Homework n°2: Exercises on Chapter 3

1. Establish the conjugation formula of the spherical dioptre with origin at the vertex S, in the paraxial approximation, using the Snell-Descartes law for refraction. Hint: use the angle $\omega = (\vec{CS}, \vec{CI})$.

2. For a spherical dioptre (vertex S and center C) between media with refraction index n and n', show that the paraxial foci F and F' are symmetric with respect to the middle of S and C, and that they cannot be between S and C.
   Hint: compare $\overline{CF}$ and $\overline{SF'}$.

3. Constructions

We consider a spherical dioptre (vertex S, center C). The ratio between the two refraction indexes is 1.5. Four cases are possible whether ($n/n' = 1.5$ or $n'/n = 1.5$) and ($\overline{SC} > 0$ or $\overline{SC} < 0$). The dioptre operates in paraxial conditions. In each case do the following:

   a. By using a construction based on S and C only, find the positions of the object and image focal points F and F'. Make your drawings to scale. Check that your construction agrees with the formulas for f and f'.
   b. Draw the incident and refracted rays for an object at infinity on axis.
   c. Draw the incident and refracted rays for an image at infinity on axis.
   d. Say whether the dioptre is converging or diverging.

4. Flat dioptre in the paraxial approximation
   a. Establish the conjugation formula for a flat dioptre, in the paraxial approximation, using the Snell-Descartes law for refraction. The object space is in air (n=1), the image space is in glass (n=N). Is your result in agreement with the conjugation formula of a spherical dioptre with an infinite radius of curvature?
   b. Calculate the transverse magnification g_γ, the angular magnification g_α and the longitudinal magnification g_x.
   c. You are standing on a river bank and by bending above the water surface you see a fish underwater at the bottom. The depth of water is 40cm. What is the apparent position of the fish and how are the proportions of the fish modified by the refraction through the water surface?

5. Flat window in the paraxial approximation

Using the conjugation formula for a flat dioptre in the paraxial approximation, calculate the algebraic distance $\overline{AA'}$ between an object point A and its paraxial image A' through a window with parallel surfaces (thickness e, refraction index N). The window is surrounded by air (n=1). Compare your result with Homework n°1 - exercise 2.

6. Constructions of refracted rays through a spherical dioptre in paraxial conditions. See appendix.

7. Constructions of the image through a spherical dioptre in paraxial conditions. See appendices.
   a. The object is at infinity.
   b. The object is in the object focal plane.
   c. n=1, n'=1.5. The position of the object is $\overline{SA} = -\frac{R}{2}$. Calculate the position of the image and the transverse magnification using the Descartes formulae. Compare to your drawing. Cross-check by using the Newton's formulae (position and transverse magnification). Is the image real or virtual?
   d. n=1.5, n'=1. The position of the object is $\overline{SA} = -R$. Same questions as 7c.

8. For a convex mirror and a real object, draw the rays that allow you to establish the following formulae:
    a. The transverse magnification with origin at the center C.
    b. The transverse magnification with origin at the vertex S.
    c. The transverse magnification using the distance $\overline{F'A'}$.
    d. The transverse magnification using the distance $\overline{FA}$.
    e. The conjugation law with origins at the foci F and F' (Newton's formula).

    Cross-check your constructions with a ruler in the particular case where $\overline{SA} = -\frac{R}{2}$ (see appendices).

9. At lunch you see your own image after reflection on a big spoon. Your eye is 25cm away from the spoon and the magnification is -0.064. Calculate the radius of curvature of the spoon. Describe quantitatively what you observe by reversing the spoon: position of the image, size, nature of the image (real or virtual). Illustrate with drawings in each case.