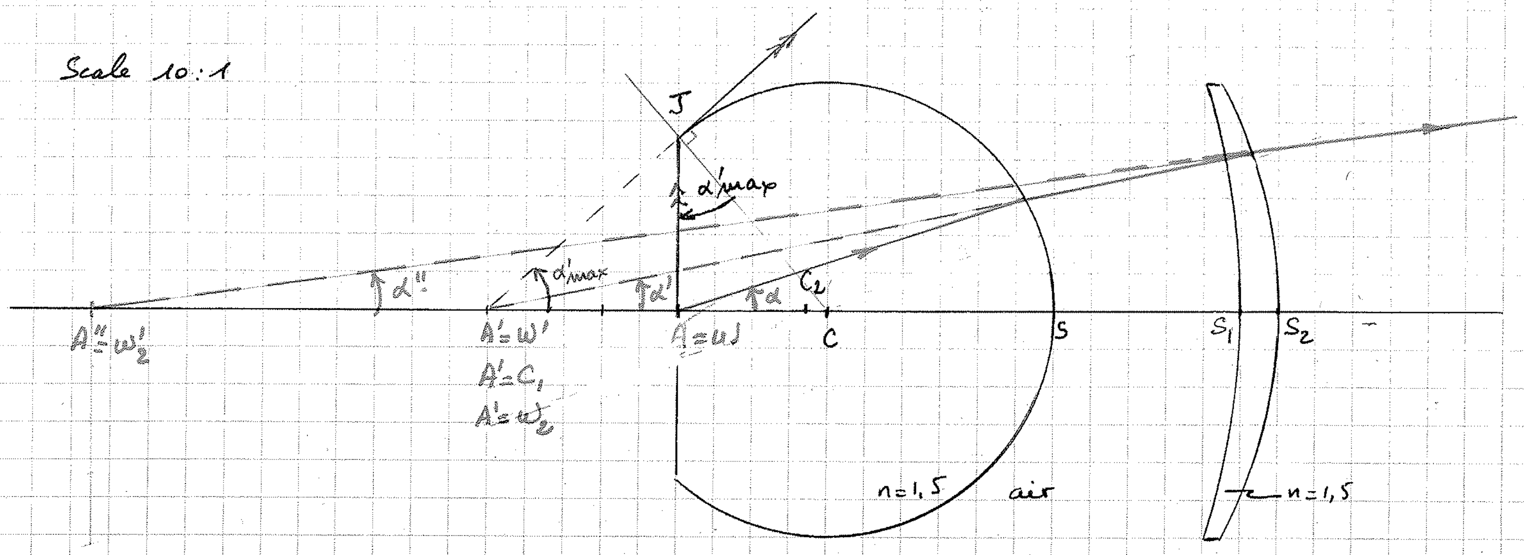
Exercise 4 - Homework 1 - Immersion microscope objective

Scale 10 : 1



a) Object A at object Weierstrass point of the glass "sphere" : $\overline{CA} = \frac{1}{n}\,\overline{SC} = \frac{2}{3}R$

$A = \omega$

so $\overline{SA} = \overline{SC} + \overline{CA} = \frac{5}{3}R = \underline{-5\,mm}$

$\underline{\overline{SA} = -5\,mm}$

b) $A = \omega \xrightarrow[\text{of glass sphere}]{1^{st}\text{ dioptre}} A' = \omega'$ = image Weierstrass point = Centre of curvature of the 1st dioptre of meniscus, C_1

Position of ω' ? $\overline{S\omega'} = \overline{SC} + \overline{C\omega'} = \overline{SC} + n\,\overline{SC} = \frac{5}{2}\,\overline{SC} = \underline{-7,5\,mm}$

$\underline{\overline{S\omega'} = -7,5\,mm}$

so, $\overline{SS_1} = \overline{S\omega'} + \overline{\omega' S_1}$

$= \overline{S\omega'} + \overline{C_1 S_1}$

$= -7,5 + 10\,mm$

$\underline{\overline{SS_1} = +2,5\,mm}$

c) A' is also the object Weierstrass point for the 2nd dioptre of the meniscus

$A' = \omega_2 \longrightarrow \omega_2' = A''$ ← final image.

so, $\overline{C_2\omega_2} = \frac{1}{n}\,\overline{S_2C_2} \Rightarrow \overline{C_2\omega_2} = -\frac{2}{3} \times 6,3\,mm = -4,2\,mm$

As ω_2 is known ($\omega_2 = A'$) this allows us to position C_2, and thus S_2

$\overline{S_1S_2} = \overline{S_1C_1} + \overline{C_1S_2} = \overline{S_1C_1} + \overline{C_1C_2} + \overline{C_2S_2} = \underline{+0,5\,mm}$ ← Thickness of the meniscus.

$-10\,mm + 4,2\,mm + 6,3\,mm$

Transverse magnification of the whole system ?

$$(g_y)_{A \to A''} = (g_y)_{A \to A'} \times (g_y)_{C_1 \to C_1} \times (g_y)_{W_2 \to W'_2}$$

Be careful : $(g_y)_{C_1 \to C_1} \neq 1$.

$$(g_y)_{A \to A''} = n^2 \times \frac{1}{n} \times n^2$$

$$\boxed{(g_y)_{A \to A''} = n^3 = \simeq +3,37}$$

Maximum angle α'_{max} of the rays that emerge from the glass sphere ?

α'_{max} corresponds to incident rays that are at grazing incidence ($\alpha = 90°$)

The emerging rays are tangential to the glass sphere.

Since $\widehat{JA'C} = \widehat{CJA} = \alpha'_{max}$ is also the incidence angle of the incident rays on the glass sphere,

and since the emerging rays make an angle 90° with (CJ),

we have : $n \sin \alpha'_{max} = \sin 90° = 1 \Rightarrow \boxed{\alpha'_{max} = \text{Arcsin}\left(\frac{1}{n}\right)}$

$$\boxed{\alpha'_{max} = 41,8° \simeq 0,73 \text{ rad}}$$

The figure shows that the emerging rays do not emerge from the whole system.

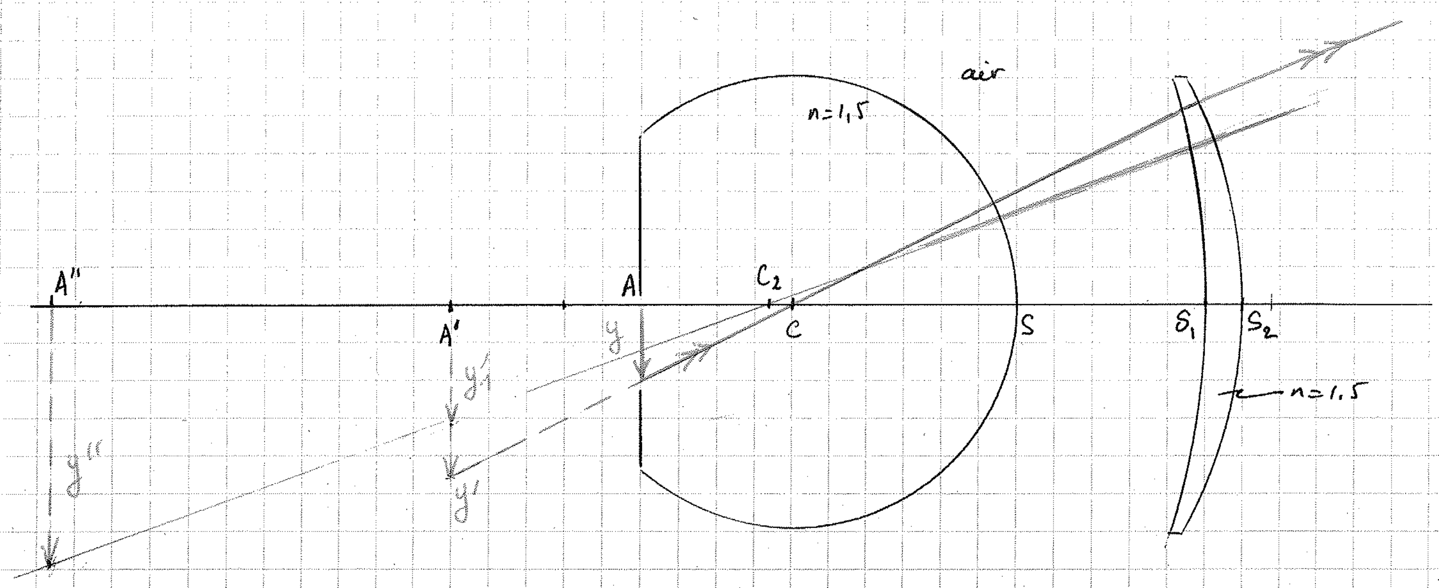
The numerical aperture, in image space, is $\sin\alpha''$

α'' is limited by the diameter of the meniscus ($\phi = 6$ mm)

so $\sin\alpha'' \simeq \frac{\phi}{2 \times \overline{\quad}} = \frac{\phi}{2\,\overline{W'_2 S_2}}$

$$\overline{S_2 W'_2} = n\,\overline{S_2 C_2} + \overline{S_2 C_2} = -\frac{5}{2} \times 6,3 \text{ mm} = \underline{-15,75 \text{ mm}}$$

so $$\boxed{\sin\alpha'' \simeq \frac{6 \text{ mm}}{2 \times 15,75 \text{ mm}} = \underline{\underline{0,19}}}$$


air
n=1,5
A''
A'
A
C₂
C
S
S₁
S₂
y
y₁'
y'
y''
n=1,5