Ray Optics Examination

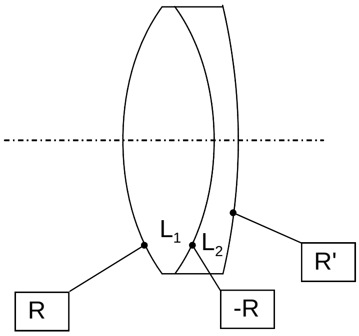
28/02/2012 – Duration: 3h

No documents allowed
Pocket calculator is allowed.

Important note: the exercises are independent. The results should be clearly underlined and the figures should be precise and clear (use the graduations of the paper). A summary of some formulae is given at the end of the text.

Exercise 1 (8.5 Points): Optical design of an achromatic doublet

We consider a cemented doublet with focal length $f' = 150mm$, made of two lenses L_1 and L_2 with focal lengths f'_1 and f'_2 (see figure below). L_1 is biconvex with radii of curvature R and $-R$ and is made of BK7 glass. L_2 is glued on L_1, i.e. the left surface of L_2 and the right surface of L_1 have the same radius of curvature, $-R$. The second surface of L_2 has a radius of curvature R'. L_2 is made of SF6 glass.



1. Assuming that all lenses are thin, what is the relationship between f', f'_1, f'_2?
2. What is the relationship between f'_1, R, and n_1 (the index of refraction of L_1)?
3. What is the relationship between f'_2, R, R' and n_2 (the index of refraction of L_2)?

The index of refraction of glasses usually depends on the wavelength of light λ. The table below summarizes the index of refraction of BK7 and SF6 at three standard wavelengths in the visible spectrum:

	BK7	SF6
$\lambda_{Red} = 656nm$	$n_{Red} = 1.5143$	$n_{Red} = 1.7961$
$\lambda_{Yellow} = 587nm$	$n_{Yellow} = 1.5168$	$n_{Yellow} = 1.8052$
$\lambda_{Blue} = 486nm$	$n_{Blue} = 1.5224$	$n_{Blue} = 1.8277$

Note that λ_{Yellow} is almost half-way between λ_{Red} and λ_{Blue}, and that for each glass $n(\lambda)$ varies by very small amounts.

To characterize the variation of $n(\lambda)$, one uses the Abbe number ν defined as follows :

$$\nu = \frac{n_{Yellow} - 1}{n_{Blue} - n_{Red}}.$$

4. For which glass does $n(\lambda)$ vary faster? Calculate the Abbe number ν_1 for BK7, and ν_2 for SF6.
5. f'_1 and f'_2 vary when the wavelength of the light varies. Explain why.

We note $\Delta f'_1 = f'_{1,Blue} - f'_{1,Red}$ the variation of f'_1 over the visible spectrum, and $\Delta f'_1 = f'_{2,Blue} - f'_{2,Red}$ the variation of f'_2.

6. Show that $\Delta f_1' = -\frac{f_1'}{\nu_1}$ and $\Delta f_2' = -\frac{f_2'}{\nu_2}$
7. How is $\Delta f'$ related to f', $\Delta f_1'$, $\Delta f_2'$, f_1' and f_2' ? Deduce a relationship between f_1', f_2', ν_1 and ν_2 in order to have $\Delta f' = 0$. This is the condition for "*achromatism*". When this condition is fulfilled, the doublet focuses the "Blue" radiation and the "Red" radiation at the same point.
8. Deduce from questions 1 and 7 the values of f_1' and f_2' (in millimeters) that allow this doublet to be achromatic.
9. Deduce from questions 2, 3 and 8 the values of R and R' (in millimeters).

Exercise 2 (9 points): Bright field of view of a microscope viewer

We consider a microscope viewer made of

- A microscope objective with focal length $f_1' = 40mm$ and a diameter $\Phi_1 = 6mm$; it magnifies a real object with a transverse magnification $|(g_y)| = 2$.
- A tube with length L.
- An eyepiece with focal length $f_2' = 20mm$ and a diameter $\Phi_2 = 4mm$.

In the following the microscope objective and the eyepiece are thin lenses.

1. Evaluate the "commercial" intrinsic magnification power $G_{i,c}$ of the eyepiece.
2. Which lens is the pupil of the system? The microscope objective or the eyepiece? Explain your answer.

The eyepiece is adjusted for visual observations with a normal eye and no accommodation.

3. Evaluate the distance L between the two lenses.
4. Draw a schematic to scale 1:1 horizontally (and 10:1 transversally) showing a bundle of rays coming for a point object on axis and propagating through the viewer.
5. Calculate the position of the exit pupil of the viewer, and its diameter. Cross-check your result by a ray construction.
6. On the same schematic, draw a bundle of rays associated to the edge of the bright field of view (the rays should propagate through the viewer, from the object plane down to the exit pupil).
7. Calculate the diameter of the bright field of view (in millimeters) in the plane of the intermediate image, and in the object plane.

We insert a reticle with small graduations in the plane of the intermediate image. The distance between two graduations is 100µm.

8. Can the eye resolve two graduations? We remind that the resolution of a normal eye is 2'.

Exercise 3 (4 points): Design of an afocal doublet

We consider a doublet with two thin converging lenses L_1 and L_2. Their focal distances are f_1' and $f_2' = 150mm$ respectively. The distance between L_1 and L_2 is e.

1. Find f_1' and e so that the doublet is afocal with a transverse magnification ratio $|g_y| = 5$.

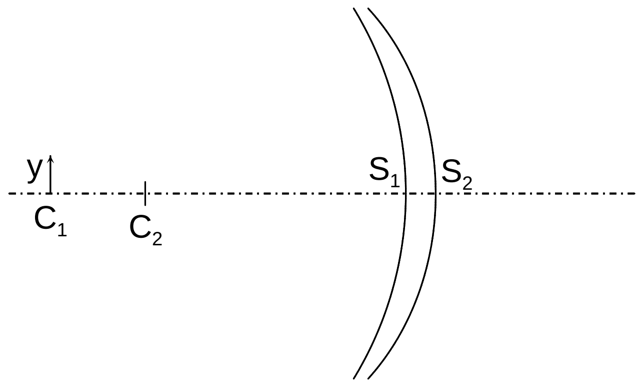
We now consider a real object at a finite distance $x_1 = -2f_1'$ from L_1.

2. Calculate the position of the image.
3. Draw a schematic to scale 1:60 and find the position of the image by ray constructions.

Exercise 4 (8 points): Aplanatic meniscus

We consider a converging meniscus lens L with radii of curvature $|R_1| = 8mm$ and $|R_2| = 6mm$, and a small object at the center of curvature C_1 of the first spherical dioptre. The diameter of the lens is

$\Phi = 10mm$ and the index of refraction of the glass is $n = 1.5$. The image through the lens is at position A' on axis.



1. Calculate the transverse magnification $(g_y)_1$ associated to the conjugation $C_1 \rightarrow C_1$ through the first dioptre. Prove that this conjugation is aplanatic.
2. We want the meniscus to be aplanatic for the conjugation $C_1 \rightarrow A'$. To which particular points of the second dioptre do C_1 and A' correspond if this is the case ?
3. Calculate the position of the image through the lens. (Note: the lens is not thin).
4. Calculate the transverse magnification $(g_y)_2$ associated to the conjugation $C_1 \rightarrow A'$ through the second dioptre.
5. Deduce from questions 1 and 4 the total transverse magnification $(g_y)_L$ of the meniscus for the conjugation $C_1 \rightarrow A'$.
6. Calculate the numerical aperture $\sin\alpha$ of the meniscus in the object space.
7. Deduce from above the numerical aperture $\sin\alpha'$ in the image space.
8. Draw a schematic to scale 10:1 showing the object with height $y = 200\mu m$ and the position of the final image A'. Show on this schematic the intermediate image y_i after refraction through the first dioptre. Find the height y' of the final image though the lens by ray constructions (note that the lens is not thin). Cross-check your result with question 5.

Summary of some non paraxial formulae:

Abbe's condition for aplanatism: $n y \sin\alpha = n' y' \sin\alpha'$

Positions of Young-Weierstrass points W and W' of a spherical dioptre $n \rightarrow n'$ with radius of curvature $R = \overline{SC}$: $\overline{CW} = \frac{n'}{n} R$ and $\overline{CW'} = \frac{n}{n'} R$

Summary of some paraxial formulae:

Descartes' formulae: $(g_y)_{A\rightarrow A'} = \frac{n}{n'} \frac{\overline{H'A'}}{\overline{HA}}$ $\qquad \frac{n'}{\overline{H'A'}} - \frac{n}{\overline{HA}} = C = \frac{n'}{f'} = -\frac{n}{f}$

Newton's formulae: $(g_y)_{A\rightarrow A'} = -\frac{\overline{F'A'}}{f'} = -\frac{f}{\overline{FA}}$ $\qquad \overline{F'A'} \times \overline{FA} = f \times f'$

Lagrange-Helmholtz invariant: $n y \alpha = n' y' \alpha'$

Longitudinal magnification: $(g_x)_{A\rightarrow A'} = \frac{n'}{n} (g_y)^2_{A\rightarrow A'}$ (true even for afocal systems)

Paraxial longitudinal invariant of the spherical dioptre: $Q_x = n\left(\frac{1}{R} - \frac{1}{x}\right) = n'\left(\frac{1}{R} - \frac{1}{x'}\right)$

with $x = \overline{SA}$, $x' = \overline{SA'}$, and $R = \overline{SC}$

Paraxial transverse invariant of the spherical dioptre: $Q_y = n \frac{y}{x} = n' \frac{y'}{x'}$

The above yields: $\frac{n'}{\overline{SA'}} - \frac{n}{\overline{SA}} = \frac{n' - n}{\overline{SC}}$ for a spherical dioptre (take $n' = -n$ for a spherical mirror).

Convergence of a thin lens surrounded by air: $C = (N - 1)\left(\frac{1}{R_1} - \frac{1}{R'_1}\right)$

Gullstrand's formula for two focal systems with intermediate medium n_i: $C = C_1 + C_2 - \frac{e}{n_i} C_1 C_2$ with $e = \overline{H'_1 H_2}$