Exercise 3 - Homework n°3 — The hemispheric lens

a) Position of F' of the lens ?

$\infty \rightarrow \infty$ through flat dioptre, so $F' = F'_2$ = image focal point of the spherical dioptre.

To find F'_2 by a graphical method, use // rays with angle i with the axis.

Construct the refracted rays using $i' = \frac{i}{1,5}$ and $i'' = 1,5\, i'$

Position of F of the lens?

$$F \xrightarrow[\text{Dioptre 1}]{} F_2 \xrightarrow[\text{Dioptre 2}]{} \infty$$

To find F_2, you can either • use ray constructions similar to above

or • remember that $F_2 = S_O(F'_2)$ with $O = m(C_2, S_2)$

(see exercise 2 - homework n° 2)

Knowing F_2, you deduce F using again Snell's law: $u' = \frac{u}{1,5}$

(see figure on next page)

Position of H' ? The image principal plane is at the intersection of

$\begin{cases} \text{incoming rays // axis} \\ \text{and emerging rays (going through } F'\text{)} \end{cases}$

Since the rays are not deviated through the flat dioptre, $\underline{\underline{H' = S_2}}$

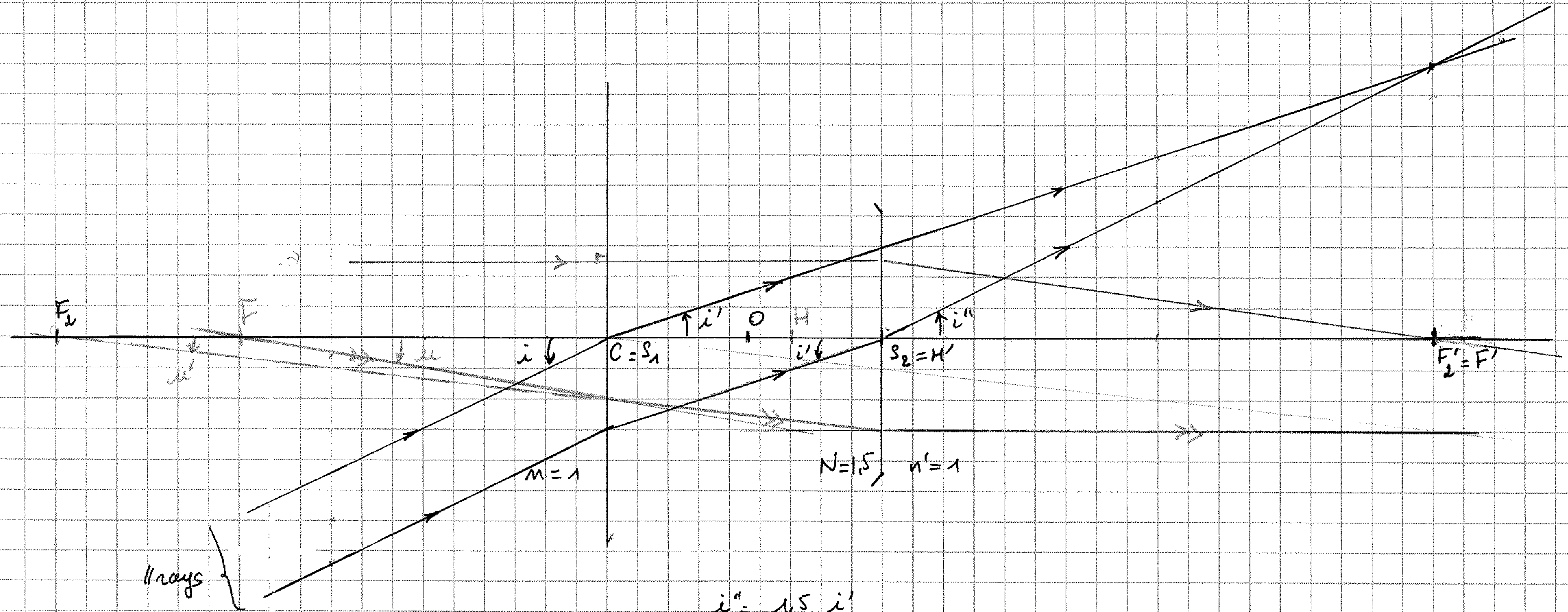
Position of H? Since the extreme media have the same $n = 1$, $\overline{H'F'} = \overline{HF}$

This constrains the position of H.

Cross-check: The object principal plane is at the intersection

of $\begin{cases} \text{incoming rays going through } F \\ \text{and emerging rays // to the axis.} \end{cases}$

Positions of N and N'? Since $n = n' = 1$, $N =$ and $N' = H'$



$i'' = 1{,}5\ i'$

$i' = \dfrac{i}{1{,}5}$

b) Position of F' ?

$F' = F'_2$ so $\overline{S_2F'} = \frac{1}{1-N}\,\overline{S_2C} = \frac{1}{1-1{,}5} \cdot (-45\text{ mm})$

$$\underline{\overline{S_2F'} = 90\text{ mm}}$$

$\underline{H' = S_2}$.

Position of F ?

$F \longrightarrow F_2$ through the flat dioptre : $\frac{\overline{S_1F_2}}{N} = \frac{\overline{S_1F}}{1}$ (conjugation formula for flat dioptre)

so $\overline{S_1F} = \frac{1}{N} \cdot (\overline{S_1S_2} + \overline{S_2F_2})$

Using $\overline{S_2F_2} = \frac{N}{N-1}\,\overline{S_2C}$, we get $\overline{S_1F} = \frac{1}{N}\left(\overline{CS_2} + \frac{N}{N-1}\,\overline{S_2C}\right)$

$$\overline{S_1F} = \frac{1}{N(N-1)}\,\overline{S_2C}$$

$$\overline{S_1F} = \frac{1}{1{,}5 \times 0{,}5}(-45\text{ mm}) = \underline{-60\text{ mm}}$$

c) Focal distance of the lens: $\underline{f' = \overline{H'F'} = 90\text{ mm}}$

<u>If the lens is no longer hemispheric</u>, the focal distance f' remains the same since H' does not change ($H' = S_2$) and F' does not change (R and N are the same). This is not a surprise: the flat dioptre plays no role is the total convergence of the lens, so you can move it anywhere. ($\overline{S_1S_2} < \overline{CS_2}$) and make the lens as thin as you want.

This is confirmed by the Gullstrand formula: $C = \frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} - \frac{e}{N}\frac{1}{f'_1 f'_2}$

Here, $f'_1 = \infty \rightarrow C_1 = 0$.

so $f' = f'_2 = \frac{1}{1-N}\,R$