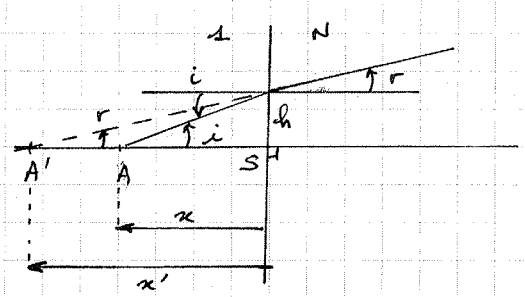
Exercise 4 - Homework n°2 - Flat dioptre in the paraxial approximation

a)



$$i = \frac{-h}{x} \qquad r = -\frac{h}{x'}$$

$$i = Nr \Rightarrow \frac{1}{x} = \frac{N}{x'}$$

Conjugation formula of the flat dioptre in the paraxial approximation: $\frac{x'}{N} = x$

This is in agreement with $\frac{N}{\overline{SA'}} - \frac{1}{\overline{SA}} = \frac{N-1}{\overline{SC}}$ using $\overline{SC} = \infty$

$$\frac{N}{x'} - \frac{1}{x} = 0.$$

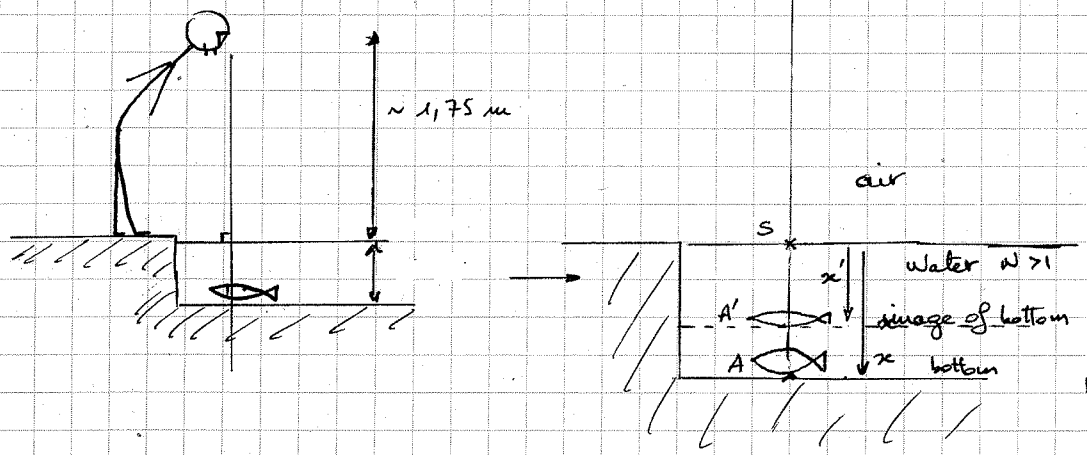
b) Using the Lagrange - Helmoltz invariant: $1 \times y \times i = N \times y' \times r$

so $\frac{y'}{y} = \frac{i}{Nr} = 1 \Rightarrow \boxed{(g_y)_{A \to A'} = +1}$

$$\boxed{(g_\alpha)_{A \to A'} = \frac{r}{i} = \frac{1}{N}}$$

$$\boxed{(g_x)_{A \to A'} = \frac{dx'}{dx} = N} \quad \text{using } x' = Nx.$$

c)



$$\frac{x'}{1} = \frac{x}{N} < x$$

so $|x'| < |x|$

↳ The fish looks nearer to water surface than it really is.

$(g_y) = 1 \Rightarrow$ The fish looks as long as it really is

$(g_x) = \frac{1}{N} < 1 \Rightarrow$ The fish looks flatter than it really is.