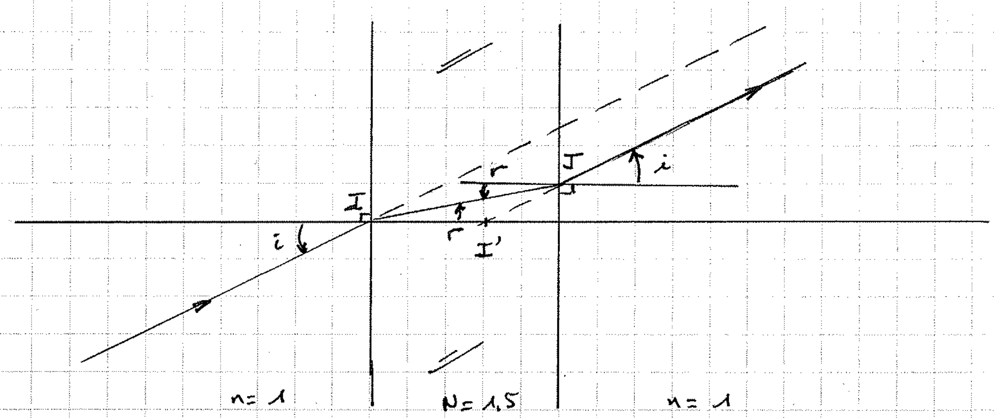
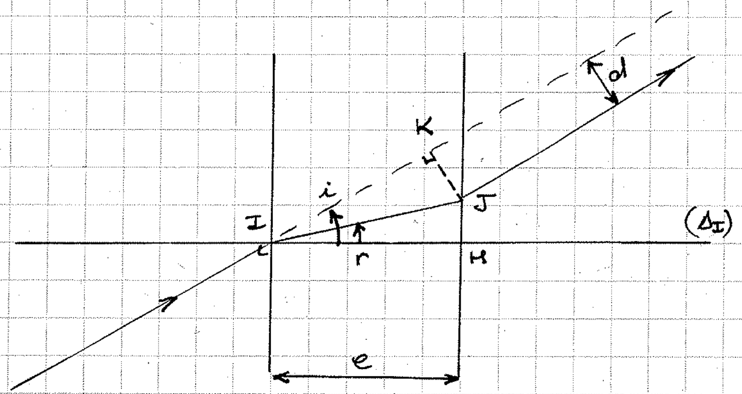
Exercise 2 - Homework 1



a) The Snell-Descartes law says that $\sin i = N \sin r$ at point I

Since the two surfaces are parallel, the incidence angle at point J is also

Thus, Snell-Descartes law says that the emerging ray also has an incidence angle i $\Rightarrow$ The incoming ray and the emerging ray are parallel.

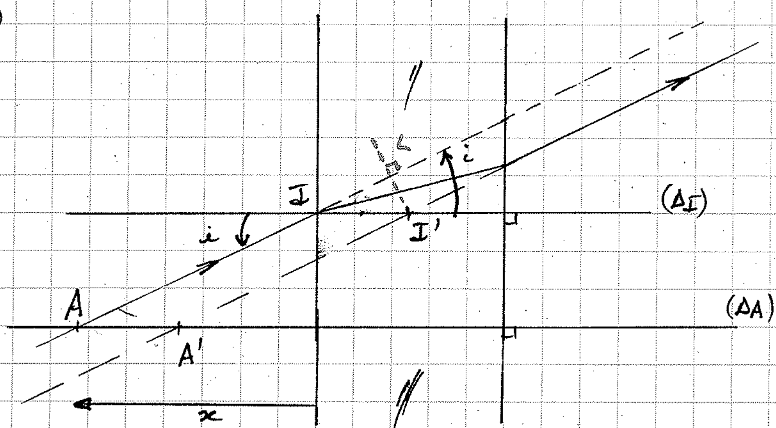


$$d = JK = IJ \sin(i-r)$$

and $IJ = \dfrac{IH}{\cos r}$

So : $$\boxed{d = e\,\dfrac{\sin(i-r)}{\cos r}}$$

b)



$(AI) \parallel (A'I')$ and $(\Delta_I) \parallel (\Delta_A)$ $\Big\}$ so $AII'A'$ is a parallelogram

so $\overline{AA'} = \overline{II'}$, independently of the position of A with respect to the window.

c) Let us calculate $\overline{II'}$.

$$\overline{II'} = \frac{I'L}{\sin i} = \frac{d}{\sin i} = e \cdot \frac{\sin(i-r)}{\sin i \cos r}$$

$$\overline{II'} = e \, \frac{\sin i \cos r - \cos i \sin r}{\sin i \cos r} \qquad \text{with } \sin r = \frac{1}{N} \sin i$$

$$\boxed{\overline{II'} = e \left[1 - \frac{1}{N} \frac{\cos i}{\sqrt{1 - \left(\frac{\sin i}{N}\right)^2}} \right]} \quad (1)$$

$i = 30^\circ \Rightarrow \sin i = 1/2$ and $\cos i = \frac{\sqrt{3}}{2}$

$N = 3/2$, $e = 1mm$

So, $\overline{II'} = 1 - \frac{\cancel{2}}{3} \frac{\sqrt{3}}{\cancel{2}} \frac{1}{\sqrt{1 - \left(\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}\right)^2}}$

$$\boxed{\overline{II'} = 0,39 \text{ mm}}$$

d) Let us develop (1) to 2^{nd} order in i:

$$\left.\begin{aligned} \cos i &\simeq 1 - \frac{i^2}{2} \\ \sin i &\simeq i \end{aligned}\right\} \quad \overline{II'} \simeq e \left\{ 1 - \frac{1}{N} \frac{\left(1 - \frac{i^2}{2}\right)}{\left(1 - \frac{i^2}{N^2}\right)^{1/2}} \right]$$

$$\simeq e \left[1 - \frac{1}{N} \left(1 - \frac{i^2}{2}\right)\left(1 + \frac{i^2}{2N^2}\right) \right]$$

$$\simeq e \left[1 - \frac{1}{N} \left(1 - \frac{i^2}{2} \left(1 - \frac{1}{N^2}\right) \right) \right]$$

To 2^{nd} order in "i": $\boxed{\overline{AA'} = \overline{II'} \simeq \underbrace{e\left[1 - \frac{1}{N}\right]}_{\text{paraxial image}} + \underbrace{\frac{e}{2N} i^2 \left(1 - \frac{1}{N^2}\right)}_{\text{spherical aberration}}}$

The first term yields the position of the paraxial image A'_0, obtained for "i" very small: $\boxed{\overline{AA'_0} = e\left(1 - \frac{1}{N}\right) = \underline{0,333mm}}$

The second term yields the real position of A' (to 2^{nd} order in i)

A' is shifted with respect to A'_0 along the axis, by $\boxed{\overline{A'_0 A'} = \frac{e}{2N} i^2 \left(1 - \frac{1}{N^2}\right)}$

This shift corresponds to the "3^{rd} order spherical aberration" of the flat window ; $\underline{\overline{A'_0 A'} = 0,0507mm}$

Note that $\overline{AA'_0} + \overline{A'_0 A'} \simeq 0,39$ mm, which is what we found in question c)

$\rightarrow$ The terms to 4^{th} order in i are negligible (corresponding to 5^{th} order spherical aberration)