Examination on Ray Optics - 03/03/2011

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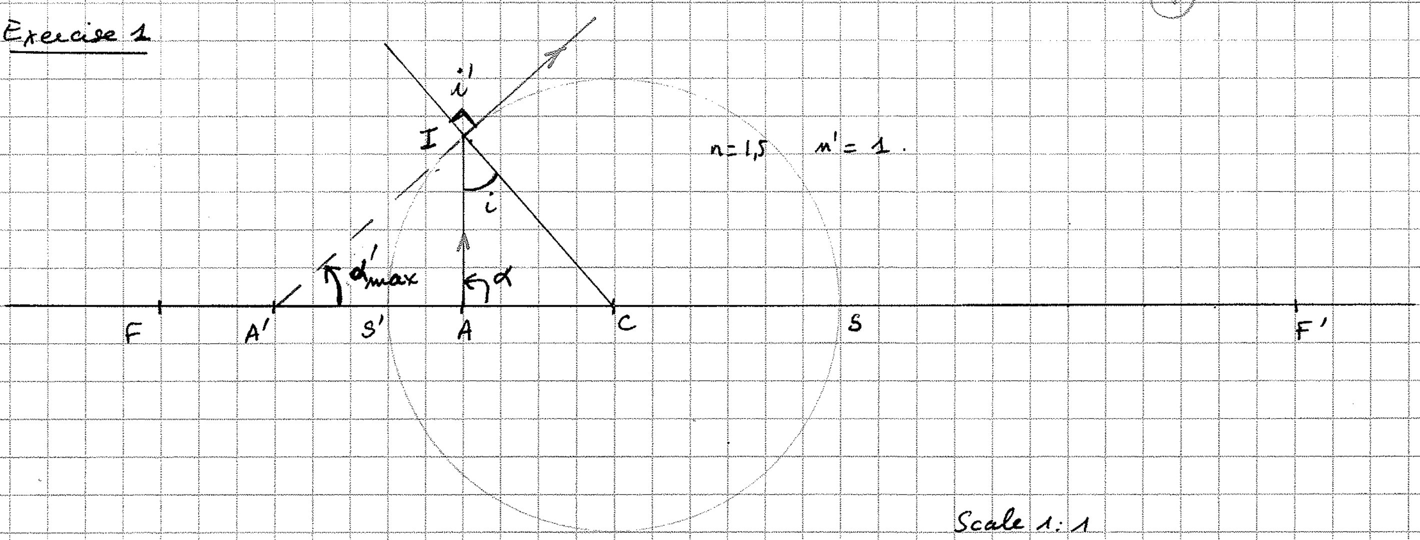
Exercise 1



Scale 1:1

1- A and A' are defined by $\overline{CA} = \frac{n'}{n}\ \overline{CS'}$ and $\overline{CA'} = \frac{n}{n'}\ \overline{CS}$

1 so $\overline{CA} = \frac{2}{3}\ \overline{CS'}$ and $\overline{CA'} = \frac{3}{2}\ \overline{CS}$

F and F' are obtained by $\overline{SF'} = f' = \frac{n'}{n'-n}\ \overline{SC} = -2\ \overline{SC}$

1 and $\overline{SF} = f = \frac{n}{n-n'}\ \overline{SC} = 3\overline{SC}$

2. $n \sin i = n' \sin i'$

Since $n > n'$, $i < i'$; the maximum refracted angle is $90°$ and corresponds to a refracted ray that is tangential to the sphere.

The incidence angle then fulfills $\sin i = \frac{1}{n}$

Besides $\frac{CA}{CS'} = \frac{1}{n}$ implies $\sin i = \frac{CA}{\overline{CS'}}$

1 → Figure and $CS' = CI$ yields $\sin i = \frac{CA}{CI}$

This means that A is the projection of I on the axis.

The incidence ray is thus perpendicular to the axis. → $\alpha = 90°$.

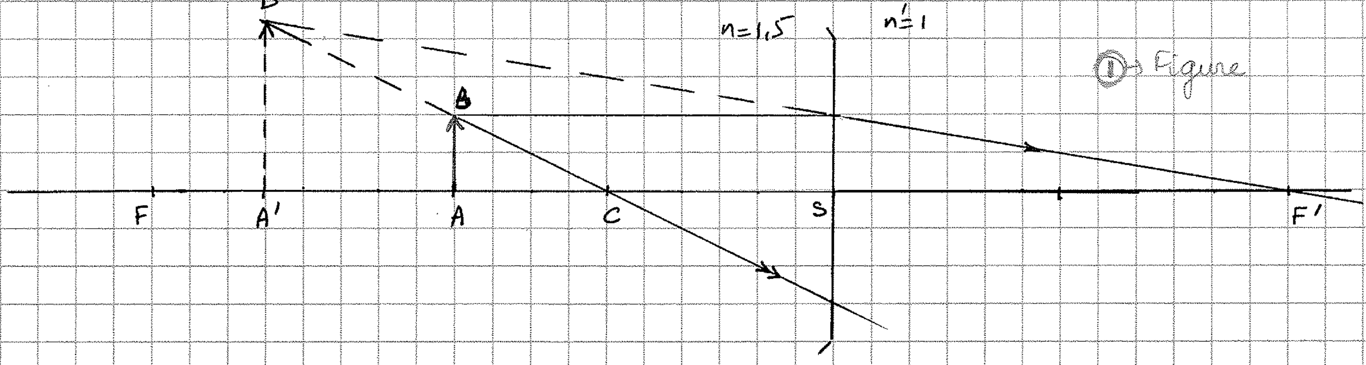
1 → $n \sin\alpha = n = \underline{1,5}$

The angle "i" can thus be found also as $\widehat{IA'C}$ ⇒ $\alpha'_{max} = i = \text{Arcsin}\,\frac{1}{n}$

1 $\underline{\alpha'_{max} = 41,8°}$

②

3. Transverse magnification : $(g_y)_{A \to A'} = n^2 = \underline{2,25}$ ← ①



Exercise 3   Ramsden eyepiece   (4)

1- $\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 - \frac{e}{1}\,\mathcal{C}_1\mathcal{C}_2 \Rightarrow \frac{1}{f'} = \frac{1}{3d} + \frac{1}{3d} - \frac{2d}{(3d)^2}$

(1)

$\frac{1}{f'} = \frac{4}{9d} \Rightarrow d = \frac{4f'}{9} \approx \underline{11,1\text{ mm}}$

2- $\infty \xrightarrow{(L1)} F'_1 \xrightarrow{(L_2)} F'$ so $\overline{F_2F'_1} \times \overline{F'_2F'} = -f_2'^2$

(1)

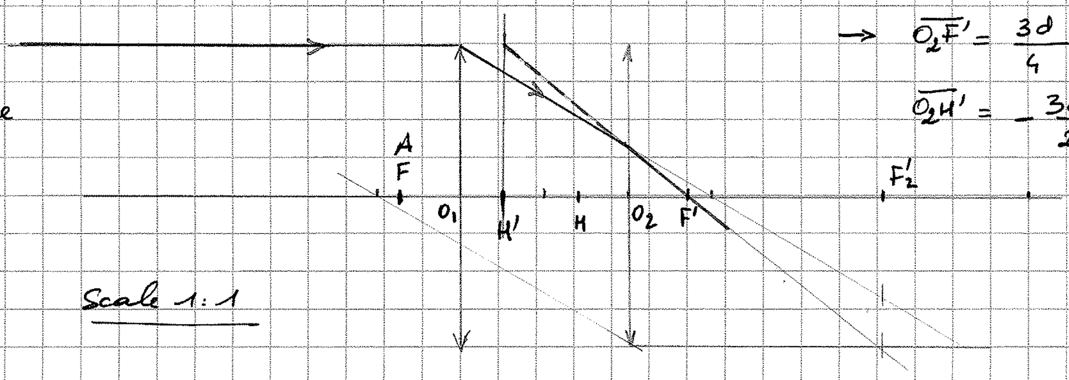
$\overline{F'_2F'} = -\frac{(3d)^2}{3d - 2d + 3d} = -\frac{9d}{4}$

$\rightarrow \overline{O_2F'} = \frac{3d}{4} \approx \underline{8,3\text{ mm}}$

$\overline{O_2H'} = -\frac{3d}{2} \approx \underline{-16,7\text{ mm}}$

3- Figure

(1)



Scale 1:1

Since the eyepiece is symmetric, $\overline{O_1F} = -\overline{O_2F'} = \underline{-8,3\text{ mm}}$

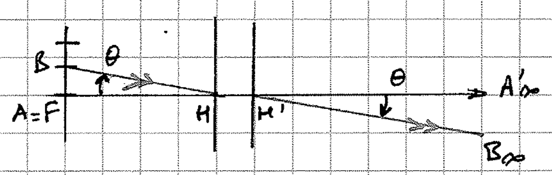
and $\overline{O_1H} = -\overline{O_2H'} = \underline{16,7\text{ mm}}$.

4- Since the eye does not accomodate, and is normal, A = F.

The image of the graduations through the eyepiece is at infinity.

(1) The angle between 2 graduations is $\theta = \frac{100\,\mu m}{f}$ in image space.

(1) $|\theta| = \frac{100\,\mu m}{25\text{ mm}} = 4\text{ mrad} \approx \underline{13,7'} > 2'$



So the eye can resolve 2 graduations

Exercise 3 Telescope with lenses

8,5

1. $G = \frac{\theta'}{\theta} = -\frac{f_1'}{f_2'}$

1

$\underline{G = -6}$

2. $G_{i,c} = \frac{250\ mm}{f_2'} = \underline{10}$

1

3- $\phi_{2\,min} = \phi_1 \times \frac{f_2'}{f_1'} \Rightarrow \phi_{2\,min} = 6 \times \frac{1}{6} = \underline{1\ mm}$

1

4- Figure : see next page

0,5

5- The aperture stop is on (L_1) ; it is thus also the entrance pupil.

$O_1 \xrightarrow{(L_2)} O_1'$

↑ exit pupil : $\overline{F_2'O_2'} \times \overline{F_2O_1} = -f_2'^2$

0,5

$\overline{F_2'O_2'} = +\frac{f_2'^2}{f_1'} = \underline{4,17\ mm}$

Diameter of the exit pupil : $\phi_1' = \phi_1 \times \frac{f_2'}{f_1'} = \underline{1\ mm}$

6. Bright field of view

We first calculate $y_i = \overline{F_1'B_i}$.

In triangles JMB_i and JIK : $\frac{MB_i}{IK} = \frac{O_1F_1'}{O_1O_2} \Rightarrow \frac{\frac{\phi_1}{2} + |y_i|}{\frac{\phi_1}{2} + \frac{\phi_2}{2}} = \frac{f_1'}{f_1' + f_2'}$

so $|y_i| = \frac{f_1'}{f_1' + f_2'} \frac{\phi_1 + \phi_2}{2} - \frac{\phi_1}{2}$.

$|y_i| = \frac{150}{150 + 25} \times \frac{6+3}{2} - \frac{6}{2}$

$|y_i| = 0,86\ mm$

0,5 figure + 1

Thus $|\theta_{bf}| = \frac{|y_i|}{f_1'} = \underline{5,7\ mrad} \simeq \underline{0,33°}$

0,5

$|\theta'_{bf}| = |\theta_{bf}| \times G = \underline{34\ mrad} \simeq \underline{1,96°}$

Alternative method Calculate θ'_{bf} using size of exit pupil (question 5)

$\theta'_{bf} = \frac{\phi_2/2 - \phi_1'/2}{O_2O_1'} = \frac{1,5 - 0,5}{25 + 4,17} \simeq 34\ mrad$

and $\theta_{bf} = \frac{\theta'_{bf}}{G}$