Reflection and transmission of light by a flat interface, Fresnel's formulae

Two media of different refractive indices have in common a planar boundary called *interface*. The optical properties of the interface depend on its *relative refractive index*, i.e. the ratio of the refractive indices of the two media. When the interface is flat, each of its faces reflects and transmits unidirectional light into one couple of directions, called *regular* or *specular directions*, attached to the reflected and refracted components. Reflection and refraction play an important role in the interaction of light with printed supports. Everyone has observed the reflection of light by the surface of a glossy photograph. At the other side of the surface, the diffuse light coming from the paper and the inks is also reflected. Thus, light travels several times in average between the paper substrate and the inks before exiting the print definitively.

1. Refractive index

The refractive index of a medium is a measure of the propagation properties of light in that medium. It is generally a complex number depending on wavelength:

$$\hat{n}(\lambda) = n(\lambda) + i\kappa(\lambda) \tag{1}$$

The real part $n(\lambda)$, called *real refractive index*, is related with the light propagation speed. The imaginary part $\kappa(\lambda)$, called *extinction index*, characterizes absorption by the medium. Table 2 gives the refractive indices of a few common materials.

Table 1. Refractive indices of materials measured at λ = 589 nm (Sodium D line)

Air	1.0003
Water (at 20°C)	1.333
Ethanol	1.36
Fused quartz $Si0_2$	1.45
Cellulose	1.47
Polypropylene	1.49
Acrylic	1.49
Polyvinyl alcohol	1.50
Plexiglass	1.51
Crown glass	1.52
Sodium Chloride (NaCl)	1.544
Amber	1.55
Polycarbonate	1.58
Polystyrene	1.59
Zircon ($ZrO_2 \cdot Si0_2$	1.923
Diamond	2.417
Rutile (TiO_2)	2.907
Gold	0.27+ 2.95 *i*
Silver	0.20 + 3.44 *i*
Copper	0.62 + 2.57 *i*
Platinum	2.63 + 3.54 *i*
Aluminium	1.44 + 5.23 *i*

Ellipsometry is the favourite technique for refractive index measurements. It is based on polarization analysis. The constraint is that the sample must be homogenous, nonscattering and very flat, which makes this technique almost impossible to use with printing materials such as inks and paper. Note that the real and imaginary functions of wavelength are related to each other by the Kramers-Kronig relations. Thus, knowing either the real index or the extinction index over the whole spectrum enables obtaining the other one for any wavelength.

For dielectric materials such as glass, plastic or paper fibres, the attenuation index is low compared to the real index. The refractive index may be considered as being real and absorption is modelled independently by an attenuation factor applied to the ray (*see* the section on Beer's law). The dependence of the real index on wavelength, being at the origin of the *dispersion* phenomenon as well as the chromatic aberrations in optical systems, is empirically modelled in the visible wavelength domain by Cauchy's law:

$$n(\lambda) = a_1 + \frac{a_2}{\lambda^2} \tag{2}$$

where the dimensionless factor a_1 and the coefficient a_2 (in m^{-2}) are to be determined for each medium. As the real index varies with respect to wavelength, rays are refracted at different angles and split white light pencils into diverging pencils, commonly called *rainbows* in the case of rain drops. However, dispersion has no significant effect when the incident light is diffuse or when the medium is diffusing, because the different spectral components superpose to each other and yield again white light in all directions. This is the reason why dispersion is ignored in the case of papers or white paints and a constant real refractive index is attached to them.

2. Snell's laws

When a light ray propagating into a given medium 1 encounters a medium 2 with different refractive index, its orientation is modified: a component is reflected back into medium 1, and a second component is refracted into medium 2. The directions of reflection and refraction satisfy *Snell's laws*: 1) the incident, reflected and refracted light rays belong to a same plane, called the *incidence plane*, which also contains the normal of the interface; 2) the angles formed by the incident ray and the reflected ray with respect to the normal of the interface are equal; 3) the angle of refraction is related to the angle of incidence according to the "sine law"

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{3}$$

where n_1 and n_2 denote the refractive indices of the two media and θ_1 and θ_2 the respective orientations of light in them (Figure 1).

Note that the wavelength of light is modified when entering the second medium. The wavelength concept is therefore dependent upon the propagation medium, being shorter in the medium with higher index. However, as light sources and detectors are generally in air, all detected rays have their original wavelength even after having traversed different media. Wavelength variation in matter is therefore ignored and only the wavelength in air is considered.

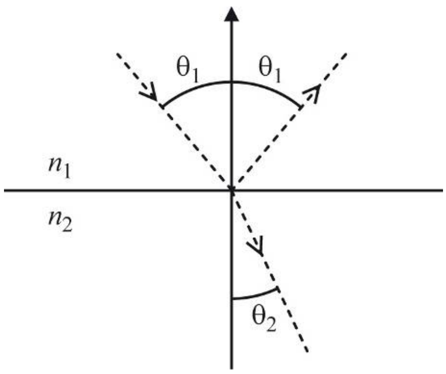

Figure 1. Reflection and refraction in the incidence plane, when $n_1 < n_2$.

3. Total reflection

Let us assume $n_1 < n_2$. When light comes from medium 1, the refraction angle is always smaller than the incidence angle. At grazing incidence, i.e. $\theta_1 = \pi/2$, the refraction angle reaches a limit value $\theta_c = \arcsin(n_1/n_2)$, called the *critical angle*. No light can be refracted into medium 2 with higher angle. When light comes from medium 2, it is refracted into medium 1 provided the incident angle θ_2 is lower than the critical angle θ_c. Otherwise, Snell's sine law (3) provides no real solution for angle θ_1, refraction does not occur and the ray is totally reflected.

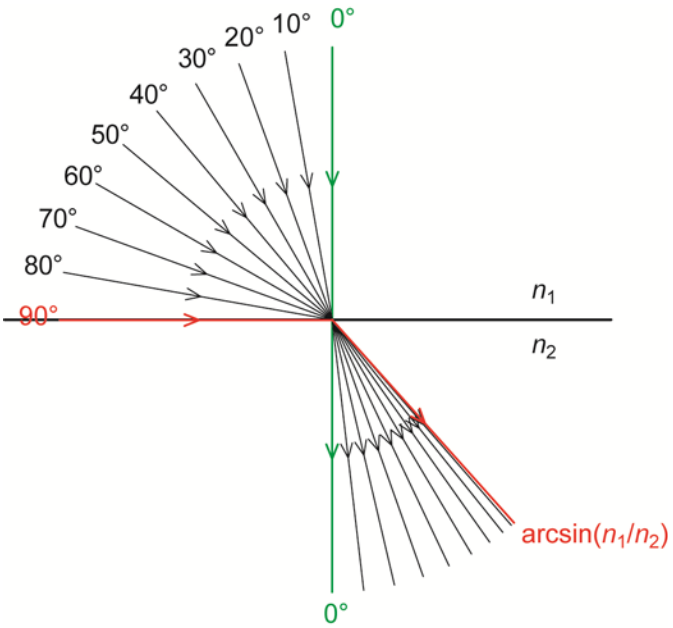

Figure 2. Correspondence between incident and refraction angles for $n_2 / n_1 = 1.5$.

4. Fresnel's formulae

The fraction of light that is reflected by the interface between media 1 and 2 is called *angular reflectance*. It is given by Fresnel's formulae, established by writing the transition equation of electromagnetic waves at the interface. It depends on the incident angle θ_1, on the relative refractive index of the interface $n = n_2 / n_1$ and on the polarization of the incident light. Most of the time, we consider unpolarized incident light which is modelled as the sum of two linearly polarized lights (see Section 1.2). Since the angular reflectance depends on the orientation of the electric field in respect to the incidence plane, we consider the cases where the electric field oscillates parallely and perpendicularly to the incidence plane. These two polarizations are respectively called "parallel" and "perpendicular" polarizations and denoted by symbols *p* and *s*.

Let us consider a light pencil coming from medium 1 with incident angle θ_1. For *p*-polarized light, the angular reflectance is

$$R_{p12}(\theta_1) = \left(\frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}\right)^2 = \left(\frac{n\cos\theta_1 - \cos\theta_2}{n\cos\theta_1 + \cos\theta_2}\right)^2 \tag{4}$$

where $\theta_2 = \arcsin(n_1 \sin\theta_1 / n_2)$ is the angle of refraction into medium 2 defined by Snell's law. For *s*-polarized light, the angular reflectance is

$$R_{s12}(\theta_1) = \left(\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}\right)^2 = \left(\frac{\cos\theta_1 - n\cos\theta_2}{\cos\theta_1 + n\cos\theta_2}\right)^2 \tag{5}$$

The variation of angular reflectance for the *p*- and *s*-polarized components are plotted in Figure 3 as a function of the incident angle θ_1 for an interface with relative refractive index $n = 1.5$.

At normal incidence, *p*-polarized, *s*-polarized and unpolarized lights have the same angular reflectance:

$$R_{p12}(0) = R_{s12}(0) = R_{12}(0) = \left(\frac{n-1}{n+1}\right)^2 \tag{6}$$

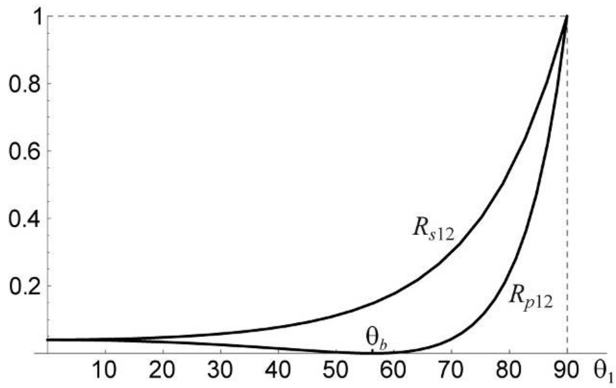

Figure 3. Angular reflectance for *p*- and *s*-polarized lights when $n = n_2 / n_1 = 1.5$. The Brewster angle is $\theta_b = \arctan(1.5) \simeq 56.3°$.

For oblique incidence, angular reflectances may expressed as functions of angle θ_1 only, by inserting $\theta_2 = \arcsin(\sin\theta_1 / n)$ into equations (4) and (5):

$$R_{p12}(\theta_1) = \left(\frac{n^2\cos\theta_1 - \sqrt{n^2 - \sin^2\theta_1}}{n^2\cos\theta_1 + \sqrt{n^2 - \sin^2\theta_1}}\right)^2 \tag{7}$$

$$R_{s12}(\theta_1) = \left(\frac{\cos\theta_1 - \sqrt{n^2 - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{n^2 - \sin^2\theta_1}} \right)^2 \quad (8)$$

Unpolarized light contains same quantity of *p*- and *s*-polarizations. Therefore, the angular reflectance for unpolarized light is the average of the two angular reflectances:

$$R_{12}(\theta_1) = \frac{1}{2}\left[R_{p12}(\theta_1) + R_{s12}(\theta_1) \right] \quad (9)$$

Except at normal incidence, the *p*- and *s*-polarized lights are reflected in different proportions. The reflected and transmitted lights are therefore partially polarized. At the angle $\theta_b = \arctan(n_{12})$, called the *Brewster angle*, *p*-polarized light is not reflected at all. The corresponding angular reflectance is zero. The reflected light is therefore totally polarized (s-polarization). Reflection at the Brewster angle is one possible method to produce linearly polarized light.

Independently of polarization, the angular reflectance is the same if light comes from medium 1 at the angle θ_1 or comes from medium 2 at the corresponding regular angle $\theta_2 = \arcsin(n_1 \sin\theta_1 / n_2)$:

$$R_{*12}(\theta_1) = R_{*21}(\theta_2) \quad (10)$$

where symbol $*$ means either *s*-polarized, *p*-polarized or unpolarized light. Figure 4 shows the variation of angular reflectances and transmittances from normal to grazing incidence in both medium 1 and 2 for an interface with relative index $n = 1.5$.

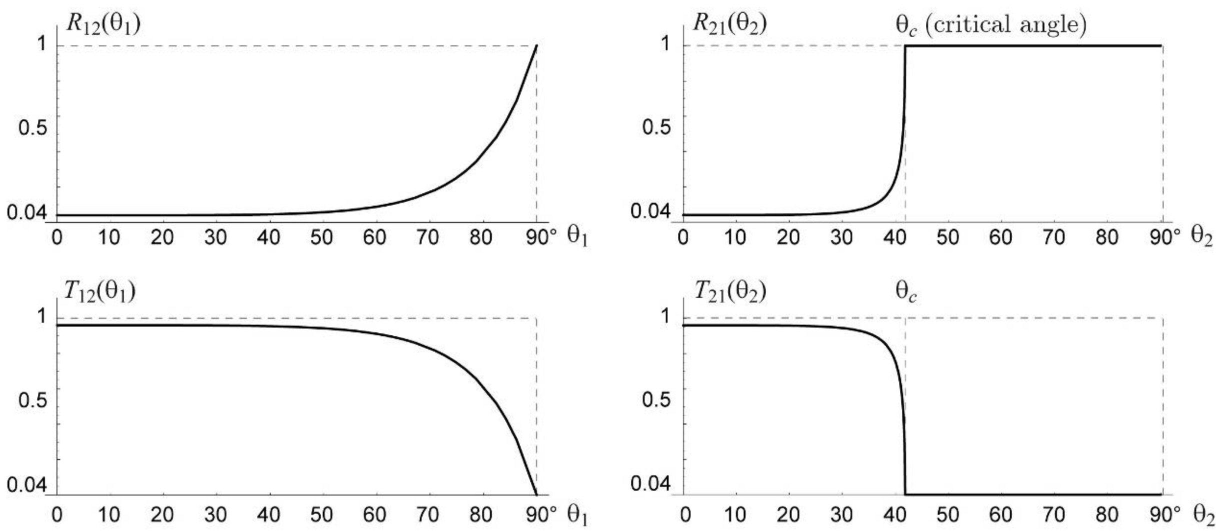

Figure 4. Angular reflectance and transmittance of an interface with relative index $n = n_2 / n_1 = 1.5$ as a function of the incident angle for natural light from medium 1 (*left*) or medium 2 (*right*).

Regarding the refracted component, since no light is absorbed at the interface, the *angular transmittance* is

$$T_{*12}(\theta_1) = 1 - R_{*12}(\theta_1) \quad (11)$$

and, as a consequence of (10), one has

$$T_{*12}(\theta_1) = T_{*21}(\theta_2) \quad (12)$$

This equality means that for a given path of light, the angular transmittance does not depend whether light transits from medium 1 to medium 2 or from medium 2 to medium 1. In case of total reflection, the angular transmittance is zero.

5. Radiance reflection and refraction

In radiometry, light pencils are described by the radiance concept. When a pencil enters a medium with different index, refraction modifies the ray's geometrical extent (Figure 5). Radiance is thus modified. The relationship between incident, reflected and refracted radiances is derived from geometrical arguments issued from Snell's laws.

The incident radiance L_1 is defined as the flux element $d^2\Phi_1(\theta_1,\varphi_1)$ coming from direction (θ_1,φ_1) through the infinitesimal solid angle $d\omega_1 = \sin\theta_1 d\theta_1 d\varphi_1$, and illuminating a an elemental area *ds*

$$L_1 = \frac{d^2\Phi_1(\theta_1,\varphi_1)}{ds\cos\theta_1 \sin\theta_1 d\theta_1 d\varphi_1} \tag{13}$$

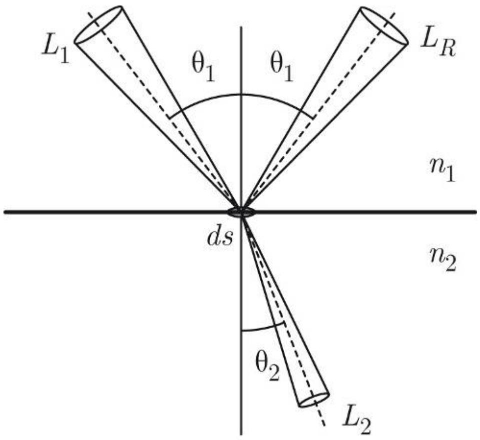

Figure 5. Incident, reflected and refracted radiances at the interface between two media of indices n_1 and $n_2 > n_1$.

The denominator in equation (13) denotes the geometrical extent of the incident pencil. Since the reflected and incident pencils form the same angle with the normal, they have the same geometrical extent. The reflected radiance L_R is therefore the incident radiance L_1 attenuated by the angular reflectance $R_{12}(\theta_1)$ of the interface

$$L_R = R_{12}(\theta_1)L_1 \tag{14}$$

Regarding the refracted pencil, the refraction and incidence angles satisfy Snell's sine law (3). By differentiating equation (3), one obtains

$$n_1 \cos\theta_1 d\theta_1 = n_2 \cos\theta_2 d\theta_2 \tag{15}$$

The incident and refracted azimuthal angles form a fixed angle π, a small variation of the one implies the same variation of the other one, i.e. $d\varphi_1 = d\varphi_2$. Hence, one has

$$n_1^2 ds\cos\theta_1 \sin\theta_1 d\theta_1 d\varphi_1 = n_2^2 ds\cos\theta_2 \sin\theta_2 d\theta_2 d\varphi_2 \tag{16}$$

i.e.

$$n_1^2 dG_1 = n_2^2 dG_2 \tag{17}$$

where dG_1 and dG_2 denote the geometrical extent of the pencil in media 1 and 2 respectively. Equation (17) shows that the geometrical extent is multiplied by a factor $(n_j / n_i)^2$ each time it goes from a medium *i* to a medium *j*, but the quantity $n_i^2 dG_i$ remains invariant. This invariance generalizes the invariance of geometrical extent stated in the previous section in the special case where the extremities of the pencil where both located in air. Finally, accounting for the changing of geometrical extent due to the refraction, the refracted radiance is

$$L_2 = (n_2 / n_1)^2 T_{12}(\theta_1) L_1 \tag{18}$$

6. Lambertian reflectance of an interface

Let us now consider that the interface is illuminated by Lambertian light. We denote as $n = n_2 / n_1$ the relative index of the interface and assume that $n > 1$. When the light comes from medium 1, the "Lambertian reflectance", denoted as r_{12}, is

$$r_{12} = \int_{\theta_1=0}^{\pi/2} R_{12}(\theta_1) \sin 2\theta_1 d\theta_1 \tag{19}$$

r_{12} depends only on the relative index *n*. It may be computed by discrete summation with a small sampling step, e.g. $\Delta\theta_1 = 0.001$ rad. Alternatively, it is given by the following analytical formula, which comes from a tedious integral calculation whose main lines are presented in Appendix

$$r_{12} = \frac{1}{2} + \frac{(n-1)(3n+1)}{6(n+1)^2} - \frac{2n^3(n^2+2n-1)}{(n^4-1)(n^2+1)} + \frac{8n^4(n^4+1)\ln(n)}{(n^4-1)^2(n^2+1)} + \frac{n^2(n^2-1)^2}{(n^2+1)^3} \cdot \ln\left(\frac{n-1}{n+1}\right) \tag{20}$$

The reflected flux fulfills the whole hemisphere but is not Lambertian anymore as the reflected radiance varies with angle. The transmitted flux is concentrated into the cone delimited by the critical angle $\theta_c = \arcsin(1/n)$. The conservation of the energy at the interface implies that the transmittance is

$$t_{12} = 1 - r_{12} \tag{21}$$

When the Lambertian light comes from medium 2, the reflectance r_{21} is similarly expressed as r_{12} with $R_{12}(\theta_1)$ replaced by $R_{21}(\theta_2)$

$$r_{21} = \int_{\theta_2=0}^{\pi/2} R_{21}(\theta_2) \sin 2\theta_2 d\theta_2 \tag{22}$$

Even though $R_{12}(\theta_1)$ and $R_{21}(\theta_2)$ are equal [*see* equation (10)], reflectances r_{12} and r_{21} are different due to total reflection which takes place in medium 2 but not in medium 1. They are related by the following formula established in Appendix B.2:

$$1 - r_{21} = \frac{1}{n^2}(1 - r_{12}) \tag{23}$$

One deduces from it the relationship of transmittances:

$$t_{21} = \frac{1}{n^2} t_{12} \tag{24}$$

For an air-glass interface of typical relative index *n* = 1.5, one has $r_{12} \simeq 0.1$, $t_{12} \simeq 0.9$, $r_{21} \simeq 0.6$ and $t_{21} \simeq 0.4$. Their values for other indices are listed in in Appendix.

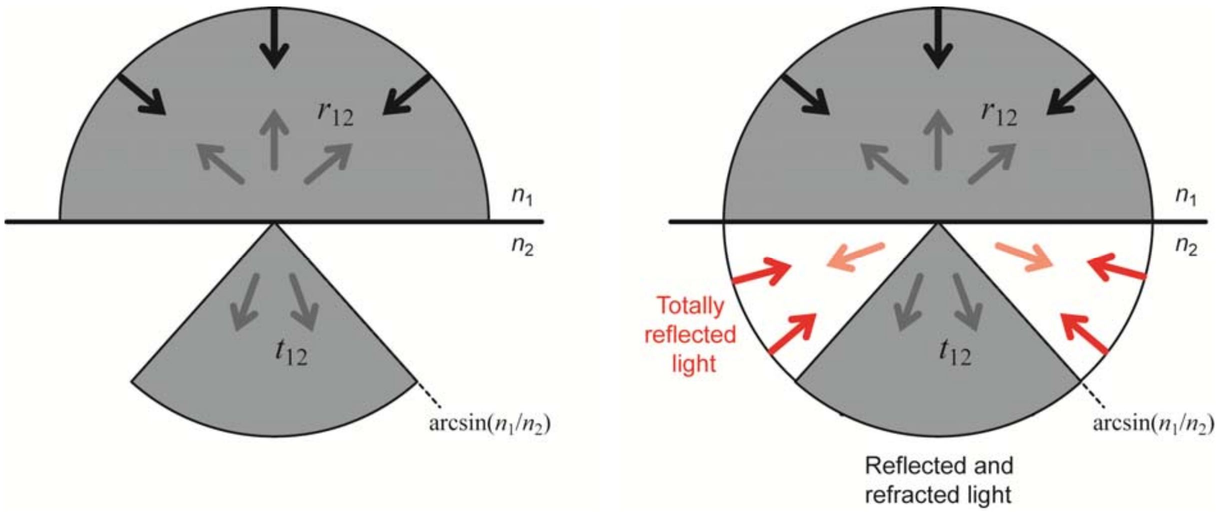

Figure 6. Reflection and transmission of diffuse light coming from medium 1 (*left*) or medium 2 (*right*) when $n_2 > n_1$.

7. Absorbing media and metals

The color of a homogenous medium comes from its capacity to absorb radiations of specific wavelengths in the visible domain, which corresponds to a nonzero extinction index κ(λ). The absorption coefficient α is related to the extinction index by the formula

$$\alpha(\lambda) = \frac{4\pi}{\lambda}\kappa(\lambda) \tag{25}$$

This relation is valid for any absorbing medium, e.g. colored glass or metal. The particularity of metals is their high extinction index, which makes them very opaque and reflecting. The angular reflectance of air-metal interfaces is given by the same Fresnel formulas (4) and (5) as for air-dielectric interfaces, but the refractive index is a complex number $\hat{n} = n + i\kappa$ including the extinction coefficient. The refraction angle θ_2 is also a complex number. Nevertheless, the angular reflectance is real and may be expanded as follows:

$$R_{s12}(\theta_1) = \frac{\left(\sqrt{a+z} - \sqrt{2}\cos\theta_1\right)^2 + a - z}{\left(\sqrt{a+z} + \sqrt{2}\cos\theta_1\right)^2 + a - z} \tag{26}$$

$$R_{p12}(\theta_1) = R_{s12}(\theta_1)\cdot\frac{\left(\sqrt{a+z} - \sqrt{2}\sin\theta_1\tan\theta_1\right)^2 + a - z}{\left(\sqrt{a+z} + \sqrt{2}\sin\theta_1\tan\theta_1\right)^2 + a - z} \tag{27}$$

with $z = n^2 - \kappa^2 - \sin^2\theta_1$ and $a = \sqrt{z^2 + 4n\kappa}$. For unpolarized incident light, the angular reflectance is the average of formulas (26) and (27).

Figure 7 illustrates how the angular reflectance increases as the extinction coefficient increases. From $\kappa = 0$ to 0.2, the angular reflectance remains close to 0.04, i.e. the value corresponding to a real index of 1.5. This justifies that for weakly absorbing dielectrics the extinction index is not taken into account in the Fresnel formulas. Beyond 0.2, the angular reflectance increases rapidly. On a spectral point of view, the angular reflectance a surface is higher at the wavelengths where the medium is the more absorbing (higher absorption coefficient). Reflected and transmitted lights therefore get complementary colors.

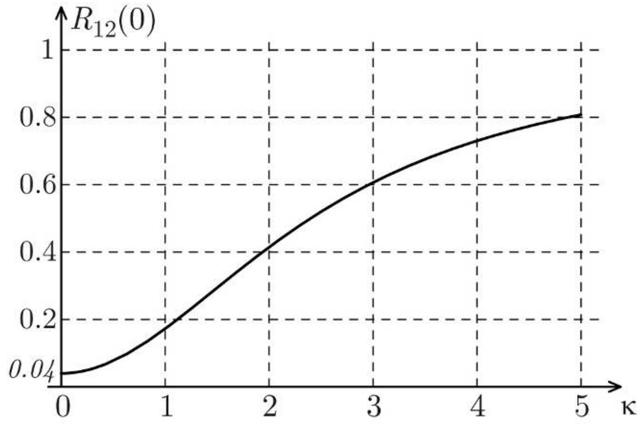

Figure 7. Variation of the angular reflectance at normal incidence of an interface with relative refractive index $\hat{n} = 1.5 + i\kappa$ as a function of κ.

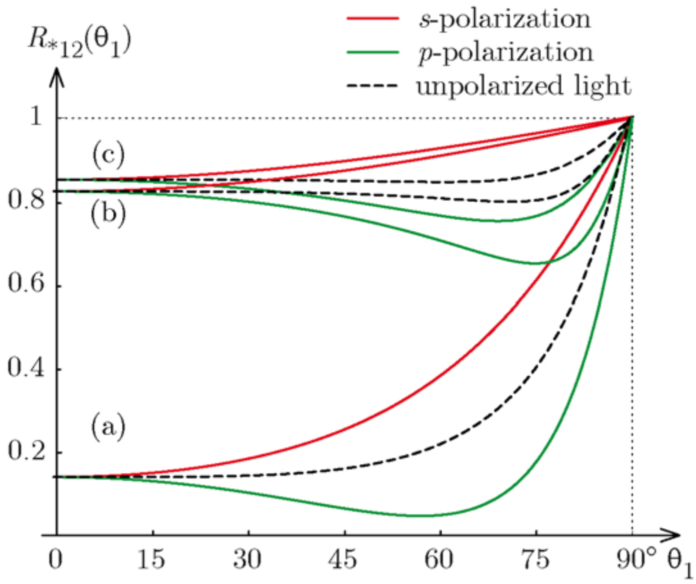

Figure 8. Angular reflectance as a function of the incident angle, for *p*-polarized, *s*-polarized and unpolarized lights, of (a) strongly absorbing glass ($\hat{n} = 1.5 + i$), (b) platinum at ($\hat{n} = 2.06 + 4.26i$) and (c) gold at 600 nm ($\hat{n} = 0.37 + 2.82i$).

The variation of the angular reflectance as a function of the incident angle is noticeably different between absorbing media and nonabsorbing media. Figure 8 shows three examples based on the refractive indices of a strongly absorbing glass ($\hat{n} = 1.5 + i$), gold at 600 nm ($\hat{n} = 0.37 + 2.82i$) and platinum at ($\hat{n} = 2.06 + 4.26i$). In the three cases, the angular reflectance for *s*-polarized light is a strictly increasing function of the incident angle, while the one for *p*-polarized light decreases to a minimum without reaching zero. The reflected light is therefore partially polarized but there is no angle at which its polarization is total. In the case of gold and platinum, the angular reflectance for unpolarized light reaches a minimum, whereas the minimum is at normal incidence for dielectrics.

Fresnel's formulae give the proportions of light that are reflected and transmitted by an interface between media with distinct refractive indices. They depend on the orientation of light and on the refractive indices of the two media.

Related references

M. Born and E. Wolf, *Principle of Optics* (Pergamon, Oxford, 7th expanded Edition, 1999), p. 47.

W. R. McCluney, *Introduction to Radiometry and Photometry*, Artech House (1994) 7–13.

D.B. Judd, "Fresnel Reflection of Diffusely Incident Light," *J. of the National Bureau of Standards*, **29** (1942) 329–332.

Appendix – Relationship between front and back Lambertian reflectances of an interface

The reflectance formula (20) is valid when the Lambertian light comes from the medium with lowest refractive index. When it comes from the other medium, the reflectance is much higher because all rays with incident angle higher than the critical angle $\theta_c = \arcsin(1/n)$ are totally reflected. There exists a simple relation between the two reflectances that we propose here to establish. As previously, we consider media of refractive indices n_1 and $n_2 > n_1$ (the relative index $n = n_2/n_1$ is therefore higher than one). Let us start from the integral expressing the reflectance at the side of medium 2 given by equation (22). We decompose it into two integrals on the intervals $[0,\theta_c]$ and $[\theta_c,\pi/2]$. In the interval $[0,\theta_c]$, relations (3), (10) and (15) yield the equality:

$$R_{21}(\theta_2)\sin 2\theta_2 d\theta_2 = R_{12}(\theta_1)\frac{1}{n^2}\sin 2\theta_1 d\theta_1 \tag{28}$$

Instead of integrating the left member of (28) according to θ_2 on the interval $[0,\theta_c]$, let us integrate the right member according to θ_1 on the corresponding interval $[0,\pi/2]$ in which we retrieve the integral expressing reflectance r_{12} [see equations (19)]:

$$\int_{\theta_2=0}^{\theta_c} R_{21}(\theta_2)\sin 2\theta_2 d\theta_2 = \frac{1}{n^2}\int_{\theta_1=0}^{\pi/2} R_{12}(\theta_1)\sin 2\theta_1 d\theta_1 = \frac{1}{n^2} r_{12} \tag{29}$$

In the interval $[\theta_c,\pi/2]$, the angular reflectance is 1. One therefore has

$$\int_{\theta_c}^{\pi/2} R_{21}(\theta_2)\sin 2\theta_2 d\theta_2 = \int_{\arcsin(1/n)}^{\pi/2} \sin 2\theta_2 d\theta_2 = 1 - \frac{1}{n^2} \tag{30}$$

The sum of the two integrals provides the relation given in equation (23).

Appendix – Numerical values of interface reflectances and transmittances

The reflectances and transmittances of flat interfaces whose numerical values are given in Table B.1 depend only the relative index *n*. The three reflectances in question, $R_{12}(0)$, r_{12} and r_{21} correspond respectively to the following three illumination geometries: directional illumination at normal incidence, Lambertian illumination from the medium with smallest index, and Lambertian illumination from the other medium. They are respectively given by the equations (6), (20) and (23). The three transmittances $T_{12}(0)$, t_{12} and t_{21}, defined by the same geometries as the reflectances, are simply given by: $T_{12}(0) = 1 - R_{12}(0)$, $t_{12} = 1 - r_{12}$, and $t_{21} = 1 - r_{21}$.

Table 2. Reflectance and transmittance of interfaces for different relative indices *n*.

n	$R_{12}(0)$	$T_{12}(0)$	r_{12}	r_{21}	t_{12}	t_{21}
1.30	0.0170	0.9830	0.0611	0.4445	0.9389	0.5555
1.31	0.0180	0.9820	0.0627	0.4538	0.9373	0.5462
1.32	0.0190	0.9810	0.0643	0.4630	0.9357	0.537
1.33	0.0201	0.9799	0.0659	0.4719	0.9341	0.5281
1.34	0.0211	0.9789	0.0675	0.4807	0.9325	0.5193
1.35	0.0222	0.9778	0.0691	0.4892	0.9309	0.5108
1.36	0.0233	0.9767	0.0706	0.4975	0.9294	0.5025
1.37	0.0244	0.9756	0.0722	0.5057	0.9278	0.4943
1.38	0.0255	0.9745	0.0737	0.5136	0.9263	0.4864
1.39	0.0266	0.9734	0.0753	0.5214	0.9247	0.4786
1.40	0.0278	0.9722	0.0768	0.5290	0.9232	0.4710
1.41	0.0289	0.9711	0.0783	0.5364	0.9217	0.4636
1.42	0.0301	0.9699	0.0799	0.5437	0.9201	0.4563
1.43	0.0313	0.9687	0.0814	0.5508	0.9186	0.4492
1.44	0.0325	0.9675	0.0829	0.5577	0.9171	0.4423
1.45	0.0337	0.9663	0.0844	0.5645	0.9156	0.4355
1.46	0.0350	0.965	0.0859	0.5711	0.9141	0.4289
1.47	0.0362	0.9638	0.0873	0.5777	0.9127	0.4223
1.48	0.0375	0.9625	0.0888	0.5840	0.9112	0.4160
1.49	0.0387	0.9613	0.0903	0.5902	0.9097	0.4098
1.50	0.0400	0.9600	0.0918	0.5963	0.9082	0.4037
1.51	0.0413	0.9587	0.0932	0.6023	0.9068	0.3977
1.52	0.0426	0.9574	0.0947	0.6082	0.9053	0.3918
1.53	0.0439	0.9561	0.0962	0.6139	0.9038	0.3861
1.54	0.0452	0.9548	0.0976	0.6195	0.9024	0.3805
1.55	0.0465	0.9535	0.0991	0.6250	0.9009	0.3750
1.56	0.0479	0.9521	0.1005	0.6304	0.8995	0.3696
1.57	0.0492	0.9508	0.1020	0.6357	0.898	0.3643
1.58	0.0505	0.9495	0.1034	0.6408	0.8966	0.3592
1.59	0.0519	0.9481	0.1048	0.6459	0.8952	0.3541