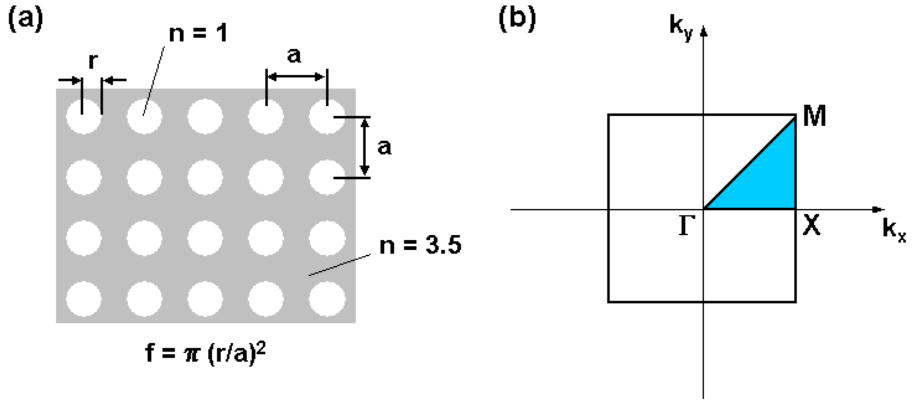
Negative refraction in 2D photonic crystals

The objective is to study light propagation in a periodic medium where the periodicity is of the order of (or slightly smaller than) the wavelength. We consider a two-dimensional photonic crystal with a square lattice in the (x,y) plane and we limit the following study to the case where light propagates in the plane, i.e., $k_z = 0$ and the problem is invariant along the z-direction (this case does not encompass photonic crystal fibers).



Two-dimensional photonic crystal. (a) Square lattice (period a) of air cylinders (radius r) in a medium with a refractive index n = 3.5. The filling factor in air is denoted by f. (b) First Brillouin zone of the reciprocal lattice and definition of the reduced zone delimited by the points Γ, X and M.

The relative permittivity describing the photonic crystal is a 2D periodic function that can be expanded in the Fourier basis,

$$\varepsilon(x, y) = \varepsilon(x + a, y + a) = \sum_{m,n} \varepsilon_{m,n}\, e^{imK_x x}\, e^{inK_y y} ,$$

where $K_x = 2\pi/a$ et $K_y = 2\pi/a$ are the basis vectors of the reciprocal lattice. We assume that the materials are non-magnetic.

1. <u>Equations of propagation, Bloch modes</u>

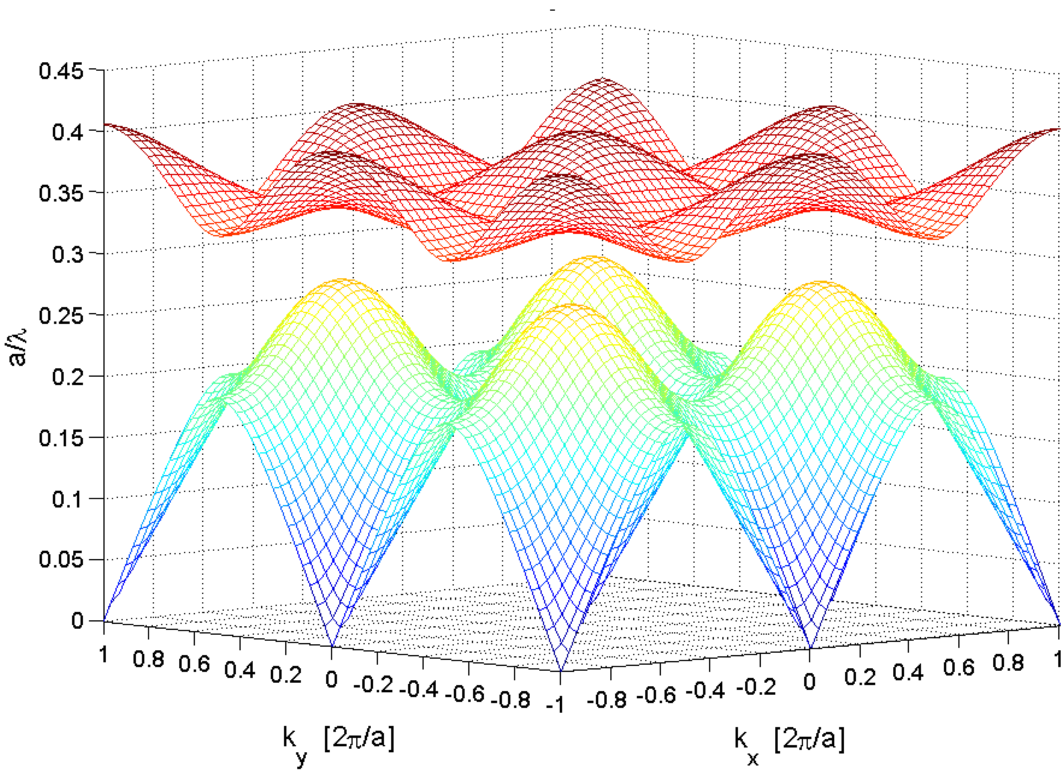
(a) Starting from Maxwell equations and using the z-invariance property of the structure ($\partial/\partial z = 0$), show that two independent polarization cases can be considered; the TE polarization with an electric field parallel to the cylinders, $\mathbf{E} = E_z\mathbf{u}_z$, and the TM polarization with a magnetic field parallel to the cylinders, $\mathbf{H} = H_z\mathbf{u}_z$. Which are the three equations satisfied by the electromagnetic field for each polarization case ?

(b) Deduce the equations of propagation satisfied by E_z and H_z for TE and TM polarization.

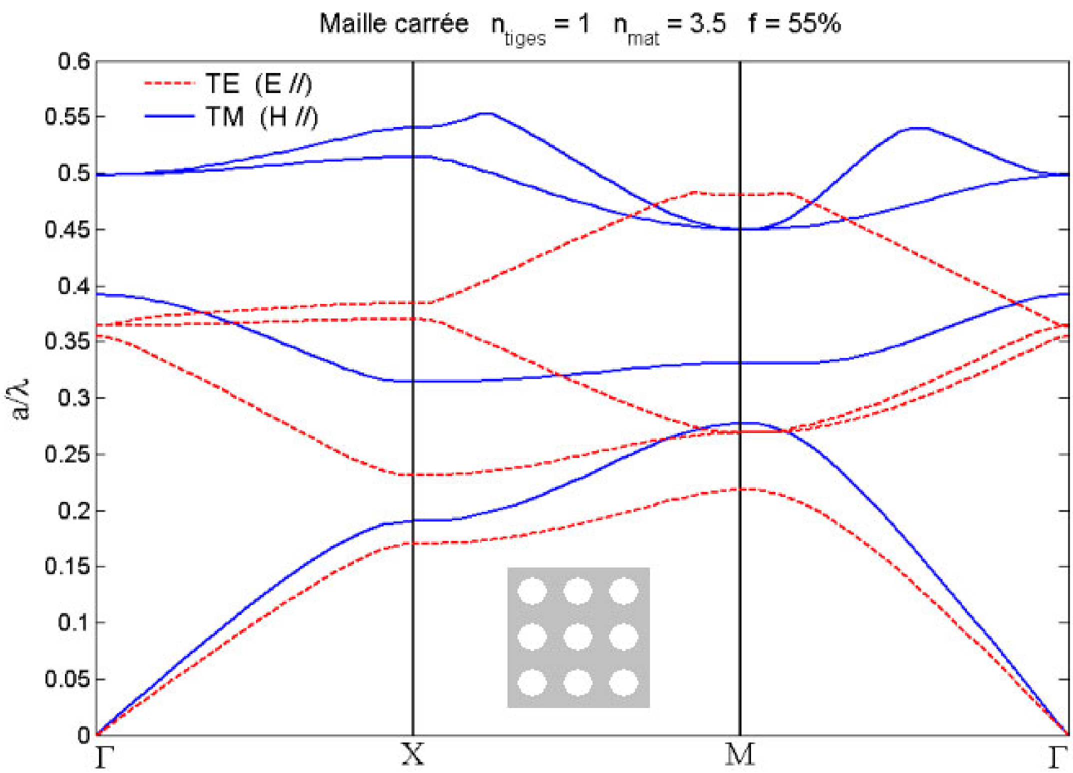
(c) Use Bloch theorem to write the expression of the field inside the photonic crystal.

2. <u>Band diagram</u>

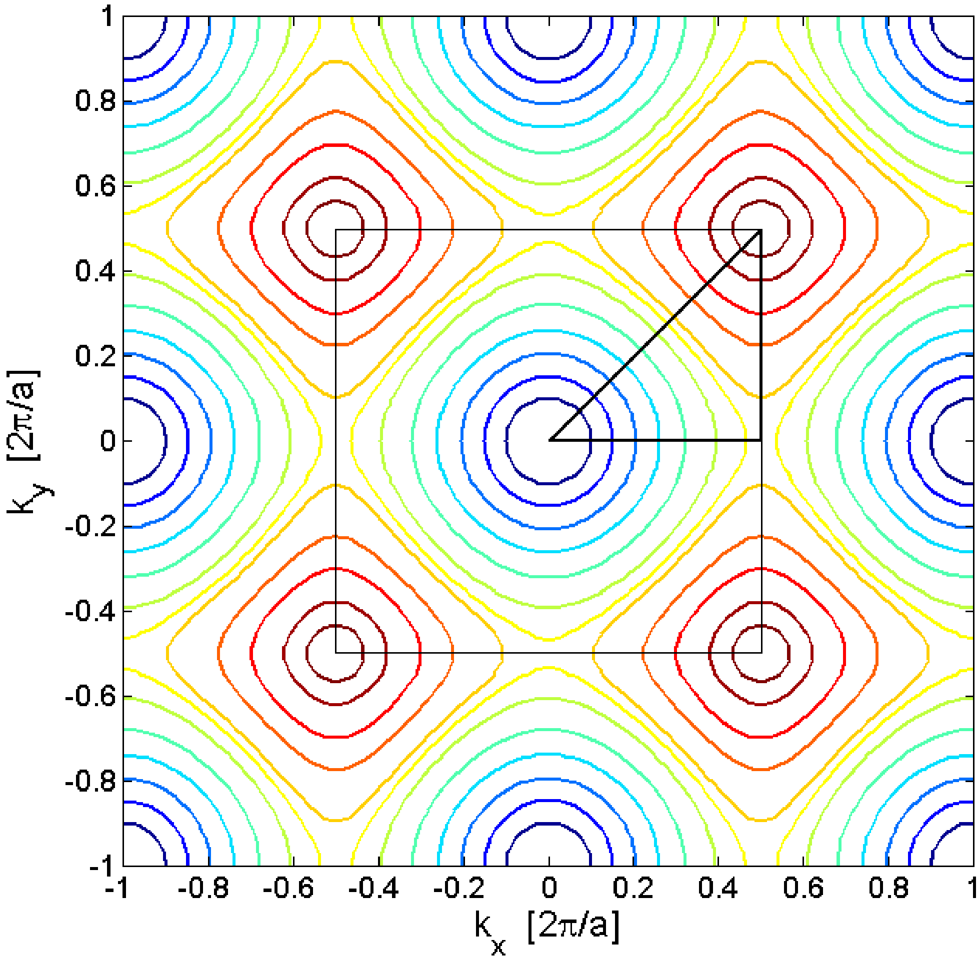
Writing the equation of propagation in the Fourier space leads to an eigenvalue problem for the frequency, as a function of fixed wavevectors k_x and k_y. Solving this eigenvalue problem gives the band diagram of the photonic crystal, i.e. the (ω,k_x,k_y) triplets that correspond to Bloch modes propagating in the periodic medium.



Band diagram $\omega(k_x,k_y)$ for TM polarization, $f = 0.55$. Only the two first bands have been represented.



Band diagram along the border of the reduced Brillouin zone for TM and TE polarizations, $f = 0.55$. The wavector (k_x,k_y) follows the highest symmetry directions ΓX, XM and ΓM.



Isofrequency curves $\omega(k_x,k_y)$ = constant for the first band. The curves are plotted for the following normalized frequencies a/λ = [0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.19, 0.2, 0.225, 0.245, 0.265, 0.275].

3. <u>Negative refraction in a photonic crystal</u>

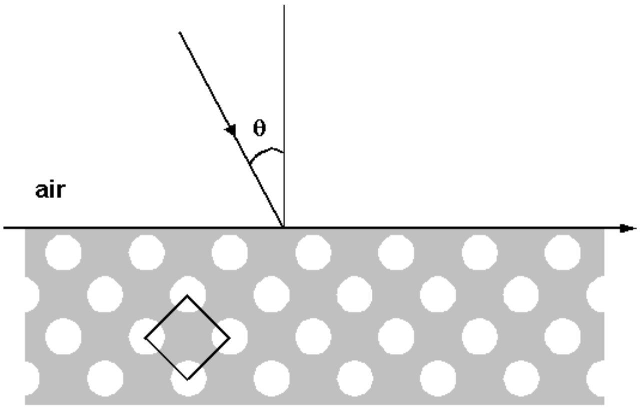
We have studied up to now the modes propagating inside the periodic medium. We consider hererafter an interface between air and the photonic crystal **with its square lattice tilted by 45° with respect to the interface**, see the figure below. The interface is illuminated by a plane wave in TM polarization with an oblique incidence defined by the angle θ. We study the propagation direction of the energy inside the photonic crystal.

It can be demonstrated that the energy transported by a Bloch mode in a periodic medium propagates with a speed $\mathbf{v}_e$ that is equal to the group velocity $\mathbf{v}_g$ of the Bloch mode,

$\mathbf{v}_e = \mathbf{v}_g = \nabla_k \omega$.

(a) Give the direction of the group velocity with respect to the isofrequency curves $\omega(k_x,k_y)$ = constant.

(b) We begin with the interface between two homogeneous media with refracticve indices n_1 and n_2. Plot the isofrequency curves in these two media. For an incident plane wave in medium 1 with an angle θ_1, determine graphically from the iso-ω curves the direction of the group velocity in medium 2.



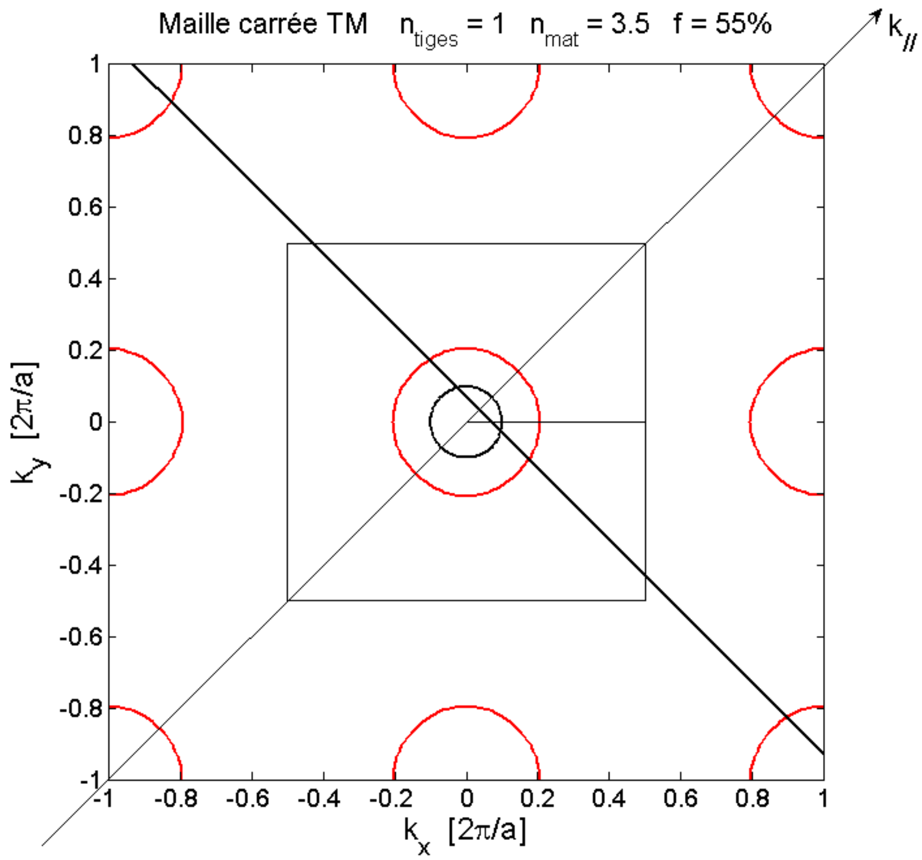
Air/photonic crystal interface. The crystal lattice is tilted by 45° with respect to the interface.

(c) Same question for the air/photonic crystal interface in the limit where the wavelength is much larger than the period $\lambda >> a$, $a/\lambda = 0.1$. What can you say about the behavior of the photonic crystal in this static limit ?

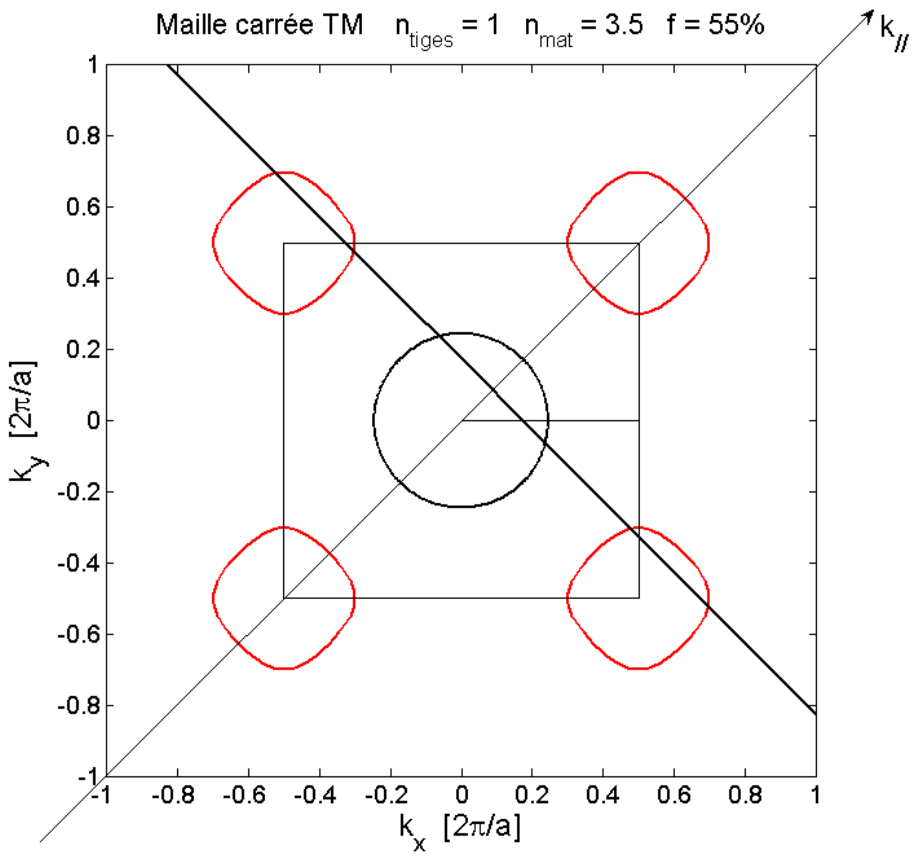
(d) Same question for a wavelength that is now close to the top of the first band, $a/\lambda = 0.245$.

4. Autocollimation in a photonic crystal

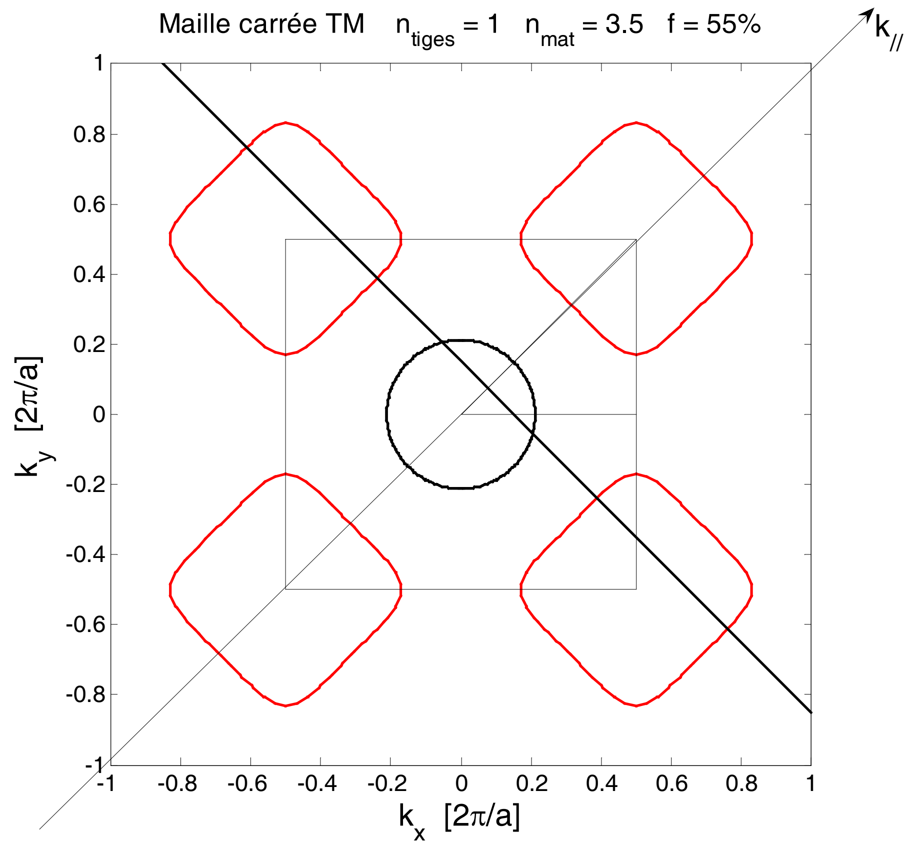
(a) Considering the same interface as previously, explain why a beam propagating at $a/\lambda = 0.212$ in the photonic crystal is collimated and propagates with almost no diffraction.



Isofrequency curve of the Bloch mode propagating inside the photonic crystal for $a/\lambda = 0.1$. The direction parallel to the air/photonic crystal interface is noted $k_{//}$.



Isofrequency curve of the Bloch mode propagating inside the photonic crystal for $a/\lambda = 0.245$. The direction parallel to the air/photonic crystal interface is noted $k_{//}$.



Isofrequency curve of the Bloch mode propagating inside the photonic crystal for $a/\lambda = 0.212$. The direction parallel to the air/photonic crystal interface is noted $k_{//}$.