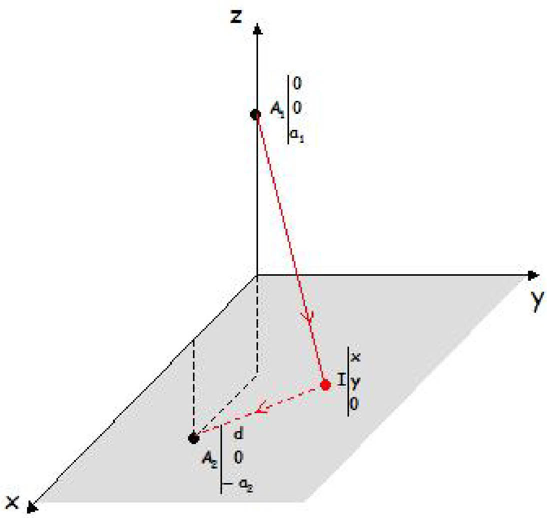
Homework n°1: Exercises on Chapters 1 & 2

1. Fermat's principle and Snell laws
   a. Reflexion

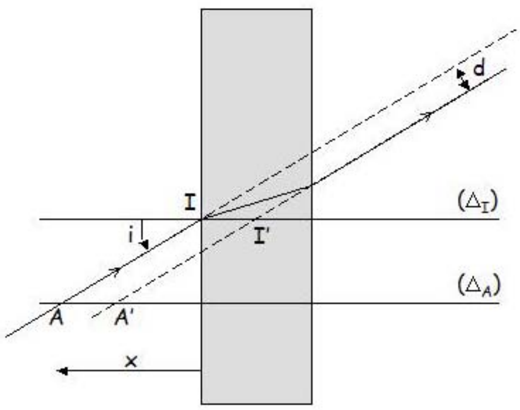
   Using Fermat's principle, show *geometrically* that for a flat mirror the absolute values of the incident and reflected angles are equal. (Hint : introduce A', the point symmetrical of A with respect to the mirror).

   b. Refraction

   Give the expression of the light propagation duration τ between point A_1 (in medium 1) and point A_2 (in medium 2) versus the (x,y) coordinates of the impact point I on the refracting flat surface, and the velocities v_1 and v_2 in the two media (see figure below). Using Fermat's principle, deduce from this expression the "extremum" condition for τ.



2. Flat window with parallel surfaces
   a. Calculate the transverse shift d of a beam falling on a flat window with an incident angle i=30° (see figure). The window is surrounded by air (n=1), and is made of glass (N=1.5) with a thickness e=1mm.
   b. Let A be an object point at a distance x away from the window, and A' the intersection point of the line (Δ_A) with the outgoing ray. Show that for a given incident angle i the position of A' on the line (Δ_A) does not depend on x.
   c. Deduce from above the algebraic distance $\overline{AA'}$ analytically and numerically.
   d. Calculate the expression of $\overline{AA'}$ to 2^{nd} order in "i". Deduce the position of the image A'_0 obtained for small values of the incident angle (the paraxial image). Evaluate $\overline{AA'_0}$ and $\overline{A'_0A'}$ for i=30°.



3. Young-Weierstrass points W_1 and W_2 of a spherical dioptre

For each of the following configurations, draw a schematic to scale 1:1 showing the spherical dioptre (radius of curvature R) that separates an object medium (index of refraction n) from an image medium (index of refraction n'), the object (size y), the image (size y') and the rays associated to the stigmatic conjugation $W_1 \rightarrow W_2$. The object is small and perpendicular to the axis.

   a. The dioptre is concave, $|R| = 3$ cm, $n = 1.5$ and $n' = 1$.
   b. The dioptre is concave, $|R| = 3$ cm, $n = 1$ and $n' = 1.5$.
   c. The dioptre is convexe, $|R| = 3$ cm, $n = 1.5$ and $n' = 1$.
   d. The dioptre is convexe, $n = 1$ and $n' = 1.5$.

   In which situations is the slope of the emerging ray smaller than for the incident ray? Calculate for each situation the transverse magnification y' / y. Which situations are interesting for the observation of a microscopic object?

4. Draw a schematic to scale 10:1 of an immersion microscope objective made of a truncated sphere of glass (radius 3mm, index of refraction n=1.5) and a meniscus lens (same glass) with radii of curvature 10mm and 6.3mm respectively. The diameter of the meniscus is 6mm. Calculate in particular
   a. the position of the object A with respect to the vertex S of the spherical dioptre of the glass "sphere",
   b. the distance between S and the vertex S_1 of the first dioptre of meniscus,
   c. the thickness of the meniscus.

   Trace the rays and the intermediate images. Calculate the transverse magnification of the whole system. Calculate the maximum angle α'_{max} of the rays that emerge from the glass "sphere". Do they actually emerge from the whole system? What is the numerical aperture in the image space?