**Homework n°3: Exercises on Chapters 3 and 4**

1. Ray constructions using the cardinal points

We consider an optical system with the following characteristics:
- Convergence C=50 diopters (1diopter=1m$^{-1}$)
- Indices of refraction n=1 (object) and n'=1.5 (image)
- Distance between the principal planes $\overline{HH'} = 1cm$

Using ray constructions draw a schematic to scale 1:1 showing:
- The cardinal points H, H', N, N', F, F'
- The image of an object point A with $\overline{FA} = 1cm$; cross-check your result by using a conjugation formula. Using relevant rays, find the expression of the transverse magnification versus $x = \overline{HA}$ and $x' = \overline{H'A'}$
- The image of a disc with radius y=15mm centred on axis in the object focal plane (on another schematic); calculate the dimension of the image
- The object point that corresponds to an image on A' defined by $\overline{H'A'} = 6cm$ (on another schematic)
- The emerging ray associated to an arbitrary incident ray
- The incident ray associated to an arbitrary emerging ray

2. Same exercise as 1, with C=-50 diopters and $\overline{HH'} = 1cm$

3. The hemispheric lens
   a. Using ray constructions (schematic to scale 1:1) determine the cardinal points of a hemispheric lens with radius of curvature $R = -4.5cm$ and index of refraction N=1.5. The medium surrounding the lens is air (n=1). The vertex $S_1$ of the flat surface is at the centre of curvature C of the sphere (C= $S_1$); $S_2$ is the vertex of the sphere
   b. Cross-check your construction by calculating the positions of H' and F' (with respect to S), H and F (with respect to C). Hint: use the conjugation formula for refractive spherical and plane surfaces.
   c. Deduce from above the focal distance of a hemispheric lens. Is the focal distance changed if the lens is no longer hemispheric (same R and N, but $S_1 \neq C$, $\overline{S_1S_2} < \overline{CS_2}$)?

4. The spherical lens
   a. Find graphically the cardinal points of a sphere with diameter 5mm and index of refraction N=1.84, when the surrounding medium is air (n=1).
   b. Calculate the focal distance of the sphere. Hint: use the Gullstrand formula, calculate the convergence C of the lens, and deduce the focal distance.
   c. Evaluate the focal distance when the surrounding medium is water (n=1.33).

5. The dentist's mirror

A dentist uses a small spherical mirror, placed at the end of a metallic rod, to observe a patient's tooth. When the mirror is at 1.5cm from the tooth, it yields an image 1.5 times larger than the tooth, and with the same orientation. Determine the characteristics of the mirror graphically and by calculation: radius of curvature, concave or convex, position of the tooth with respect to F'?

6. The Cassegrain telescope

A Cassegrain telescope is composed of two mirrors ($M_1$) and ($M_2$) as shown below. The object is a star at infinity. In this configuration, the image through the primary mirror (vertex $S_1$, centre $C_1$, focal point $F'_1$) is a virtual object for the secondary mirror (vertex $S_2$, centre $C_2$, focal point $F'_2$), and the image through the whole telescope is real and detected on a CCD camera. A central hole is usually made through the primary mirror to let the rays focus on the CCD.

a. What should be the shapes of the mirrors to ensure that the telescope is perfectly stigmatic on axis?
b. In the paraxial regime, the mirrors can be approximated by spherical surfaces with the following characteristics:
   - primary mirror: $|R_1| = 1000\text{mm}$, diameter $\Phi_1 = 140\text{mm}$
   - secondary mirror: $|R_2| = 720\text{mm}$, diameter $\Phi_2 = 60\text{mm}$

   Find the positions of the characteristic points of the telescope so that the focal distance of the whole telescope is $f' = 1000\text{mm}$. In particular, calculate the transverse magnification $(g_\gamma)_2$ for the conjugation of the secondary mirror, the (algebraic) distance $\overline{S_1S_2}$ between the two mirrors, and the position of the image focal point $F'$ of the telescope.
c. Draw a schematic of the telescope (scale 1:10 along the axis, 1:2 transversely), showing the propagation of an incident ray parallel to the axis through the telescope. Show the position of the image principal point H', the centres of curvature $C_1$ and $C_2$, the focal points $F'_1$ and $F'_2$. Show on the same drawing the propagation of a incident ray that makes an angle $\theta$ with the axis, and cross-check the value for the magnification $(g_\gamma)_2$.