens, or hundreds, of reflexions,
hence a macroscopic path of
100's or 10,000's of capillary diameters

TYPICAL EXAMPLES

- **Small** index contrast :
  - Silica/Ge-doped silica
  - $LiNbO_3$/Ion-exchanged $LiNbO_3$
- **Medium** index contrast : -GaAs/AlGaAs ; InP/GaASInP ;
  - GaN/AlGaN ; ZnSe/CdTe
  - Silicon nitride/silica
  - silica/water
- *In-between* : Silica/air « nanofibers »
- **High** index contrast :
  - Silicon/silica
  - Silicon air
  - Silicon nitride/air
  - Ga(Al)As/air
  - InP/air

« membranes » (→ Silicon air, Ga(Al)As/air, InP/air)

HOW "NANOPHOTONICS" COMES IN WAVEGUIDES

- Simplest periodicity :
- same symmetry, $k_{//}$ « good quantum number »
- periodicity allow k_x (vertical)

  2^{nd} « good quantum number » only in 1st Brillouin Zone

- Simplest technology :
- breaks invariance

  $k_{//}$ « good quantum number » only in 1st Brillouin zone