Course: Ray Optics – Master Erasmus Mundus *OpSciTech* (1st year)
Lecturer: Y. Sortais

Examination
03/03/2011 – Duration: 3h

No documents allowed
Pocket calculator is allowed.

Important note: the exercises are independent and their relative weights for the evaluation are indicated. The results should be clearly underlined and the figures should be precise and clear. A summary of some paraxial formulae is given at the end of the text.

Exercise 1 (7 points): Transverse magnification at the Young-Weierstrass points

We consider a spherical dioptre with radius of curvature $R = \overline{SC} = -30mm$ between an object medium (n=1.5) and an image medium (n'=1).

1. Calculate the positions of the Young-Weierstrass points A, A', and of the focal points F and F'. Using a compass, draw a schematic to scale 1:1 showing A, A', F and F'.
2. What is the maximum aperture angle α'_{max}, in the image space, of rays that originate from A and that emerge from the dioptre (give your result in degrees)? Draw the corresponding rays on your schematic. Evaluate the numerical aperture $n \sin \alpha$ in this configuration.
3. Calculate the transverse magnification associated to the conjugation A → A'. Cross-check your result by drawing a second schematic, in the paraxial approximation, to scale 1:1 (along the axis) and 10:1 (perpendicular to the axis showing). The object height is 1mm.

Exercise 2 (4 points): Ramsden eyepiece

We consider a 3-2-3 eyepiece with two thin lenses (Ramsden configuration). This means that $f_1' = f_2' = 3d$ and the distance between the lenses is $e = 2d$ ($d > 0$). The focal distance of the eyepiece is $f' = 25mm$. The lenses are surrounded by air.

1. Calculate d.
2. Calculate the positions of the focal points F and F' of the eyepiece, and the positions of the principal points H and H' with respect to the centers O_1 and O_2 of the lenses.
3. Draw a schematic to scale 1:1. Using ray constructions, cross-check the positions of H, H', F and F'.

A normal eye observes through the eyepiece with no accommodation an object perpendicular to the axis. The object is a ruler with small graduations and the distance between two graduations is 100µm.

4. Where is the object? Can the eye resolve two graduations (the resolution of a normal eye is 2')?

Exercise 3 (8.5 points): Telescope with lenses

We consider a telescope with two thin lenses: an objective and an eyepiece. The objective has a focal length $f_1' = 150mm$ and a diameter $\phi_1 = 6mm$. The eyepiece has a focal length $f_2' = 25mm$. The object is at infinity and the eyepiece is adjusted for observations with a normal eye and no accommodation. The lenses are surrounded by air.

1. Define and calculate the magnification power G of the telescope.
2. Evaluate the "commercial" intrinsic magnification power $G_{i,c}$ of the eyepiece.
3. We want that the objective limits the aperture of the telescope. What should be the minimum value for the diameter of the eyepiece?

We now take the diameter of the eyepiece equal to $\phi_2 = 3mm$.

4. Draw a schematic of the telescope to scale 1:1 (along the axis), and 10:1 (perpendicular to the axis). Show the rays propagation for an object at infinity on axis.
5. Calculate the position and the diameter of the exit pupil. Draw the exit pupil on your schematic.
6. Calculate the bright field of view angles in object and image spaces (give your results in mrad and in degrees). Draw the rays associated to the edge of the bright field of view on your schematic.

7. Draw the rays associated to the edge of the total field of view on your schematic. (Do not make any calculations in this question).

The "contour" field is defined as the field between the bright field of view and the total field. When we move out from the bright field of view, the illumination decreases progressively and reaches zero at the edge of the total field. For this reason, observations are not accurate in the contour field and it is often preferred to suppress the contour field. To do that, we introduce a diaphragm that brings the illumination abruptly to zero at the edge of the bright field of view.

8. Where should we place this diaphragm? Calculate its diameter? Indicate this diaphragm on your schematic and cross-check its size graphically.
9. In this question the position of the eyepiece with respect to the objective remains unchanged but the eye accommodation is 4δ. For simplicity, we assume that the eye pupil is placed at the image focal point F_2' of the eyepiece.
    a. What is the position of the image in the intermediate space then?
    b. What is the position of the nearest object (with respect to the objective) that can be observed using this telescope?

Exercise 4 (7 points): The Cassegrain telescope

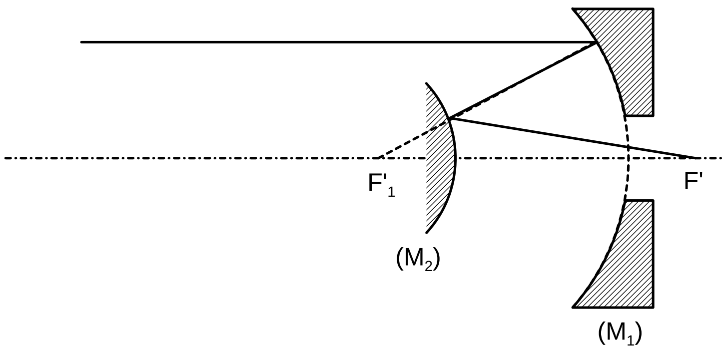
A Cassegrain telescope is composed of two mirrors (M_1) and (M_2) as shown below. The object is a star at infinity. In this configuration, the image through the primary mirror (vertex S_1, centre C_1, focal point F_1') is a virtual object for the secondary mirror (vertex S_2, centre C_2, focal point F_2'), and the image through the whole telescope is real and detected on a CCD camera. A central hole is made through the primary mirror to let the rays focus on the CCD. The image focal point of the telescope is F'.

1. What should be the shape of (M_1) to ensure that (M_1) is perfectly stigmatic on axis? What should then be the shape of (M_2) to ensure that the whole telescope is perfectly stigmatic on axis? To what remarkable points of mirrors (M_1) and (M_2) do F_1' and F' correspond?
2. In the paraxial regime, the mirrors can be approximated by spherical surfaces with the following characteristics:
    - primary mirror: $|R_1| = 1000mm$, diameter $\Phi_1 = 140mm$
    - secondary mirror: $|R_2| = 720mm$, diameter $\Phi_2 = 60mm$

   We want that the focal distance of the telescope is $f' = 1000mm$.
    a. Calculate the transverse magnification $(g_y)_2$ for the conjugation of the secondary mirror.
    b. Calculate the (algebraic) distance $\overline{S_1S_2}$ between the two mirrors.
    c. Calculate the position of the image focal point F' of the telescope with respect to S_2.
3. Draw a schematic of the telescope (scale 1:10 along the axis, 1:2 transversely), showing the propagation of an incident ray parallel to the axis through the telescope. Show the position of the image principal point H', the centres of curvature C_1 and C_2, the focal points F_1' and F_2', and the focal point F' of the telescope.
4. Show on a second schematic the propagation through the telescope of an incident ray that makes an angle θ with the axis, and cross-check the value for the magnification $(g_y)_2$.

The CCD detector is a 6mm x 6mm square centred on axis; the pixels are 10µm x 10µm squares.

5. Calculate the field angle θ (in the object space) associated to a point in the corner of the detector? Give your result in minutes of arc.
6. Calculate the angular resolution of the telescope (in the object space) associated to a pixel size of 10µm? Give your result in seconds of arc.



Summary of some paraxial formulae:

Descartes' formulae: $(g_y)_{A\to A'} = \frac{n}{n'}\frac{\overline{H'A'}}{\overline{HA}}$

$$\frac{n'}{\overline{H'A'}} - \frac{n}{\overline{HA}} = C = \frac{n'}{f'} = -\frac{n}{f}$$

Newton's formulae: $(g_y)_{A\to A'} = -\frac{\overline{F'A'}}{f'} = -\frac{f}{\overline{FA}}$

$$\overline{F'A'} \times \overline{FA} = f \times f'$$

Lagrange-Helmholtz invariant: $ny\alpha = n'y'\alpha'$

Longitudinal magnification: $(g_x)_{A\to A'} = \frac{n'}{n}(g_y)^2_{A\to A'}$ (true even for afocal systems)

Paraxial longitudinal invariant of the spherical dioptre: $Q_x = n\left(\frac{1}{R} - \frac{1}{x}\right) = n'\left(\frac{1}{R} - \frac{1}{x'}\right)$

with $x = \overline{SA}$, $x' = \overline{SA'}$, and $R = \overline{SC}$

Paraxial transverse invariant of the spherical dioptre: $Q_y = n\frac{y}{x} = n'\frac{y'}{x'}$

The above yields: $\frac{n'}{\overline{SA'}} - \frac{n}{\overline{SA}} = \frac{n'-n}{\overline{SC}}$

For the spherical mirror, take $n' = -n$ in the above formulae.

Convergence of a thin lens surrounded by air: $C = (N-1)\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)$

Gullstrand's formula for two focal systems with intermediate medium n_i: $C = C_1 + C_2 - \frac{e}{n_i}C_1C_2$

with $e = \overline{H_1'H_2}$