Introduction to radiometry - Mathieu Hébert

1h30 – No document permitted

Exercise 1

A lamp emits a flux of 0.3 W. Assuming that its spectral flux is uniform in the spectral band 432-437 nm and zero in the rest of the spectrum, what is the spectral flux in the spectral emission band?

The power of 0.3 W is uniformly distributed over a waveband of 5 nm. The spectral flux is therefore 0.3 / 5 = 0.06 W/nm.

Using the table of V(λ) in appendix, what is the visual flux emitted by this lamp?

Being given the spectral flux $F(\lambda)$, the visual flux F_v is :

$$F_v = K_m \int_0^\infty S(\lambda)V(\lambda)d\lambda = K_m S(435\text{ nm})V(435\text{ nm})\Delta\lambda \text{ (in lm)}$$

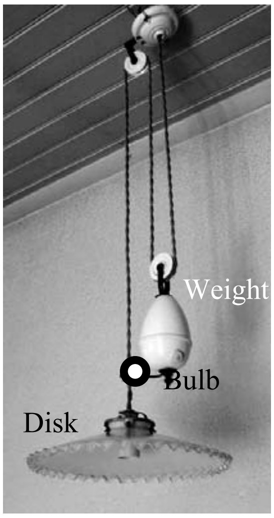
with K_m = 683.002, $S(435\text{ nm})$ = 0.3 W/nm, $V(435\text{ nm})$ = 0.2625, $\Delta\lambda$ =5 nm.

Exercise 2

N photons of wavelength 500 nm form a flux F.
How many photons of wavelength 700 nm form the same flux?

The number of photons is 700 N / 500 = (7/5) N. Since the photons at 700 nm are less energetic, more photons are needed to make the same flux F.

Problem: balance weight lamp.



Balance weight lamps were frequently used in the past by dressmakers to adjust the illuminance from a light bulb on their work. The light bulb is assumed to be an isotropic point source, of electrical power P. Its radiating efficiency is q (in %), and its luminous efficacity is α (in lm/W).
As shown on the picture, the bulb is topped by a white glass disk aiming at reflecting more light towards the table. We assume that this disk is a Lambertian reflector of radius R and reflectance ρ. The distance between the bulb and the center of the white glass disk is y.

1. What is the energetic flux F_e emitted by the bulb?

$$F_e = qP \text{ (in W)}$$

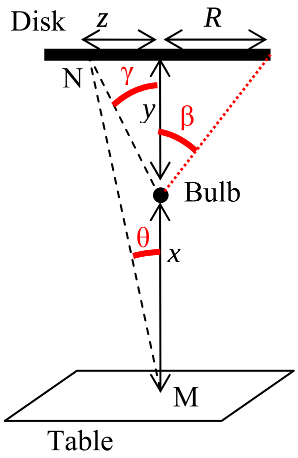
2. What is the visual flux F_v emitted by the bulb?

$$F_v = \alpha F_e = \alpha q P \text{ (in lm)}$$

3. What is the visual intensity emitted by the bulb in a given direction?

Since the source is isotropic, the visual intensity is

$$I_v = \frac{F_v}{4\pi} = \frac{\alpha q P}{4\pi} \text{ (in cd)}$$



4. In absence of disk, what is the irradiance E on the table at point M below the bulb at a distance x from the bulb? How does the irradiance E vary when the lamp-to-table distance x is doubled?

Assuming that the table is far enough from the bulb (i.e. x is large enough), the irradiance on the table at the vertical of the bulb is

$$E_v = \frac{I_v}{x^2} = \frac{\alpha q P}{x^2 4\pi} \text{ (in lux)}$$

If x is doubled, E_v is divided by four.

5. What is the solid angle subtended by the disk from the bulb?

Let us denote as β the angle defined as : $\beta = \arctan(R/y)$

The solid angle ω subtended by the disk is not infinitesimal. It is given by the formula (retrieved by integrating the infinitesimal solid angles subtended by small areas over the disk)

$$\omega = 2\pi(1-\cos\beta) = 2\pi\left(1-\frac{y}{\sqrt{R^2+y^2}}\right)$$

6. What is therefore the total flux received by the disk? and the total flux reflected by the disk? 7. Is the irradiance on the disk uniform over its whole area?

Since the bulb is an isotropic source, the flux received by the whole disk is

$$F = I_v\omega = \frac{\alpha q P}{2}\left(1-\frac{y}{\sqrt{R^2+y^2}}\right) \quad \text{(in lm)}$$

and the reflected flux is $F_{reflected} = \rho F$ by definition of reflectance.

But the irradiance on the disk is not uniform: it is larger on the center of the disk where the distance to the bulb is smaller.

8. Consider a point N on the disk at a distance z from its center. What is the irradiance at this point?

Let us as γ the angle between the line (bulb-N) and the vertical axis : $\gamma = \arctan(z/y)$

The irradiance of a small area ds around N is

$$E(z) = \frac{dF(z)}{ds} \text{ where } dF(z) \text{ is the flux element received by } ds$$

$$= \frac{I_v d\omega}{ds} \text{ where } d\omega = \frac{ds\cos\theta}{z^2+y^2} \text{ is the solid angle subtended by } ds$$

$$= I_v\frac{\cos\theta}{z^2+y^2}, \text{ and we have : } \cos\theta = \cos(\arctan(z/y)) = \frac{y}{\sqrt{y^2+z^2}}$$

$$= \frac{\alpha q P}{4\pi}\frac{y}{(z^2+y^2)^{3/2}} \quad \text{(in lux)}$$

9. Which radiance flows between N and the point M on the table?

The exitance at point N is $\rho E(z)$, and since the disk is Lambertian, the radiance emitted in every direction is

$$L(z) = \frac{\rho E(z)}{\pi} = \frac{\rho\alpha qP}{4\pi^2}\frac{y}{\left(z^2+y^2\right)^{3/2}} \quad \left(\text{in cd/m}^2\right)$$

10. What is the total irradiance in M by considering the direct illumination from the bulb and the total light reflected by the disk?

The total irradiance in M is the irradiance by direct illumination from the bulb, expressed in question 4, and the sum of all components issued from the points of the disk.

Each point on the disk, denoted by its radial coordinates (z, φ), emits towards point M a radiance $L(z)$ expressed in the previous question (depending on its distance z from the center of the disk). This radiance generates an irradiance component in M.

Let us first consider the points of the disk, by starting with the one located at the center of the disk, emitting the radiance $L(0)$. By defining an elemental area around this central point N, ds, and another elemental area around M, ds', knowing that these two areas are distant from each other by a length $x + y$, the geometrical extent is

$$d^2G = \frac{ds\,ds'}{(x+y)^2}$$

The flux flowing in this extent is $d^2F = L(0)d^2G$, and the irradiance in M is

$$dE_M = \frac{ds}{(x+y)^2}L(0)$$

Let us consider another point N on the disk, at a distance z from the center of the disk. The angle between the line (MN) and the vertical axis is

$$\theta = \arctan\left(\frac{z}{x+y}\right) \quad \text{which gives} \quad \cos\theta = \frac{x+y}{\sqrt{(x+y)^2+z^2}}$$

The geometrical extent becomes, according to the $\cos^4$ law:

$$d^2G_z = \frac{ds\,ds'}{(x+y)^2}\cos^4\theta$$

and we can express the elemental area ds around (z, φ) as $ds = z\,dz\,d\varphi$. The elemental irradiance in M issued from this point N is therefore

$$\begin{aligned}
dE_M &= d^2G_z L(z) = \\
&= \frac{z\,dz\,d\varphi\cos^4\theta}{(x+y)^2}L(z) \\
&= zL(z)\left(\frac{x+y}{(x+y)^2+z^2}\right)^2 dz\,d\varphi \\
&= \frac{\rho\alpha qPy(x+y)^2}{4\pi^2}\frac{z}{\left(z^2+y^2\right)^{3/2}\left[(x+y)^2+z^2\right]^2}dz\,d\varphi
\end{aligned}$$

Finally, the total irradiance in M issued from the disk is

$$E_M = \frac{\rho\alpha qPy(x+y)^2}{4\pi^2}\int_{\varphi=0}^{2\pi}\int_{z=0}^{R}\frac{z}{\left(z^2+y^2\right)^{3/2}\left((x+y)^2+z^2\right)^2}dzd\varphi$$

$$= \frac{\rho\alpha qPy(x+y)^2}{2\pi}\int_{z=0}^{R}\frac{z}{\left(z^2+y^2\right)^{3/2}\left((x+y)^2+z^2\right)^2}dz$$

and the total irradiance in M, including the direct illumination from the bulb, is

$$E_{Mtotal} = \frac{\alpha qP}{x^2 4\pi}+\frac{\rho\alpha qP(x+y)^2 y}{2\pi}\int_{z=0}^{R}\frac{z}{\left(z^2+y^2\right)^{3/2}\left((x+y)^2+z^2\right)^2}dz$$

$$= \frac{\alpha qP}{4\pi}\left[\frac{1}{x^2}+2\rho(x+y)^2 y\int_{z=0}^{R}\frac{z}{\left(z^2+y^2\right)^{3/2}\left((x+y)^2+z^2\right)^2}dz\right]$$

NB: as R tends to infinity, the integral tends to

$$\frac{1}{x^2(x+2y)^2}\left[\frac{1}{y}+\frac{y}{2(x+y)^2}-\frac{3}{2\sqrt{x(x+2y)}}\left(\frac{\pi}{2}-\arcsin\left(\frac{y}{x+y}\right)\right)\right]$$