[ENGLISH]

Test of Radiometry

1.5h – No document permitted.

A) Radiometric quantities and units –

A1. Give the name of the following radiometric quantities and the corresponding unit in the energetic, visual and photonic systems:

a) Density of flux per unit wavelength.

Energetic : spectral flux, in $W.nm^{-1}$ (or $W.\mu m^{-1}$)
Visual : does not exist
Photonic : spectral (photon) flux, in $s^{-1}.nm^{-1}$

b) Flux accumulated during one second.

Energetic : energy, in J.
Visual : integrated flux, in lum.s
Photonic : number of photons (no unit).

A2. What is the radiance L reflected in any direction by a 10 cm × 10 cm Lambertian surface of reflectance ρ, when it is uniformly illuminated under an angle θ with a collimated flux F ?
The square (of area denoted as A) receives the flux F. The irradiance is therefore (independently of the illumination angle):
$E = F / A$
The emittance after reflection is
$M = \rho\, E$
and finally the radiance in every direction is

$$L = \frac{\rho F}{\pi A}$$

B) Problem – A house is equipped with two water warming panels, of dimensions 2 × 1.5 m. The first one, in the garden, is horizontal; the second one, located on the roof, is inclined by an angle $\theta = 30°$. At noon, the sunlight is perpendicular to the ground; the radiance from the sun is $L = 2\times10^9$ cd.m^{-2}, and it fulfills a solid angle $\omega = 6\times10^{-5}$ sr.

B1. Express the irradiance on each panel as a function of the panel area A, the sun radiance L, the solid angle ω, the angle θ between the panel and a horizontal plane. Then give numerical evaluation and unit.

The irradiance, in lux, over the panel area A (but independent on A) is

- $E = L\omega$ for the horizontal panel,
- $E = L\omega\cos\theta$ for the oblique panel.

B2. Express the total flux received by each panel, then give numerical evaluation and unit.

The flux, in lumen, received by each panel is

- $F = E \times A = LA\omega$ for the horizontal panel,
- $F = E \times A = LA\omega\cos\theta$ for the oblique panel.

B3. Express the integrated flux received by each panel in one hour, then give numerical evaluation and unit.

Let τ be the duration. The integrated flux e, in lum.s, is

- $e = F\tau = LA\omega\tau$ for the horizontal panel,
- $e = LA\omega\tau\cos\theta$ for the oblique panel.

Now, we consider the same problem with the energetic radiance, instead of the visual radiance.

B4. What is the unit for energetic radiance? and for the total flux received by each panel?

Energetic radiance is in $W.m^{-2}.sr^{-1}$ and energetic flux is in W.

B5. What is the equivalent for the integrated flux concept in this case? Which unit?

The flux accumulated in a given duration corresponds to energy, in J.

B6. In this thermal problem, do you think it is preferable to consider energetic or visual radiance? Explain why.

It is of course preferable to assess light quantities in energetic units, because thermal computations have nothing to do with visual perception. Moreover, visual units do account for all radiation, especially infrared radiation which are important for thermal applications.

C) Problem – Three white LEDs are aligned on the ceiling of a room, equally spaced by a distance $d = 2$ m. Each one emits 4 lum over the hemisphere. They are assumed to be point sources, but not isotropic. Their intensity diagrams are given by:

$$I(\theta,\varphi) = I_0 \cos^2(\theta)$$

where I_0 is the intensity along the normal of the LED, θ the angle from the normal, and φ the azimuthal angle around the normal (Figure 1).
The distance between the floor and the ceiling is $h = 4$ m. We are interested in the irradiance on the floor in point P at the vertical of the central LED (Figure 2).

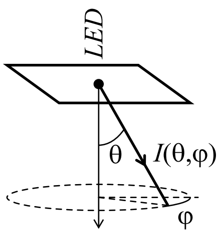

Figure 1.

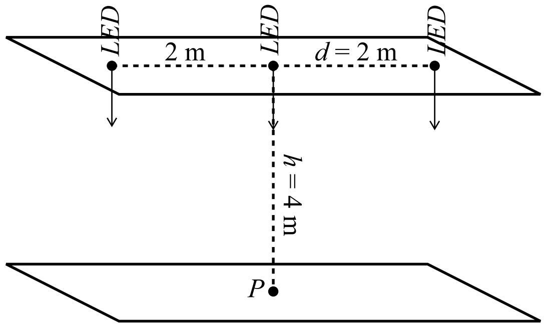

Figure 2.

C1. Knowing the total flux F emitted by one LED in the hemisphere, calculate the intensity along the normal, I_0.

We obtain the total flux by summing up the intensity over the hemisphere:

$$F = \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_0 \cos^2\theta \sin\theta \, d\theta \, d\varphi$$

$$= 2\pi I_0 \int_{\theta=0}^{\pi/2} \cos^2\theta \sin\theta \, d\theta$$

$$= 2\pi I_0 \int_{u=0}^{1} u^2 du$$

$$= \frac{2\pi}{3} I_0$$

Hence, the intensity I_0 is:

$$I_0 = \frac{3F}{2\pi} \text{ (in cd)}$$

C2. Calculate the irradiance E_P received in *P* from the central LED, then from one lateral LED, then from the three LEDs jointly.

Define a small area *ds* around point P.

- Viewed from the central LED, this area *ds* subtends a solid angle :

$d\omega = ds / h^2$.

The elemental flux flowing through this solid angle is $dF = I_0 d\omega$ and the irradiance around point P is:

$$E = \frac{dF}{ds} = \frac{I_0 d\omega}{ds} = \frac{I_0}{h^2} \text{ (in lux)}$$

- Viewed from one lateral LED, the small area *ds* subtends a solid angle:

$$d\omega' = \frac{ds\cos\theta}{h^2 + d^2} \text{ with } \cos\theta = \frac{h}{\sqrt{h^2 + d^2}}$$

We can write

$$d\omega' = \frac{ds\, h}{\left(h^2 + d^2\right)^{3/2}} = \frac{ds}{h^2}\cos^3\theta$$

The irradiance around point P is therefore

$$E' = \frac{I(\theta,0)}{h^2}\cos^3\theta = \frac{I_0}{h^2}\cos^5\theta \text{ (in lux)}$$

Finally, the irradiance issued from the three LEDs is

$$E_{total} = E + 2E' = \frac{I_0}{h^2}\left(1 + 2\cos^5\theta\right) = \frac{I_0}{h^2}\left(1 + 2\frac{h^5}{\left(h^2 + d^2\right)^{5/2}}\right) \text{ (in lux)}$$