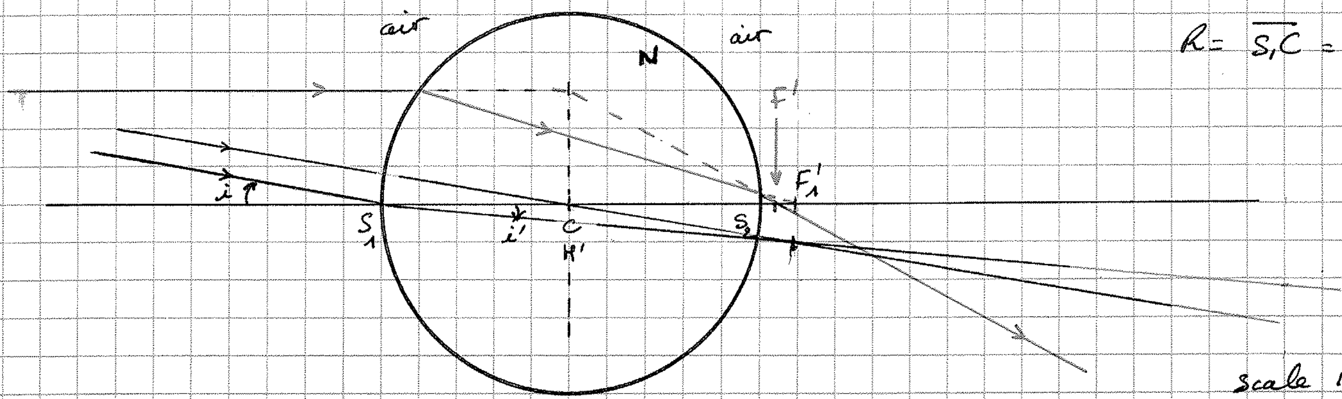
Exercise 4 - Homework n°3 The spherical lens

$N = 1{,}84$

$R = \overline{S_1C} = 2{,}5\ mm$



Scale 10 : 1

a) Positions of N and N' : a ray going through C is not deviated $\Rightarrow$ $N = N' = C$.

H and H' : Since the extreme media are the same ($n = 1$), $H = H' = N = N' = C$

To find F_1' graphically, use // rays with angle i and trace the refracted rays through the 1st dioptre using Snell's law: $i' = \frac{i}{1{,}84}$

F' ? To find F', use the image principal plane to place the intersection of the incoming ray (// to axis) and the emerging ray

b) Focal distance of the sphere: $\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 - \frac{e}{N}\mathcal{C}_1\mathcal{C}_2$ (Gullstrand)

$$\mathcal{C}_1 = \frac{N}{f_1'} = \frac{N(N-1)}{N\ R} = \frac{N-1}{R}.$$

$$\mathcal{C}_2 = \frac{1}{f_2'} = \frac{1-N}{(-R)} = \mathcal{C}_1 \quad \text{(not a surprise!)}$$

$$e = \overline{S_1S_2} = 2R$$

so $$\mathcal{C} = \frac{1}{f'} = 2\frac{N-1}{R} - \frac{2R}{N}\left(\frac{N-1}{R}\right)^2$$

$$\mathcal{C} = 2\frac{N-1}{R}\left(1 - \frac{R}{N}\frac{N-1}{R}\right)$$

$$\mathcal{C} = 2\frac{N-1}{N}\frac{1}{R}$$

$$\boxed{f' = \frac{N}{N-1}\frac{R}{2} \simeq 2{,}74\ mm}$$

c) When the sphere is surrounded by water ($n = 1{,}33$)

Now $\mathcal{C}_1 = \frac{N}{f_1'} = \frac{N}{R}\frac{N-n}{N} = \frac{N-n}{R}$ so $\mathcal{C} = \frac{n}{f'} = 2\frac{N-n}{R} - \frac{2R}{N}\left(\frac{N-n}{R}\right)^2$

$$\mathcal{C} = 2\,\frac{N-n}{R}\left(1 - \frac{R}{N}\,\frac{N-n}{\cancel{R}}\right)$$

$$\frac{n}{f'} = 2\,\frac{N-n}{R}\,\frac{n}{N}$$

so $$\boxed{f' = \frac{N}{N-n}\,\frac{R}{2} \simeq \underline{4{,}5\ \text{mm}}}$$

Note : the relevant factor is $\frac{N}{n}$, which defines the way rays are refracted ($Ni' = ni$) and thus the focal length.

$$\boxed{f' = \frac{N/n}{\frac{N}{n} - 1}\,\frac{R}{2}}$$